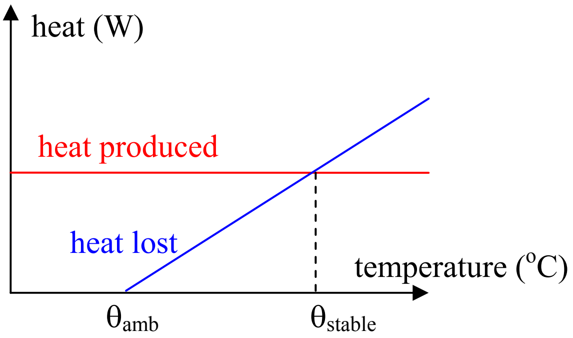
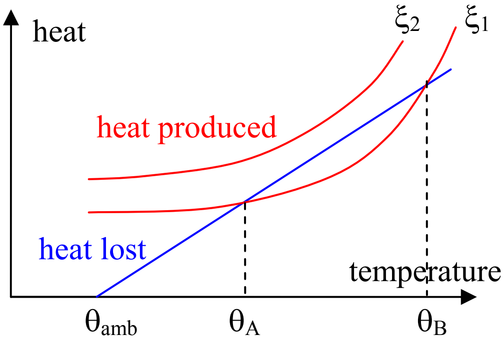
UEE 403 – High Voltage Engineering – Answers

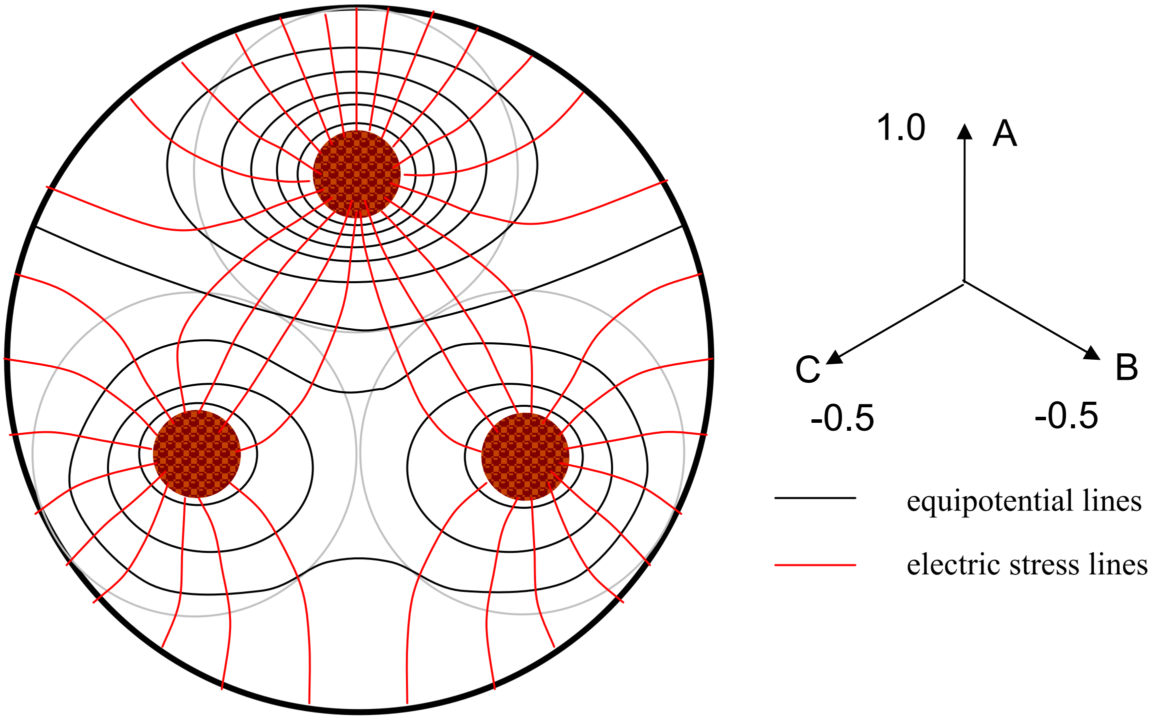
1. (a) Thermal mechanism of breakdown in an energised dielectric.

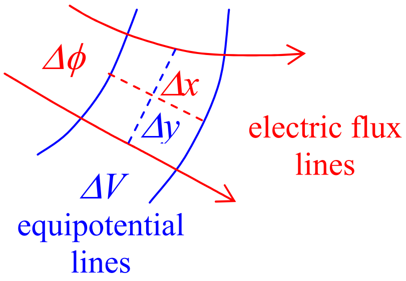




In a dielectric, when energised the power loss $V^2 C\,\omega \tan\delta$ (or $\xi^2\,\varepsilon\,\omega \tan\delta$ per unit volume). If the heat produced by the power loss is taken as constant a temperature θ_{stable} would be obtained. However as the power loss actually increases with temperature, the heat produced would be non linear as shown. If the field is ξ_1 then provided the temperature is not allowed to go above θ_B a stable temperature θ_A would exist. If the field is increased to ξ_2 then there would be no stable temperature and the temperature would keep on increasing until the material physically breaks down. [2 marks]

(b) Sketch of field pattern in a 3 core cable for a particular instant of time [3 marks]





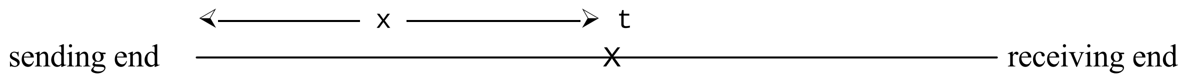
The electric flux lines and the equipotential lines are perpendicular to each other. Further for constant differences, Δϕ (or Δq) and ΔV are constants. Thus for the elemental figure shown, capacitance is constant.

$C = \dfrac{A\varepsilon}{d} = \dfrac{\Delta y . l\, \varepsilon}{\Delta x}$ = constant. i.e. Δy/Δx = constant (usually chosen as 1 for convenience of drawing). Thus curvilinear squares are formed in the sketch. [1 mark]

(c) The transient behaviour of a transmission line is governed by the partial differential equation (show brief derivation) (where $a^2 = 1/l.c$)

$$a^2 \frac{\partial^2 v}{\partial x^2} = \frac{\partial^2 v}{\partial t^2}$$

This is satisfied by the expression $v = f(x-at) + F(x+at)$



Consider the function f(x-*a*t)

at the point (x_o, t_o), function has value f (x_o - a t_o)

if it moves forward at constant velocity *a*, then after time t, it would have travelled a distance *a.t* so that its co-ordinates would be (x_o+ *a*t, t_o+t)

at the point (x_o+ *a*t, t_o+t), function has value f (x_o+ *a*t - *a* (t_o+t)) or f (x_o - *a* t_o).

Thus the function f(x-*a*t) remains constant when travelling forward at velocity *a*. Thus it represents a forward travelling wave.

Similarly, F(x+at) represents a reverse travelling wave.

Thus the surge on a transmission line can be represented by a forward travelling wave and a reverse travelling wave [3 marks]

The corresponding surge current is given by (show derivation)

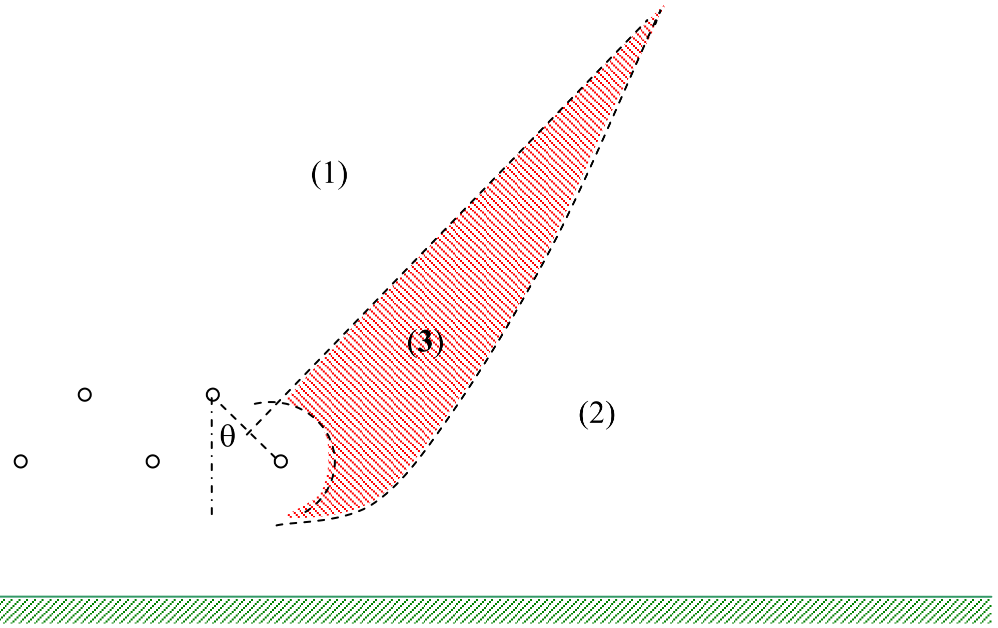
$i = 1/Z_o\,[f(x-at) - F(x+at)]$ or $Z_o\,.i = [f(x-at) - F(x+at)]$

Thus $v + Z_o\,.i = f(x-at)$ = constant for a forward wave

similarly, $v - Z_o\,.i = F(x+at)$ = constant for a reverse wave

Thus forward waves and reverse waves can be represented by lines of slope – Z_o and + Z_o respectively on a *v* versus *i* diagram (Bergeron diagram) [2 marks]

(d)



[1 marks]

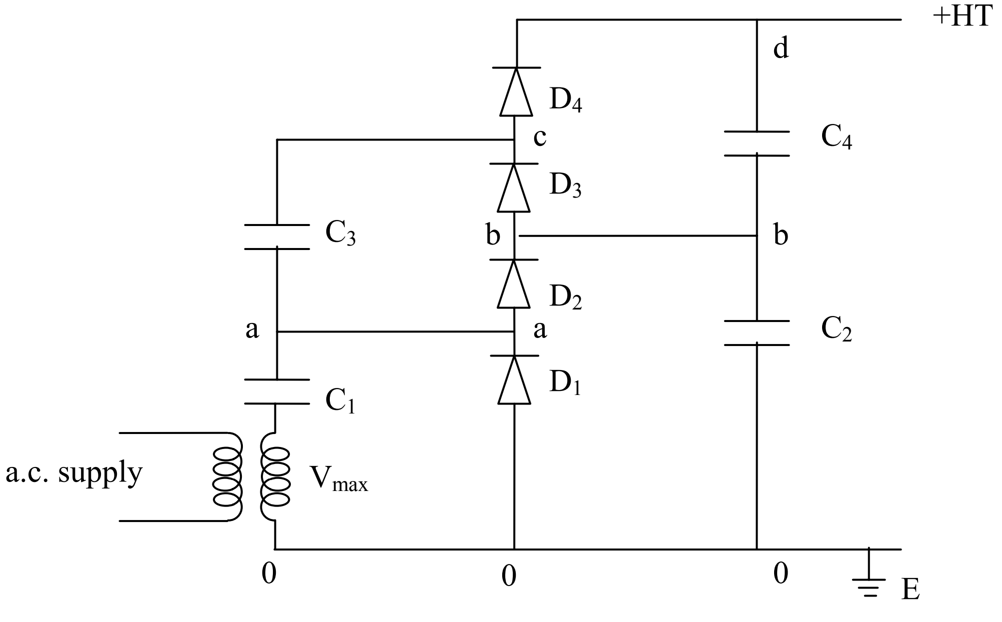
The region (1) represents the region in which lightning will most likely strike the earth wire and thus provide protection against direct strikes. The locus of the lower boundary of this region is approximately defined by the perpendicular bisector of the line joining the phase conductor (the outermost for a horizontal arrangement and the uppermost in the case of a vertical arrangement) and the earth wire.

The region (2) represents the region in which lightning avoids both the overhead conductor as well as the earth wire but strikes some nearby object. The region has the upper boundary defined approximately by a parabolic locus. This locus is taken as equidistant from both the earth plane as well as the phase conductor. {This assumption is not exactly true as the phase conductor is a better attractor of lightning due to its sharper configuration).

Depending on the strength of the charge on the leader core, lightning will be initiated at a distance away from the object struck. Thus if the leader core could approach very close to the phase conductor before it discharges, then that particular stroke will be weak. This defines a minimum region within which lightning strikes terminating on the line does not do any damage. This region thus has a circular locus around the conductor, which is not be considered in risk evaluation.

The region (3) is the balance region, demarcated by the locus of region (1), the locus of region (2) and the circular locus where the stroke is too weak to cause damage. In this region (3) the lightning stroke is most likely to terminate on the phase conductor. The area (3) is thus a measure of the efficiency of the earth-wire protection. The smaller this region is the better the shielding provided by the overhead earth wires. For the same semi-vertical shielding angle θ, the taller the tower the lesser the efficiency of protection provided by the earth wire. Further if the semi-vertical angle of shielding is reduced, the area (3) reduces giving better protection. Thus to obtain the same degree of protection, taller towers require smaller protection angles. [2 marks]

(e)



[1 marks]

V_{max} is the peak value of the secondary voltage of the high voltage transformer. To analyze the behaviour, consider that charging of capacitors actually takes place stage by stage rather than somewhat simultaneously.

Consider first, the part of circuit containing the diode **D_1**, the capacitor **C_1**, and the secondary winding. During the first negative half cycle of the applied voltage, the capacitor **C_1** charge up to voltage **V_{max}**.

During the positive half cycle which follows, the diode **D_1** is reverse biassed, the capacitor **C_1** will not discharge and the peak of this half cycle, the point **a** will be at **$2\,V_{max}$**.

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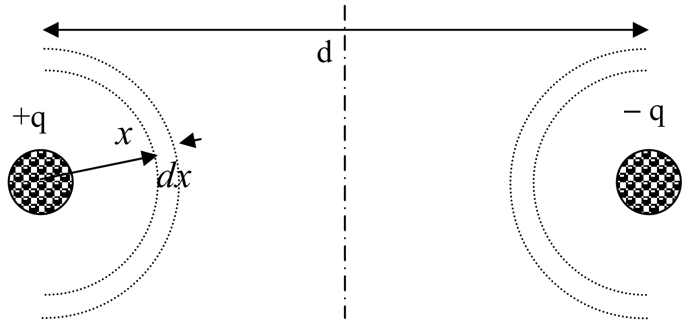
During the following cycles, the potential at **a** will vary between **0** and **2 V_{max}**, depending on whether the secondary voltage and the capacitor voltage are opposing or assisting.

Initially, capacitor C_2 would be uncharged, and the voltage at **b** would be zero. Thus as the voltage at **a** varies between 0 and 2 V_{max}, the diode D_2 is forward biassed, and the capacitor C_2 would charge to 2 V_{max}. Once the voltage at **b** has reached 2 V_{max}, the voltage at **a** would be less than or equal to the voltage at **b**. Thus once C_2 has charged up, this diode too would be reverse biassed and the capacitor C_2 would not discharge. The voltage at **b** would now remain constant at 2 V_{max}. C_3 is also initially assumed uncharged. Since the voltage at **a** varies between 0 and 2 V_{max}, the diode D_3 would initially be forward biassed for almost the whole cycle. Thus the capacitor C_3 charges until it reaches 2 V_{max} when **b** is 2 V_{max} and **a** is 0. As the voltage at **a** again increases to 2 V_{max}, the voltage at **c** increases, and thus the diode D_3 is reverse biassed and C_3 would not discharge. Now as **a** reaches 2 V_{max} the voltage at **c** rises to 4 V_{max}, as C_3 has not discharged.

Thus after charging up has taken place, the voltage at **c** varies between 2 V_{max} and 4 V_{max}. Assuming C_4 also to be initially uncharged, since the voltage at **b** is a constant at 2 V_{max} and the voltage at **c** varies between 2 V_{max} and 4 V_{max} initially, during most of the cycle, the diode D_4 is forward biassed and C_4 charges up to the maximum difference between **d** and **b** (i.e. to 2 V_{max}). This occurs when the voltage at **c** is 4 V_{max} and the voltage at **d** would now be 4 V_{max}. As the voltage at **c** falls from 4 V_{max} to 2 V_{max}, since the capacitor C_4 has charged up it would not discharge, since there is no discharge path. Thus once the capacitors are charged up the voltage at **d** remains constant at 4 V_{max}. [2 mark]

(f)



$$D = \varepsilon\,\xi\,,\quad \xi = \frac{dV}{dx} = \frac{D}{\varepsilon} = \frac{q}{A\varepsilon} = \frac{q}{\varepsilon\ 2\pi x \cdot l} \quad \text{giving} \quad dV = \frac{q}{2\pi\varepsilon\ x \cdot l}dx$$

$$\int dV = \int_r^d \frac{q}{2\pi\varepsilon\ x\cdot l}dx \quad \text{giving}\quad V = \frac{q}{2\pi\varepsilon}\ln\frac{d}{r}\ \text{across the 2 conductors due to +q}$$

There will be an equal voltage difference due to the charge –q.

The voltage to the neutral would be half the sum, giving $V = \frac{q}{2\pi\varepsilon}\ln\frac{d}{r}$.

therefore $\xi = \frac{q}{\varepsilon\ 2\pi x\cdot l} = \frac{V}{x\cdot\ln\frac{d}{r}}$ per unit length, giving $V = \xi\cdot x\cdot\ln\frac{d}{r}$

maximum stress occurs at minimum radius (i.e. x = r) and critical stress (peak value) for air is 30 kV/cm at NTP (30^oC, 760 torr). If the irregularity factor is m_o for corona inception, and δ is the air density correction factor

Thus rms value of critical $V_o = \frac{30}{\sqrt{2}}\cdot r\cdot m_o\cdot\delta\cdot\ln\frac{d}{r}$

i.e. disruptive critical voltage = $21.2\ r\cdot m_o\cdot\delta\cdot\ln\frac{d}{r}$ [3 marks]

(g) *Type tests* are done on equipment to establish that the particular design is suitable for a particular purpose . They are normally done once on new designs and when specifically requested by consumers purchasing in bulk quantities.

Ex: One minute rain test on porcelain insulators where the insulator is sprayed throughout the test with artificial rain.

Sample tests done on equipment for the purpose of verifying certain characteristics on equipment which might change during the course of manufacture. These generally involve destructive tests which cannot be done routinely.

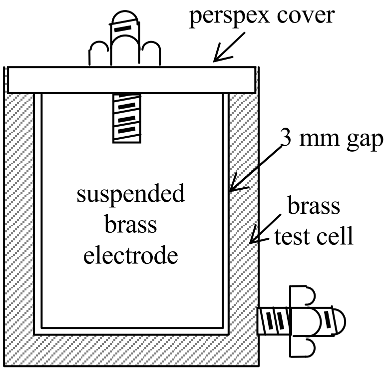
Ex: Porosity test on porcelain insulators which needs freshly broken pieces of porcelain to show no dye penetration.

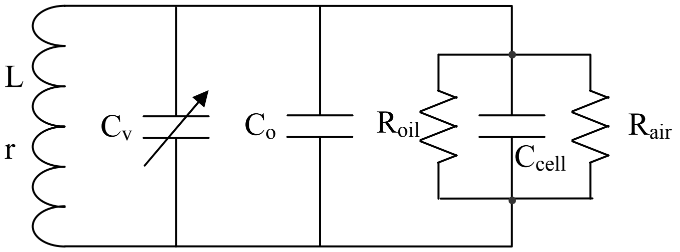
Routine tests are done on equipment for the purpose of eliminating equipment with manufacturing defects by non-destructive tests. These are generally easily verifiable.

Ex: Mechanical loading of porcelain insulators with a load 20% in excess of maximum working load of the insulator.

[2 marks]

(h)





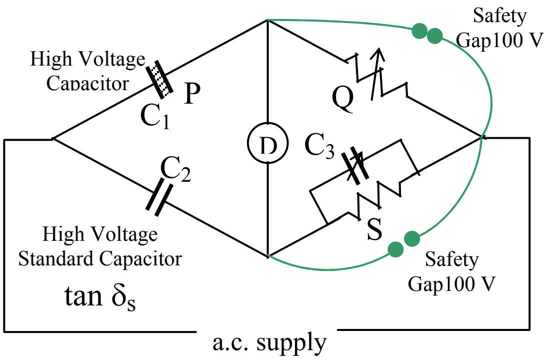
Equivalent circuit

The cell is connected to a circuit containing a variable capacitance C_v so as to form a resonant circuit with the inductance (L, r) from a constant frequency supply. The effect of the stray capacitance C_o is eliminated in the measurements. By varying the stray capacitance the resonance condition and the half power points are obtained for the following cases.

i) inner electrode removed (only stray capacitance)
ii) inner electrode in place but no oil (air as dielectric)
iii) inner electrode in place with oil as dielectric

Expressions are obtained for the relative permittivity and the loss tangent as a ratio of a combination of capacitances. [2 marks]

(i)



It can be shown that this bridge is frequency independent, and that at balance

$$\frac{C_2}{C_1} = \frac{Q}{S}, \quad \text{also} \quad \frac{P}{Q} = \frac{C_3}{C_2}$$

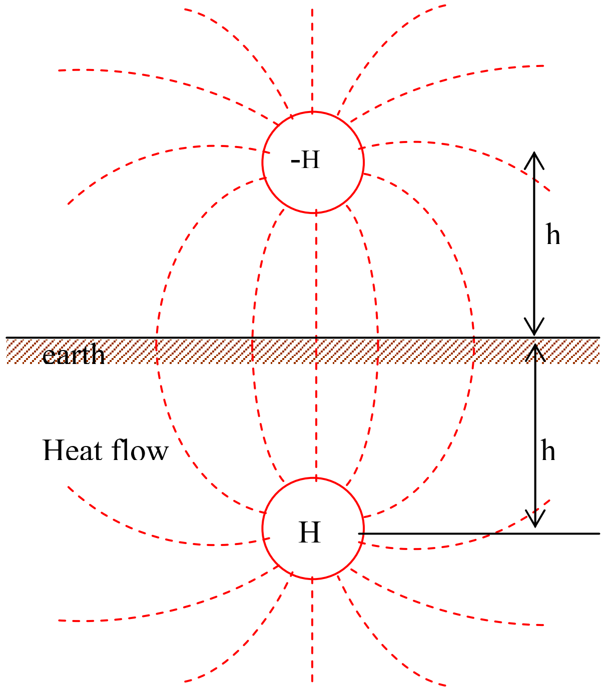
$\theta \approx \tan\theta$, $\delta_s \approx \tan\delta_s$, $\delta \approx \tan\delta$

$\delta - \delta_s = \theta$ giving $\tan\delta \approx \tan\delta_s + \tan\theta$

i.e. $\tan\delta = \tan\delta_s + \omega C_3 S$

and $C_1 = \dfrac{S}{Q} C_2$ [4 marks]

2



The effect of the earth can be replaced by an equal and opposite heat source on the opposite side of the surface at the same distance from the surface.

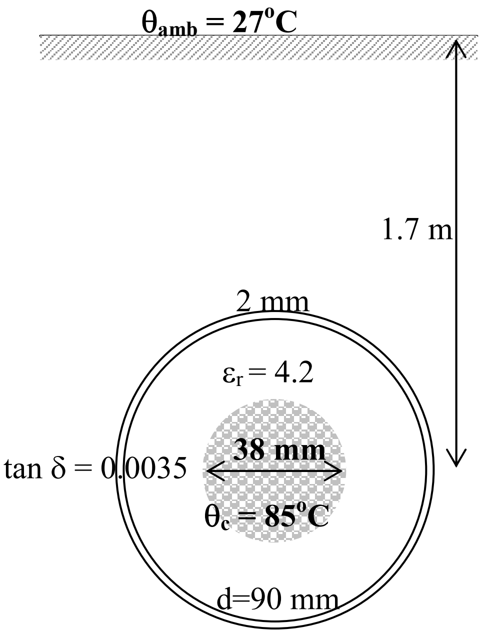
The effects of the charge +H and -H can now be separately considered, and the results superimposed. Each heat source considered separately will give rise to radial flux lines.

The temperature difference between the charges caused by one of the heat sources is given by $\theta = \dfrac{kH}{2\pi}\log_e\dfrac{2h}{r}$

Hence thermal resistance is given by $S = \dfrac{\theta}{H} = \dfrac{k}{2\pi}\log_e\dfrac{2h}{r}$ [give derivation]

Since the earth surface is not at exactly a constant temperature, a empirical factor of 2/3 is used.

Thus Thermal resistance = $S = \dfrac{k}{3\pi}\log_e\dfrac{2h}{r}$ [5 marks]



single phase voltage = 220/√3 = 127 kV

derivation of $I = \sqrt{\dfrac{\theta - W_d(S+G)}{R(S+G)}}$ [2 marks]

$$C = \frac{2\pi\varepsilon}{\ln(R/r)} = \frac{2\pi\ 4.2\times 8.854\times 10^{-12}}{\ln(45/19)} = \frac{2.3365\times 10^{-10}}{0.8622}$$

$= 270.99\times 10^{-12} = 271$ pF/m = 0.271 μF/km

[2 marks]

dielectric loss = $V^2 C\,\omega\tan\delta$

$= (127000)^2\times 0.271\times 10^{-6}\times 100\pi\times 0.0035 = 4806$ W/km

[2 marks]

thermal resist of insulat^n = $\dfrac{5.0}{2\pi}\ln\dfrac{45}{19} = 0.685$ °C m/W

[2 marks]

therm resis of ground= $\dfrac{2}{3}\times\dfrac{1.4}{2\pi}\ln\dfrac{2\times 1.7}{0.047}$ =0.636 °Cm/W

[2 marks]

total thermal resistance = 0.685+0.636 = 1.321 °Cm/W

temperature rise = 85 – 27 = 58 °C

Current rating of cable = $\sqrt{\dfrac{58 - 4.806\times 1.321}{0.016\times 10^{-3}\times 1.321}} = 2065 = \underline{\underline{2.065\text{ kA}}}$ [3 marks]

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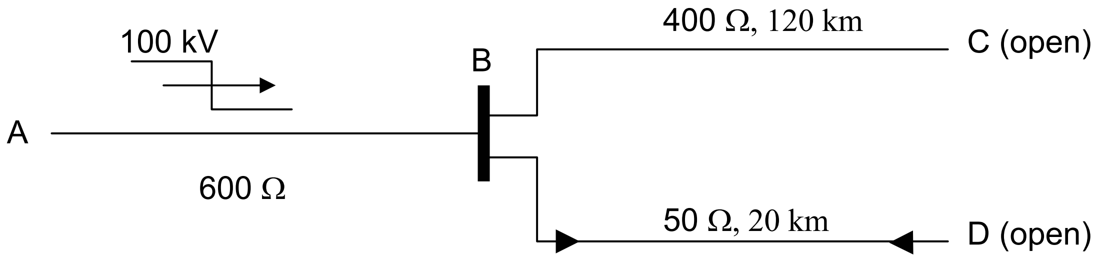
3. (a)



travel time of BC = $120/3\times10^5 = 400\ \mu s$

travel time of BD = $20/2\times10^5 = 100\ \mu s$

for surge along AB at B, effective outgoing =400//50 = 44.44 Ω

$$\text{reflection coefficient} = \frac{44.44-600}{44.44+600} = -0.862$$

transmitted surge into BC and BD at t=0 is (1-0.862)×100 = 13.8 kV

reflection coefficients at C and D (both open) are 1.0

voltage at D at t=100 μs is 2×13.8 = 27.6 kV

surge arriving at B from DB at time 200 μs is 13.8 kV

for this surge, effective outgoing = 400//600 = 240 Ω,

$$\text{and reflection coefficient} = \frac{240-50}{240+50} = 0.655$$

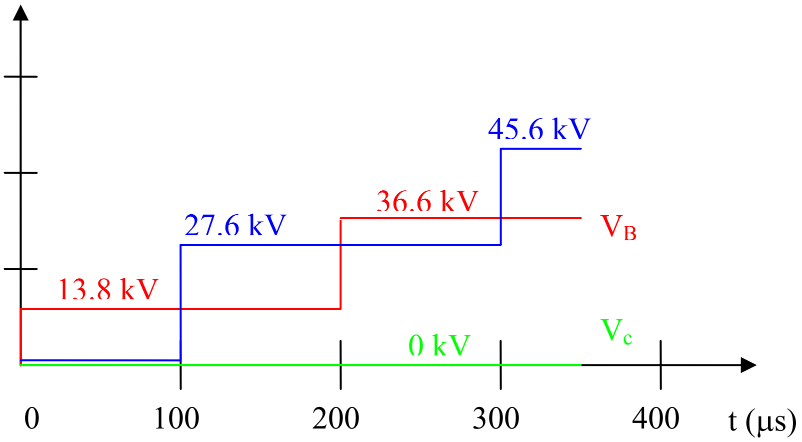
surge reflected back to BD = 13.8×0.655 = 9.0 kV

surge transmitted to BA and BC = 13.8×1.655 = 22.8 kV

reflected surge arrives back at D at time 300 μs and gives a voltage of 2×9.0 = 18.0 kV

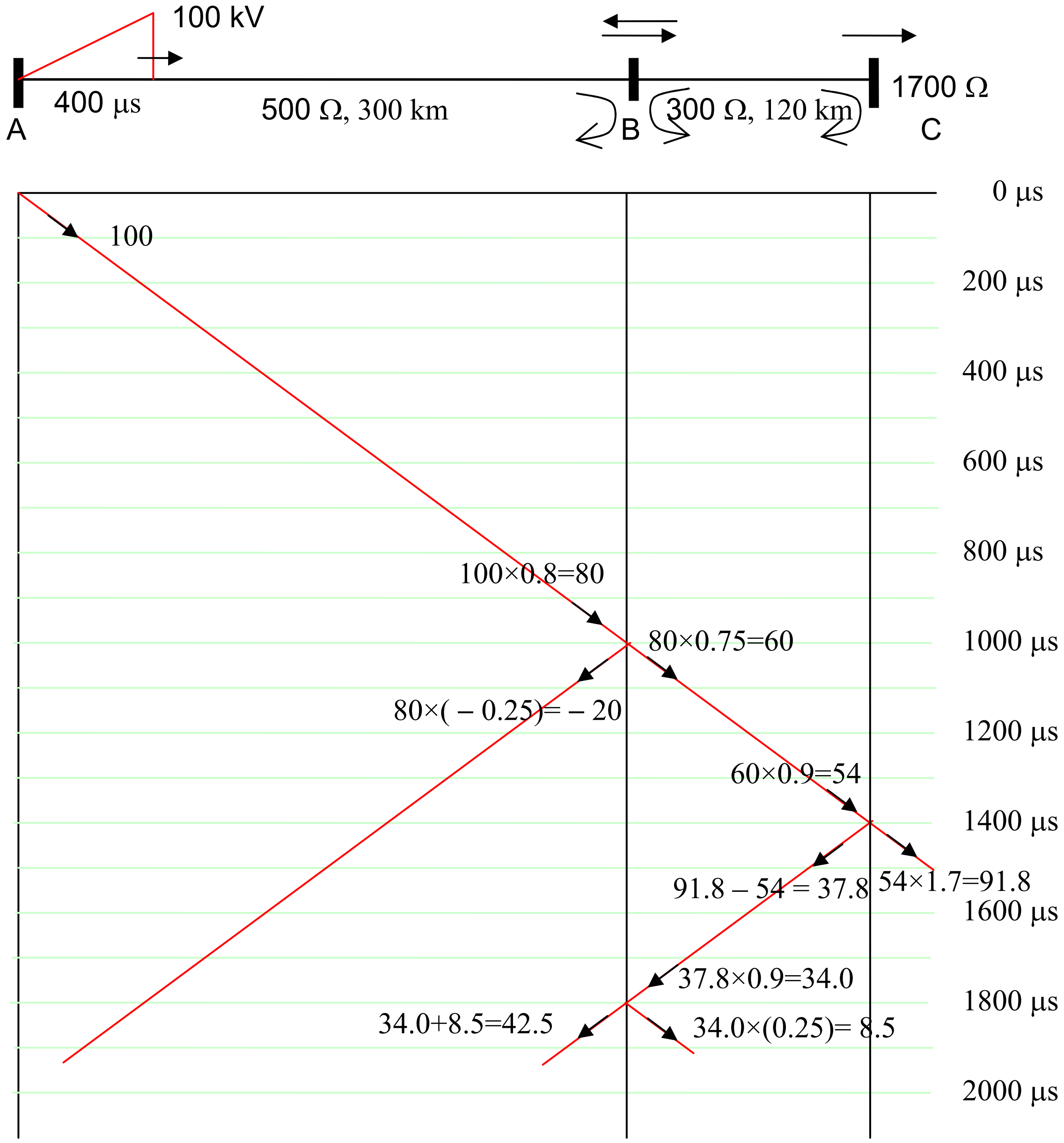
No voltages would arrive at C during the first 350 μs

[4 marks]



[5 marks]

(b)



reflection coefficient at B, for surge from AB $\dfrac{300-500}{300+500} = -0.25$

and for surge from CA $\dfrac{500-300}{300+500} = 0.25$

transmission coefficient at B = 1 - 0.25 = 0.75 and 1 + 0.25 = 1.75

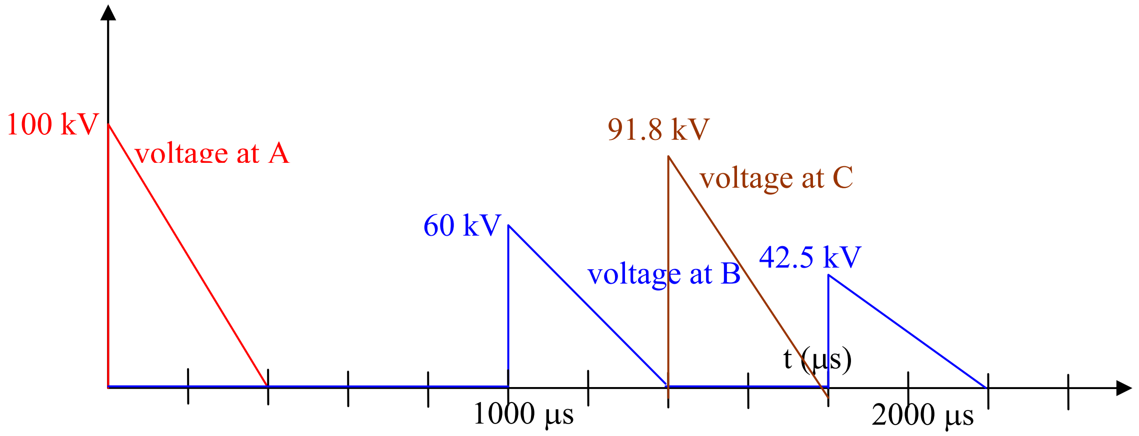
reflection coefficient at C, $\dfrac{1700-300}{1700+300} = 0.7$

transmission coefficient at C = 1+ 0.7 = 1.7

travel time of AB = 300/300 = 1 ms or 1000 µs

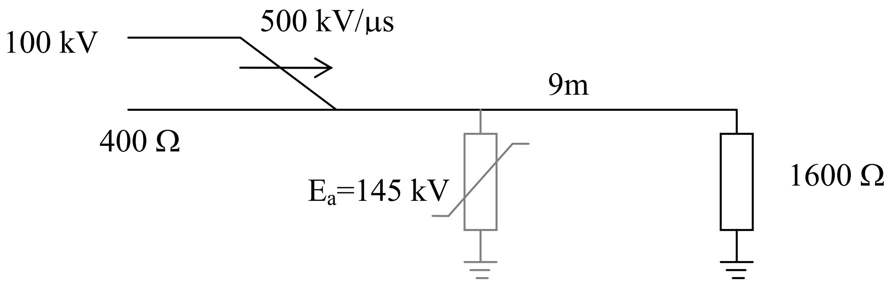
travel time of BC = 120/300 = 0.4 ms or 400 µs [6 marks]

Thus the waveforms of the voltages are



[3 marks]

4



Wavefront time = 100/500 = 0.2 μs

Assuming the velocity of propagation in the overhead line as 3×10^8 m/s,

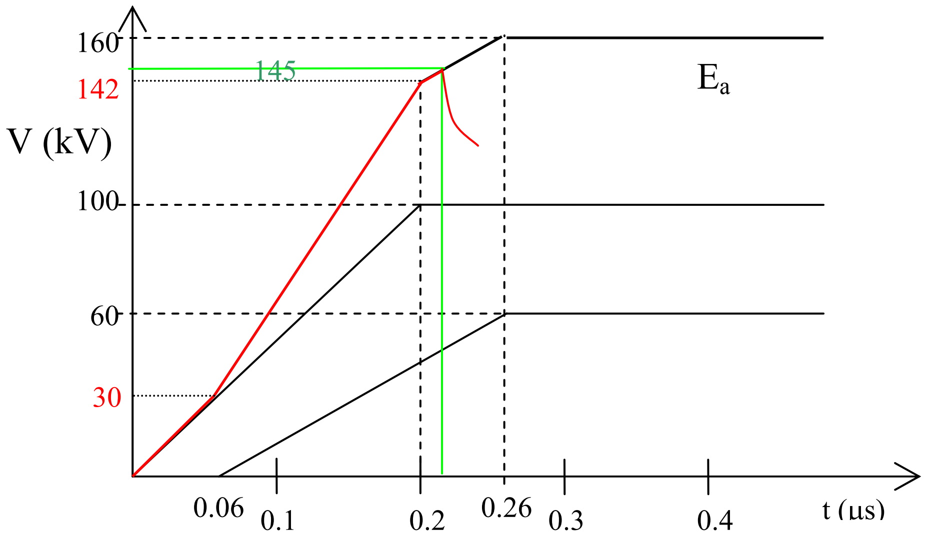
travel time of section between arrester and terminal device = $9/3\times10^8$ = 0.03 μs

reflection coefficient at device = (1600 – 400)/(1600 + 400) = 0.6

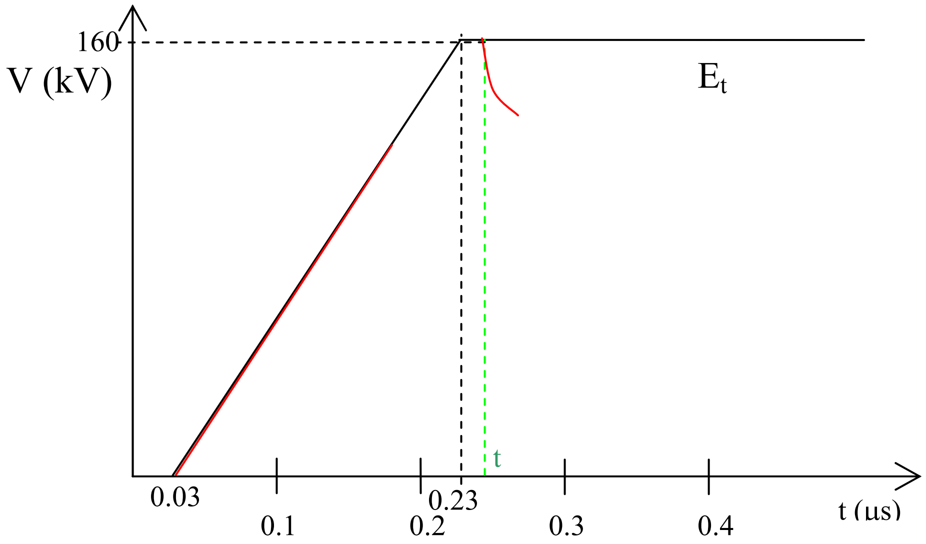
defining t=0 as instant at which surge first arrives at arrester location,

peak value of reflected surge = 0.6×100 = 60 kV

reflected surge arrives back at arrester location at t = 0.06 μs



[10 marks]



[4 marks]

The arrester operates at 145 kV (between 142 kV and 160 kV)

at time t (between 0.2 and 0.26 μs)

where the slope of the characteristic is 300 kV/μs

thus (t – 0.2)/(0.26 – 0.2) = (145 – 142)/(160 – 142)

therefore t = 0.2 + 3*0.06/18 = 0.21 μs

time at which arrester operates = 0.21 μs [2 marks]

This arrives at device at time = 0.21 + 0.03 = 0.24 μs > 0.23 μs

Thus the voltage arriving at the device would have reached its peak and

voltage to which device will rise = 160 kV [2 marks]

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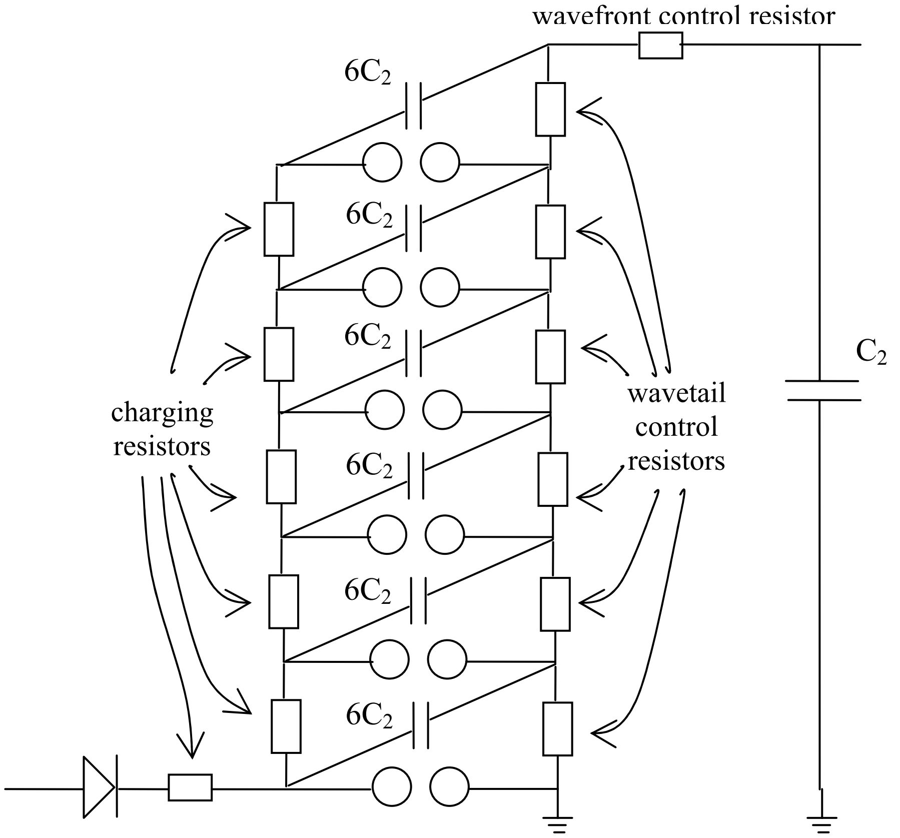
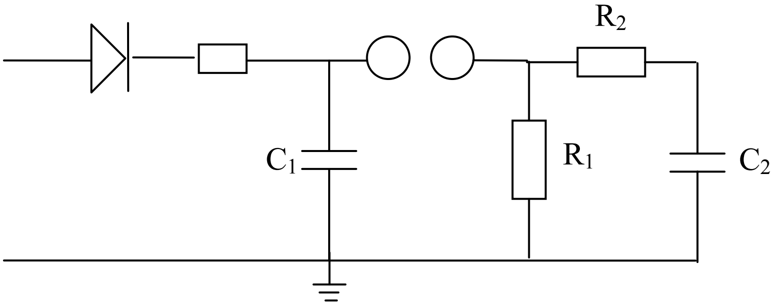
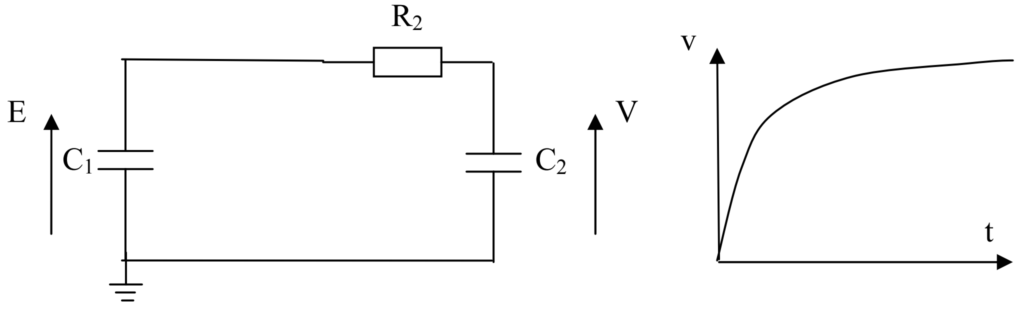


[3 marks]

The impulse generator can be reduced to the form



During wavefront, since $R_1 >> R_2$, the approximate charging circuit is



giving a charging time constant $1/\beta = R_2 .(C_1//C_2) = R_2\, C_1C_2/(C_1+C_2) = \eta\, R_2\, C_2$

voltage efficiency = $\eta = \dfrac{C_1}{C_1 + C_2} = 0.9$

giving a charging time constant $1/\beta = R_2 .(C_1//C_2) = R_2 C_1C_2/(C_1+C_2) = \eta R_2 C_2$

and an expression $v = V_{\max}\left(1 - e^{-\beta t}\right)$

$V_m = \eta E = 600$ kV

input voltage required = 600/0.9 = 666.7 kV

energy = $\frac{1}{2} C_1 V^2 = \frac{1}{2} C_1 (666.7\times10^3)^2 = 2.5\times10^3$

therefore $C_1 = 0.012\ \mu F = 11.25$ nF

and $C_2 = 11.25/9 = 1.25$ nF [4 marks]

defining wavefront based on 30% to 90% and extrapolation

$0.3 V_m = V_m (1- e^{-\beta t1})$ giving $0.7 = e^{-\beta t1}$

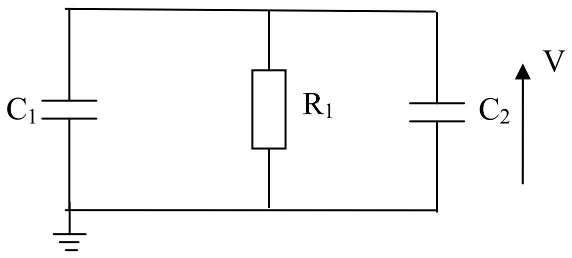
$0.9 V_m = V_m (1- e^{-\beta t2})$ giving $0.1 = e^{-\beta t2}$

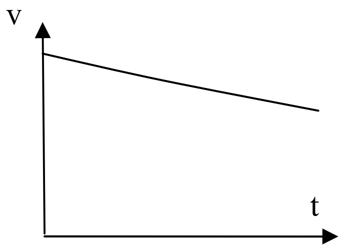
therefore, $7 = e^{\beta(t2-t1)}$ giving $t_2 – t_1 = (\ln 7)/\beta = \eta R_2 C_2 1.946$

wavefront time = $(t_2 – t_1)/(0.9 – 0.3) = 3.243 \eta R_2 C_2 = 1.2\ \mu s$

therefore $R_2 = 1.2/(3.243\times0.9\times1.25e^{-3}) = 329\ \Omega$ [4 marks]

Similarly, during wavetail, since $R_2 << R_1$, the approximate charging circuit is





giving a discharging time constant $1/\alpha = R_1 .(C_1+C_2) = R_1 C_1/\eta$

and an expression $v = V_{\max} e^{-\alpha t}$

at wavetail $0.5 V_m = V_m e^{-\alpha t}$ giving $\alpha t_t = \ln (2)$

therefore $t_t = 0.693/\alpha = 0.693 R_1 C_1/\eta = 50\ \mu s$

i.e. $R_1 = 50\times0.9/(0.693\times11.25e^{-3}) = 5772\ \Omega$ [4 marks]

Thus the components of the circuit are

1 wavefront control resistor = 329 Ω

6 wavetail control resistors each of value = 5772/6 = 962 Ω

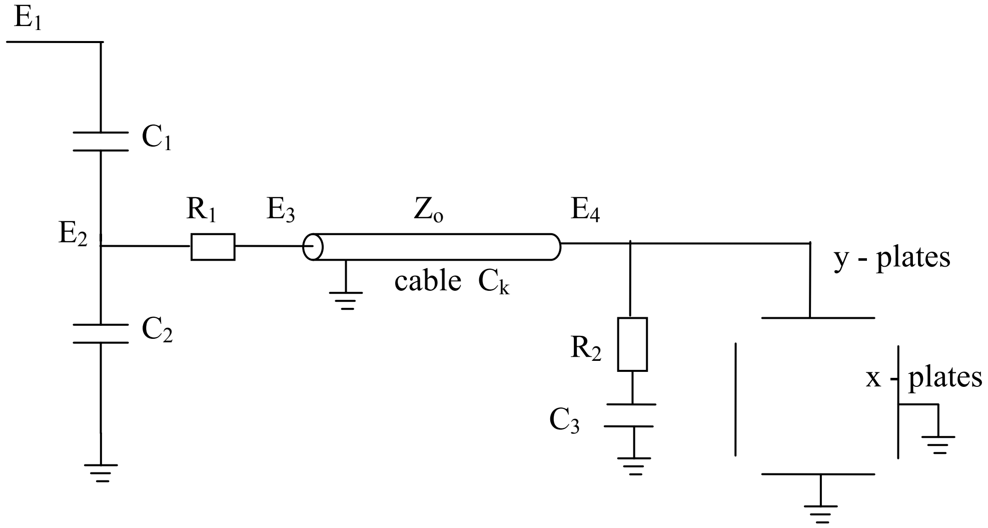
6 capacitors each of value ($6C_1$) = 6×11.25 = 67.5 nF

1 capacitor of value (C_2) = 1.25 nF

Select the charging resistors as about 1000 larger than the wavetail control resistors

charging resistors each of value = 1 MΩ. [3 marks]

6 (a)



cable may be matched at both the sending end and the receiving end for both initial conditions of the transient and the steady state conditions.

For initial condition matching (effective frequency is high)

at divider end, $R_1 = Z_o$ and at oscilloscope end, $R_2 = Z_o$

with cable matched at both ends,

$$E_2 = \frac{C_1}{C_1 + C_2} E_1, \quad E_4 = E_3 = \tfrac{1}{2} E_2 = \tfrac{1}{2} \frac{C_1}{C_1 + C_2} E_1$$

For steady state conditions, the capacitances are all fully charged (effective low frequency)

Circuit behaves as capacitive divider with C_1 on upper arm and ($C_1 + C_2 + C_k + C_3$) on lower arm giving

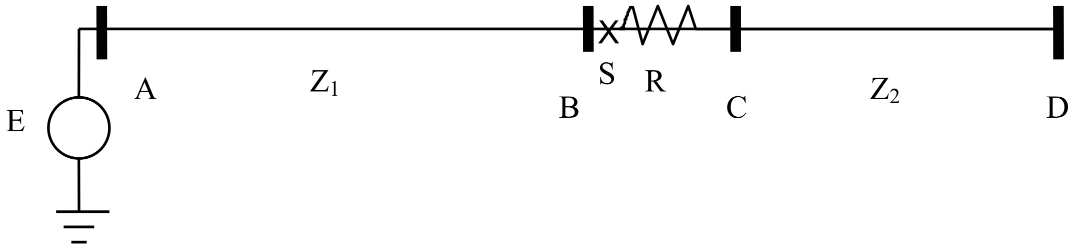
$$E_4 = \frac{C_1}{C_1 + C_2 + C_k + C_3} E_1$$

For undistorted output, the initial and final values must be the same.

Thus $2(C_1 + C_2) = (C_1 + C_2 + C_k + C_3)$

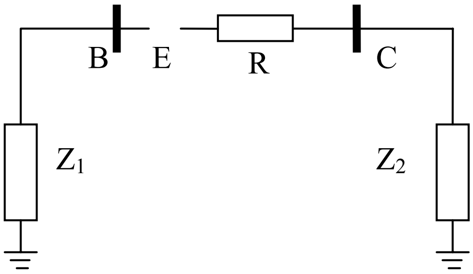
or $C_1 + C_2 = C_k + C_3$ [5 marks]

(b)



Since the relation between the surge voltage and surge current can be represented by $V = Z_o I$, the closure of the switch can be represented inthe following manner.

Voltage across the switch prior to closure = E, so that closure corresponds to a step change E. Thus changes can be analysed using the circuit



Using potential divider action, the step change E causes the following

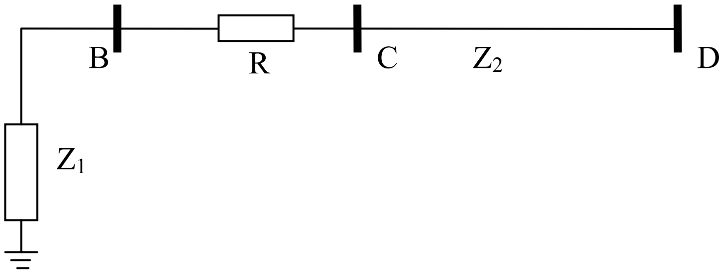
a surge current of $E/(Z_1 + R + Z_2)$

a surge voltage of $E\dfrac{Z_2}{Z_1+R+Z_2}$ into line CD

a surge voltage of $-E\dfrac{Z_1}{Z_1+R+Z_2}$ in line BA [7 marks]

Surge on CD is reflected at D and has an unchanged value (reflection coefficient of 1 at open circuit)

For surge returning from D, the equivalent circuit can be viewed as

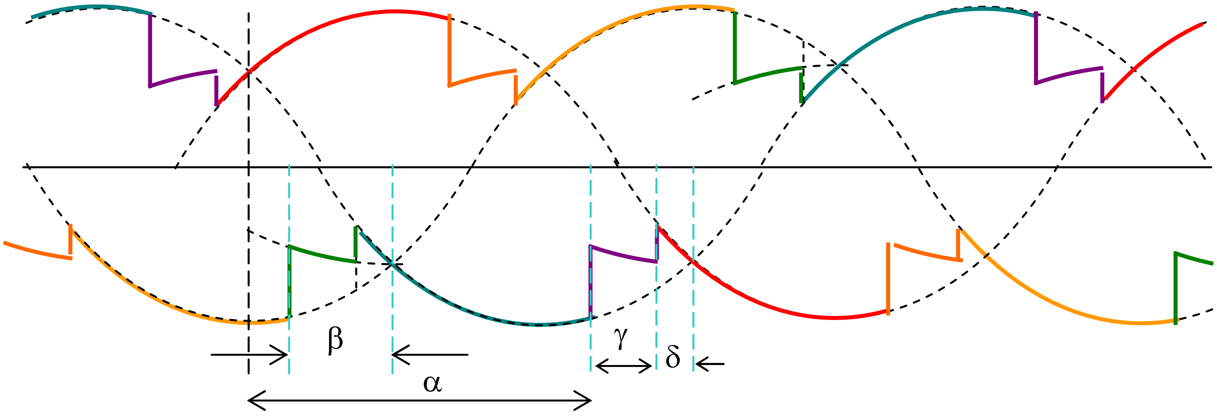


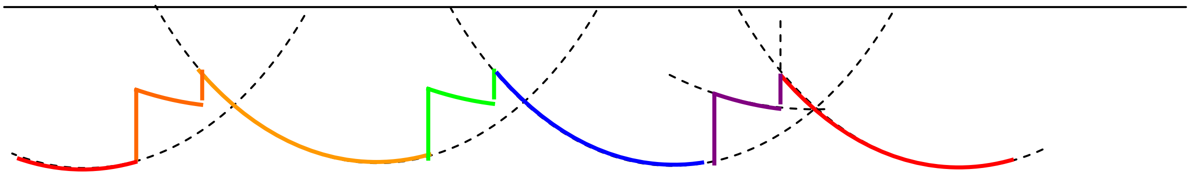
From equivalent circuit, transmitted coefficient into BA is given by

$$\alpha = \frac{2\,(Z_1+R)}{Z_2+(Z_1+R)} \times \frac{Z_1}{(Z_1+R)} = \frac{2\,Z_1}{Z_2+(Z_1+R)}$$

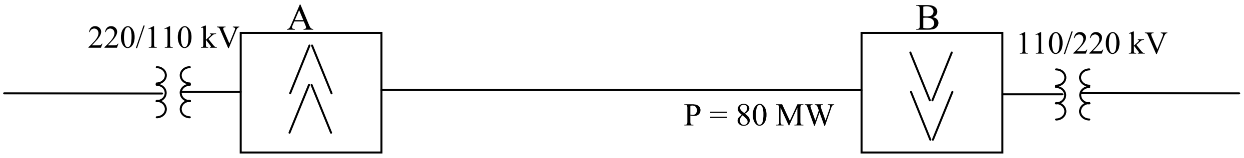
therefore the surge transmitted to BA is $E\dfrac{Z_2}{Z_1+R+Z_2} \times \dfrac{2Z1}{Z_1+R+Z_2}$ [6 marks]

7





[4 marks]



$\delta = 8 + 5 = 13^o$

$X_c = 14\ \Omega$

Derivation of relevant equations [4 marks]

$$V_{do} = \frac{3\sqrt{2}E}{\pi} = \frac{3\sqrt{2}\times 110}{\pi} = 148.55 \text{ kV}$$

$$V_d = V_{do}\cos\delta - \frac{3X_c}{\pi}I_d = 148.55\times 10^3 \times \cos 13^o - \frac{3\times 14}{\pi}I_d = 144.74\times 10^3 - 13.37 I_d$$

$V_d\, I_d = \text{P} = 80\times 10^6$

substituting for I_d gives

$$V_d = 144.74\times 10^3 - 13.37\times\frac{80\times 10^6}{V_d} \text{ giving } V_d^{\,2} - 144.74\times 10^3 \times V_d + 13.37\times 80\times 10^6 = 0$$

which has solutions V_d = 136.93 kV ~~and 7.81 kV~~ (not acceptable)

i.e. direct voltage = 136.9 kV

direct current = 80/136.93 = 0.584 kA or 584 A

also, $V_d = V_{do}\cos\beta + \frac{3X_c}{\pi}I_d = 148.55\times 10^3 \times \cos\beta + 13.37\times 584 = 136.93\times 10^3$

therefore cos β = 0.8692 giving $\beta = 29.63^o = \gamma + \delta = \gamma + 13^o$

commutation angle $\gamma = 16.6^o$

power factor = 0.5 (cos β + cos δ) = 0.5×(0.8692 + 0.9744) = 0.922, equivalent $\varphi = 22.81^o$

reactive power requirement = P tan φ = 80×tan 22.81^o = 33.6 M var

[10 marks]