EE 402 – Insulation Co-ordination – Answers

1. (a) Derivation of $\frac{\partial^2 e}{\partial x^2} = l.c\frac{\partial^2 e}{\partial t^2}$, $a^2\frac{\partial^2 e}{\partial x^2} = \frac{\partial^2 e}{\partial t^2}$, $a = \frac{1}{\sqrt{l.c}}$ **[1 mark]**

The solution of the second order partial differential equation is of the form

$e = f(x - a\,t) + F(x + a\,t)$ = forward wave + reverse wave

with velocity of propagation equal to $a = \frac{1}{\sqrt{l.c}}$ **[1 mark]**

considering only the forward wave,

$$l\frac{\partial i}{\partial t} = -\frac{\partial e}{\partial x} = -f'(x - a\,t)$$

$$\therefore i = \frac{1}{a\,l}f\,(x - at) = \frac{1}{a\,l}.e = \sqrt{\frac{c}{l}}.e$$

$$\text{i.e. } e = \sqrt{\frac{l}{c}}.i = Z_0.i$$

The relationship between the surge voltage v and the surge current i is the surge impedance $Z_0 = \sqrt{\frac{l}{c}}$ **[1 mark]**

Also $e = \sqrt{\frac{l}{c}}.i$ gives $c\,e^2 = l\,i^2$

Energy in surge is stored in both the electrostatic field and the electromagnetic field and can be given as

Energy stored = $\tfrac{1}{2}li^2 + \tfrac{1}{2}ce^2 = \tfrac{1}{2}ce^2 + \tfrac{1}{2}ce^2 = ce^2$ **[1 mark]**

(b) $v = f(x\text{-}at) + F(x\text{+}at)$

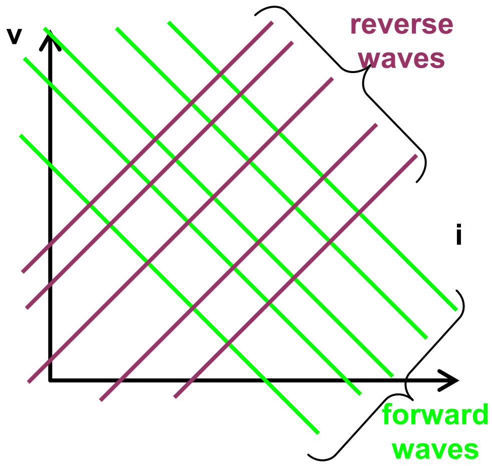
$Z_o\,i = f(x\text{-}at) - F(x\text{+}at)$

$v + Z_o\,i = 2\,f(x - at)$ = constant for forward wave

$v - Z_o\,i = 2F(x\text{+}at)$ = constant for reverse wave

These 2 equations represent the forward wave and the reverse wave on the v vs i Bergeron diagram.

[2 marks]



(c)

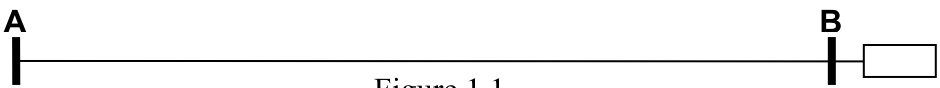


Figure 1.1

line surge impedance	= 400 Ω
constant voltage source	= 100 kV
surge impedance of source	= 100 Ω at end A
non-linear load	v = 7000 i for v < 140 kV
	v = 140 kV for v > 140 kV

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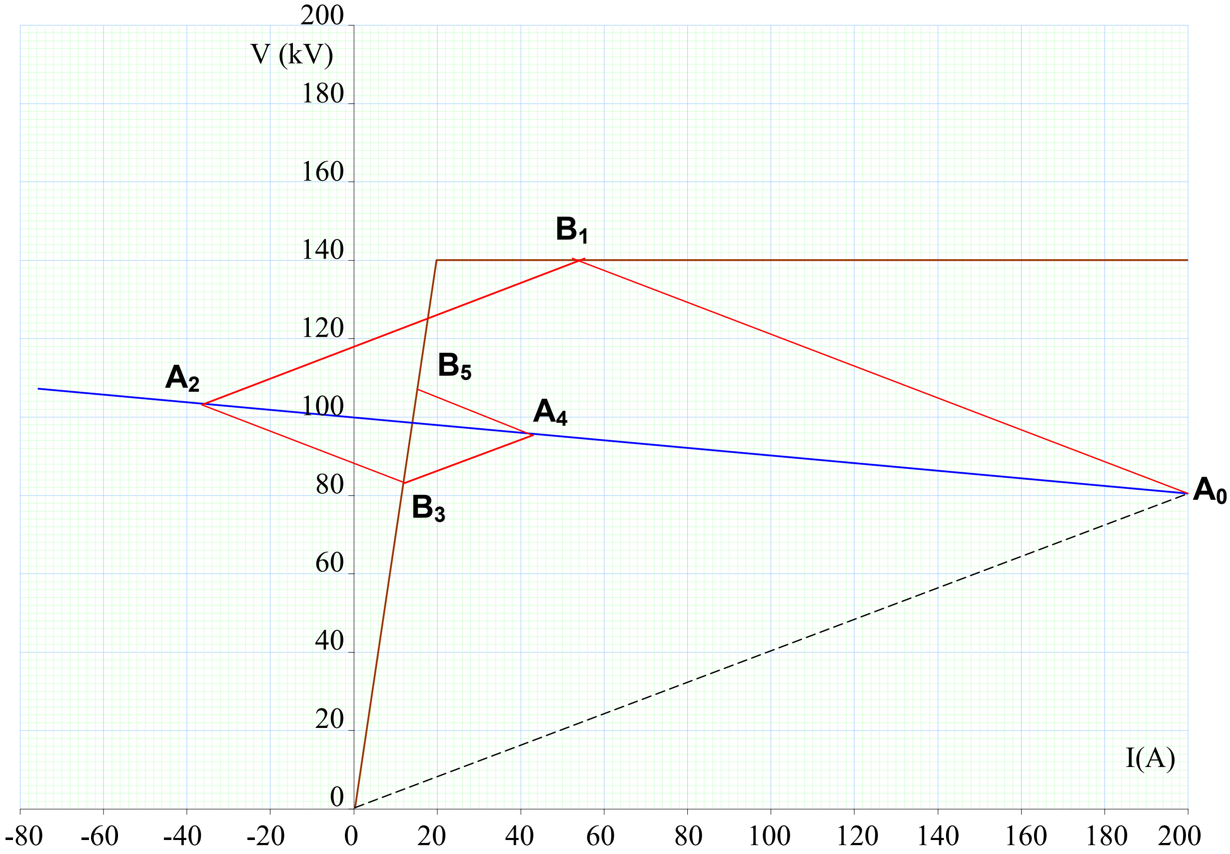
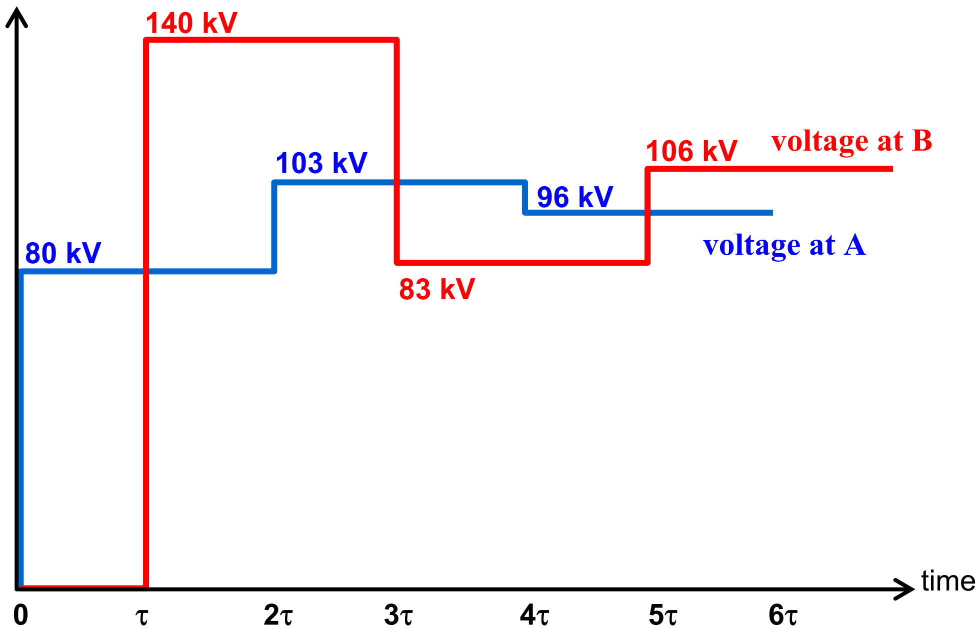


[4 marks]



[3 marks]

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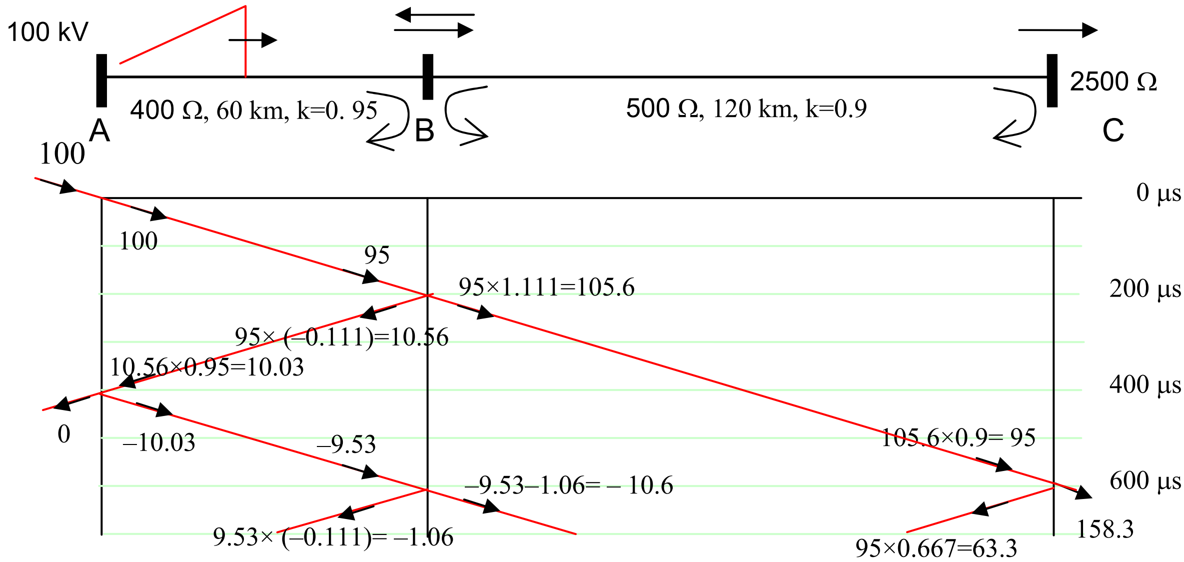


reflection coefficient at B, for surge from AB $= \dfrac{500-400}{500+400} = 0.111$

and for surge from CA $= -\ 0.111$

corresponding transmission coefficients at B $= 1 + 0.111 = 1.111$ and $1 - 0.111 = 0.889$

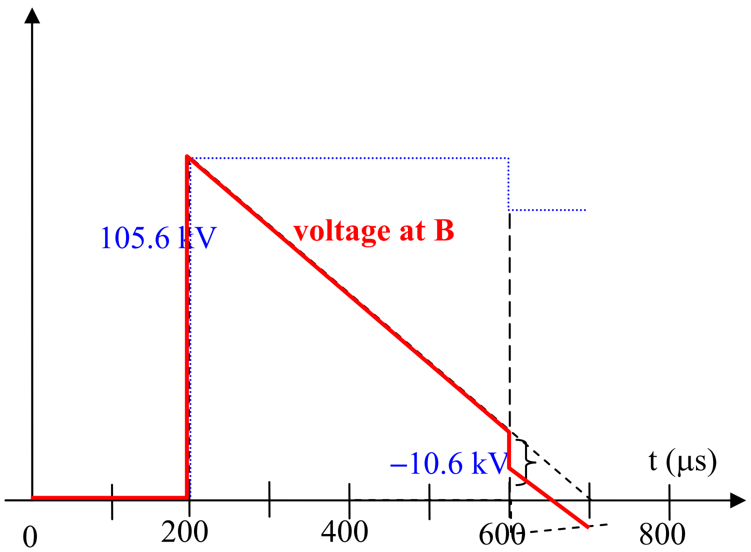
reflection coefficient at C, $= \dfrac{2500-500}{2500+500} = 0.667$

transmission coefficient at C $= 1 + 0.667 = 1.667$

travel time of AB = 60/300 = 200 μs

travel time of BC = 120/300 = 400 μs **[5 marks]**

(e) Thus the waveforms of the voltages are



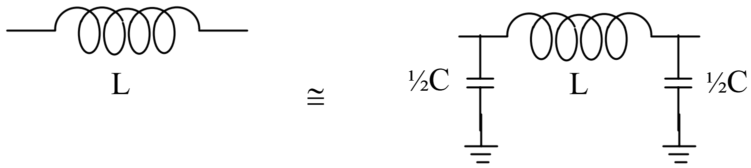
[2 marks]

EE 402 – Insulation Co-ordination – Answers

2. (a) Inductances and capacitances could be represented, by considering them as very short lines or stub lines. This is done by assuming that an inductance has a distributed capacitance of negligible value to earth, and that shunt capacitances have a negligible series inductance. These assumptions will make the lumped elements stub lines with negligible transmission times.

It is usual to select the transmission times corresponding to the minimum time increment Δt or ½ Δt

For the lumped inductance connected in series

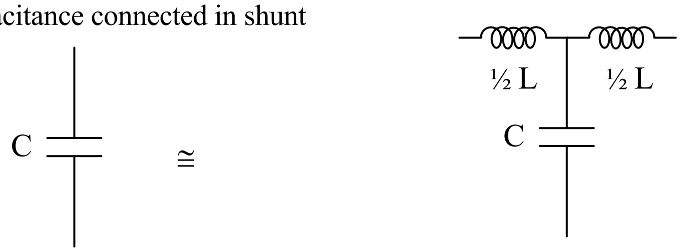


If the travel time of the line is selected corresponding to $\tau = \Delta t$

$$\tau = \sqrt{LC} \quad \text{so that} \quad C = \frac{\tau^2}{L} = \frac{(\Delta t)^2}{L}, \qquad Z_0 = \sqrt{\frac{L}{C}} = \frac{L}{\tau} = \frac{L}{\Delta t}$$

A lumped inductance may be represented by a stub line of transit time **Δt** and surge impedance **L/Δt**.

For the lumped capacitance connected in shunt

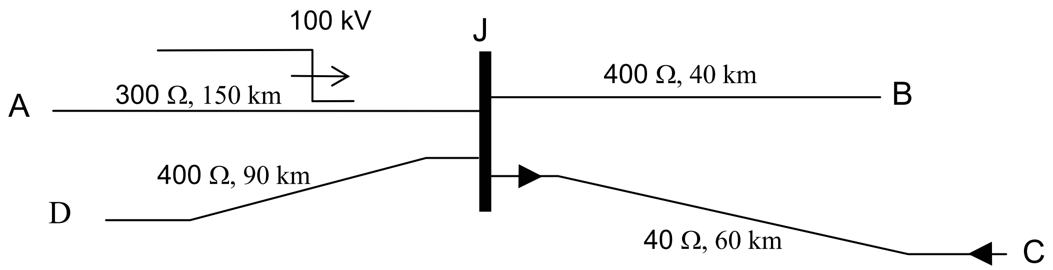


$$\tau = \sqrt{LC} \quad \text{so that} \quad L = \frac{\tau^2}{C} = \frac{(\Delta t)^2}{C}, \qquad Z_0 = \sqrt{\frac{L}{C}} = \frac{\tau}{C} = \frac{\Delta t}{C}$$

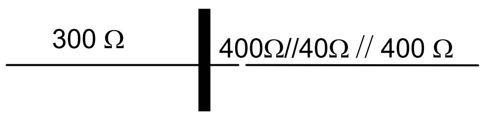
A lumped capacitance may be represented by a stub line of transit time **Δt** and surge impedance **Δt/C**.

[3 marks]

(b)



Derivation of the equivalence of the above circuit as



effective outgoing = 400//40//400 = 33.33 Ω

reflection coefficient = $\dfrac{33.33 - 300}{33.33 + 300}$ = – 0.800

transmission coefficient = 1– 0.8 = 0.2

voltage and current reflected to line JA → 100 × (-0.8) = – 80 kV, – (– 80/300) = 266.7 A

voltage and current transmitted to line JB → 100 × 0.2 = 20 kV, 20/400 = 50 A

voltage and current transmitted to line JC → 100 × 0.2 = 20 kV, 20/40 = 500 A

voltage and current transmitted to line JD → 100 × 0.2 = 20 kV, 20/400 = 50 A

[4 marks]

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(c) travel time of AJ = $150/3\times10^{-5} = 666^2/_3$ μs

travel time of BJ = $40/3\times10^{-5} = 133^1/_3$ μs

travel time of CJ = $60/3\times10^{-5} = 200$ μs

travel time of DJ = $90/3\times10^{-5} = 300$ μs

Only the reflections from BJ will arrive back at B within given time period

reflection coefficients at B (open) is 1.0

voltage at B at t=$133^1/_3$ μs is 2×20 = 40 kV

surge arriving at J from BJ at time $266^2/_3$ μs is 40 kV

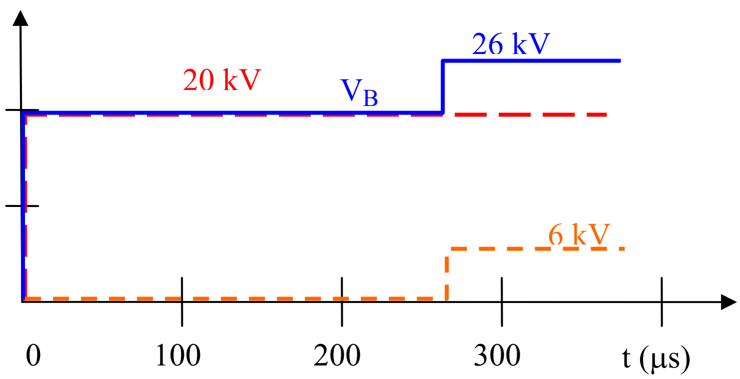
for this surge, effective outgoing = 300//40//400 = 32.43 Ω,

and reflection coefficient = $\dfrac{32.43-400}{32.43+400} = -0.85$

surge reflected back to JB = – 40×0.85 = – 34 kV

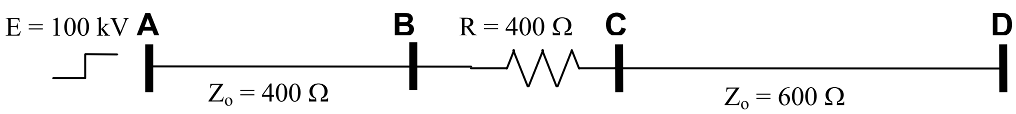
surge transmitted to JA and JC = 40×0.15 = 6 kV

No more surges would arrive at J during the first 300 μs **[3 marks]**

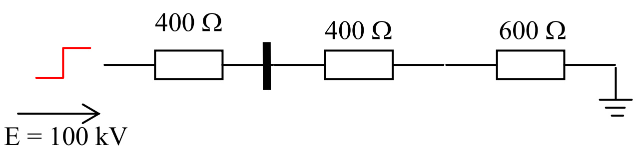


[2 marks]

(d)



Since the relation between the surge voltage and surge current can be represented by $V = Z_o I$, the line CD can be represented by the equivalent circuit, when looking from B



Thus effective reflection coefficient at B has $Z_1 = 400$ and $Z_2 = (400+600)$

i.e. reflection coefficient at B, $\beta = \dfrac{1000-400}{1000+400} = 0.429$; transmission coefficient at B is 1+0.429 = 1.429

Thus the first surge reflected back to BA = 0.429 × 100 = 42.9 kV

Only the fraction indicated by the potential divider action would actually be transmitted to CD.

Thus the first surge transmitted to CD = 1.429 × (600/1000) × 100 = 25.7 kV

[3 marks]

3 (a)

Figure shows a thundercloud cloud located above a overhead transmission line.

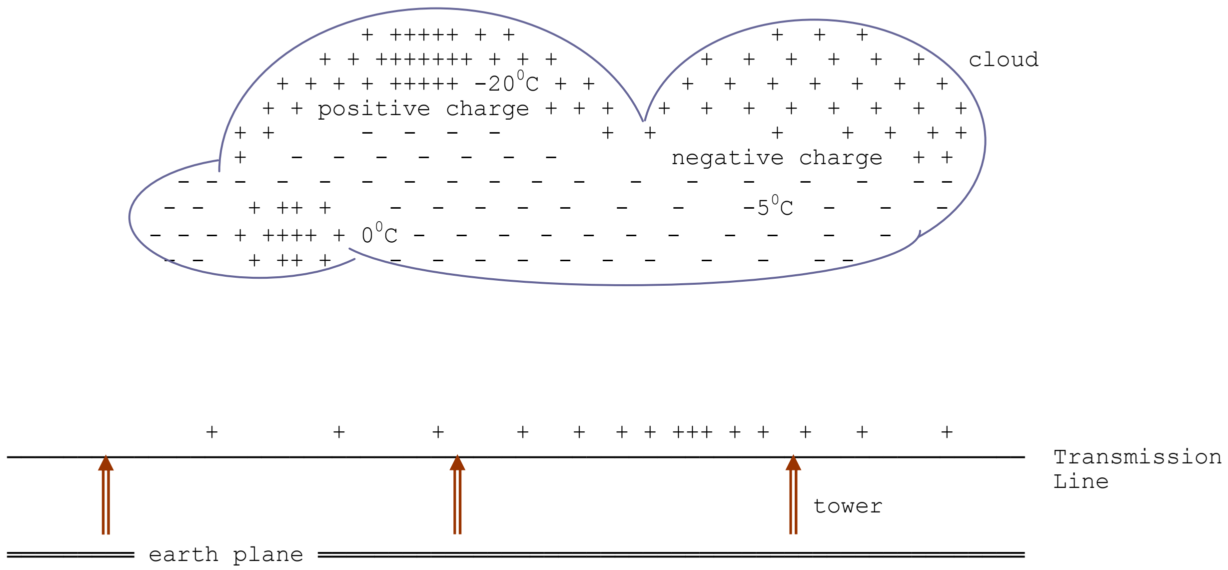


Figure - Induced charges on transmission line

Under the influence of sufficiently strong fields, large water drops become elongated in the direction of the field and become unstable, and streamers develop at their ends with the onset of corona discharges. When the electric field in the vicinity of one of the negative charge centres builds up to the critical value (about 10 kV/cm), an ionised channel (or streamer) is formed, which propagates from the cloud to earth with a velocity that might be as high as one-tenth the speed of light. Usually this streamer is extinguished when only a short distance from the cloud.

Forty micro-seconds or so after the first streamer, a second streamer occurs, closely following the path of the first, and propagating the ionised channel a little further before it is also spent. This process continues a number of times, each step increasing the channel length by 10 to 200 m. Because of the step like sequence in which this streamer travels to earth, this process is termed the **stepped leader**. This process is shown diagrammatically in the figure.

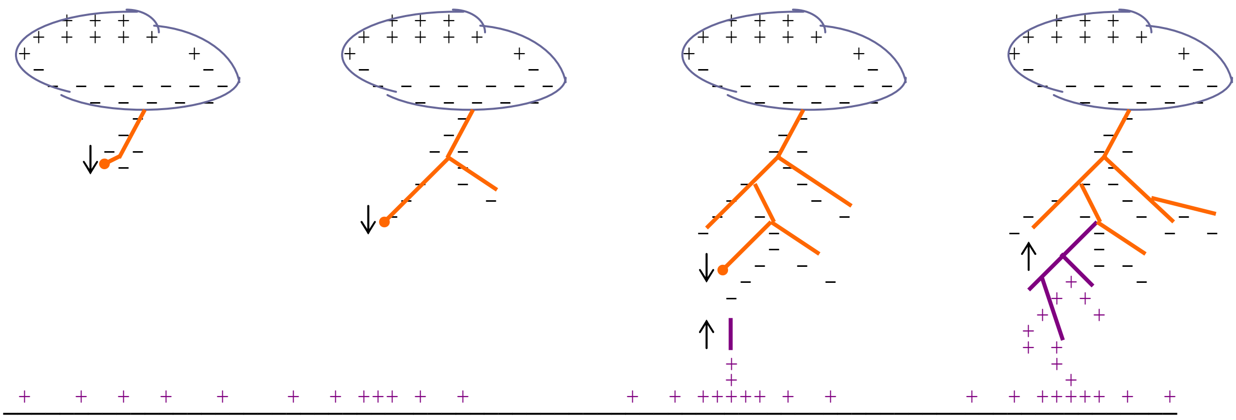


Figure - Propagation of lightning channel

When the stepped leader has approached to within 15 to 50 m of the earth, the field intensity at earth is sufficient for an upward streamer to develop and bridge the remaining gap. A large **neutralising** current, termed the **return** stroke, flows along the ionised path, produced by the stepped leader with an average current of about 20 kA.

The luminescence of the stepped leader decreases towards the cloud and in one instances it appears to vanish some distance below the cloud. This would suggest that the current is confined to the stepped leader itself. Following the first, or main stroke and after about 40 ms, a second leader stroke propagates to earth in a continuous and rapid manner and again a return stroke follows. This second and subsequent leader strokes which travel along the already energised channel are termed **dart leaders**.

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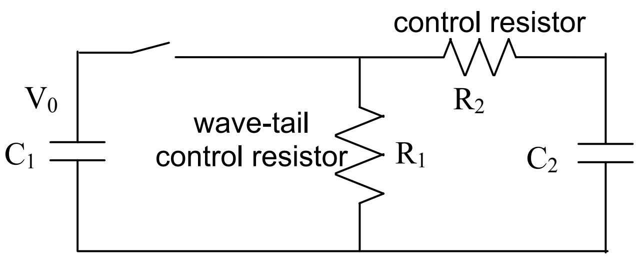
What appears as a single flash of lightning usually consist of a number of successive strokes, following the same track in space, at intervals of a few hundredths of a second. The average number of strokes in a multiple stroke is four, but as many as 40 have been reported. The time interval between strokes ranges from 20 to 700 ms, but is most frequently 40-50 ms. The average duration of a complete flash being about 250 ms.
The approximate time durations of the various components of a lightning stroke are summarised as follows.

Stepped leader	=	10 ms
Return stroke	=	40 μs
period between strokes	=	40 ms
duration of dart leader	=	1 ms

For the purpose of surge calculations, it is only the heavy current flow during the return stroke that is of importance.

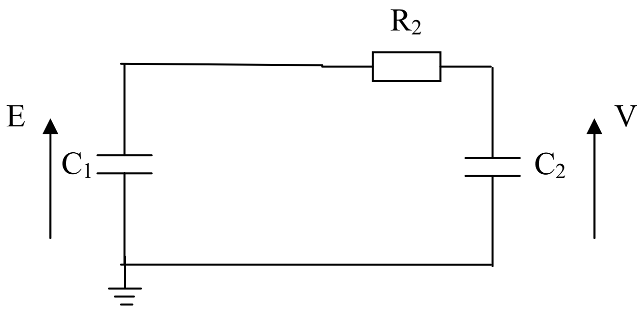
[2 marks]

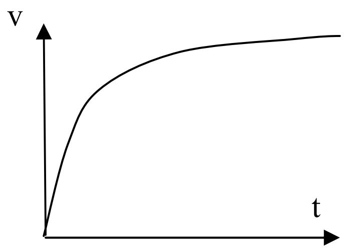
(b)



[1 mark]

(c) During wavefront, since $R_1 >> R_2$, the approximate charging circuit is





peak input voltage to impulse generator E

$V_{max} = \eta$ E

(i) voltage efficiency = $\eta = \dfrac{C_1}{C_1 + C_2}$ **[1 mark]**

(ii) nominal energy capacity = $\frac{1}{2} C_1 E^2$ **[1 mark]**

(iii) also, charging time constant $1/\beta = R_2 .(C_1 \text{ series } C_2) = R_2 C_1C_2/(C_1+C_2) = \eta R_2 C_2$

and an expression $v = V_{max}\left(1 - e^{-\beta t}\right)$

defining wavefront based on 30% to 90% and extrapolation

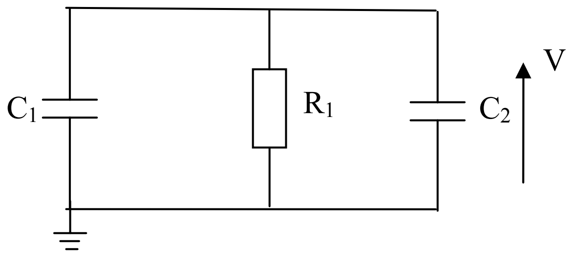
$0.3\ V_m = V_m\ (1- e^{-\beta t1})$ giving $0.7 = e^{-\beta t1}$

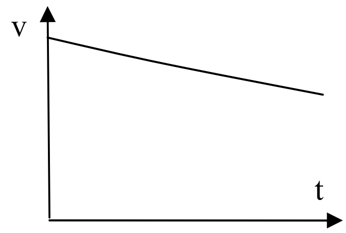
$0.9\ V_m = V_m\ (1- e^{-\beta t2})$ giving $0.1 = e^{-\beta t2}$

therefore, $7 = e^{\beta(t2-t1)}$ giving $t_2 - t_1 = (\ln 7)/\beta = \eta R_2 C_2 1.946$

wavefront time = $(t_2 - t_1)/(0.9 - 0.3) = 3.243 \eta R_2 C_2$ or $3.243 C_1C_2/(C_1+C_2)$ **[3 marks]**

(iv) Similarly, during wavetail, since $R_2 << R_1$, the approximate charging circuit is





giving a discharging time constant $1/\alpha = R_1 .(C_1+C_2) = R_1 C_1/\eta$

and an expression $v = V_{max} e^{-\alpha t}$

at wavetail $0.5\, V_m = V_m\, e^{-\alpha t}$ giving $\alpha\, t_t = \ln (2)$

therefore wavetail time $t_t = 0.693/\alpha = 0.693\, R_1 C_1/\eta$ or $0.693\, R_1 .(C_1+C_2)$ **[2 marks]**

(d) E = 300 kV, energy = 1 kJ

$\frac{1}{2} C_1 E^2 = \frac{1}{2} C_1 \times (300 \times 10^3)^2 = 1 \times 10^3$

C_1 = 22.2 nF, giving 4 capacitors each of value 22.2 × 4 = 88.9 nF for the 4 stages

If C_2 is taken as the capacitance of the device, C_2 = 1 nF giving a voltage efficiency of

$\eta = \frac{22.2}{22.2+1} = 95.7\%$ which is acceptable, giving peak output voltage = 300×0.957 = 287 kV

the standard impulse voltage waveform is 1.2/50 μs

wavefront time $= 3.243\, \eta\, R_2 C_2$, i.e. $1.2 \times 10^{-6} = 3.243 \times 0.957 \times R_2 \times 0.001 \times 10^{-6}$

giving R_2 = 387 Ω

also $t_f = 3.243/\beta$ giving $\beta = 3.243/1.2 = 2.70\ (\mu s)^{-1}$

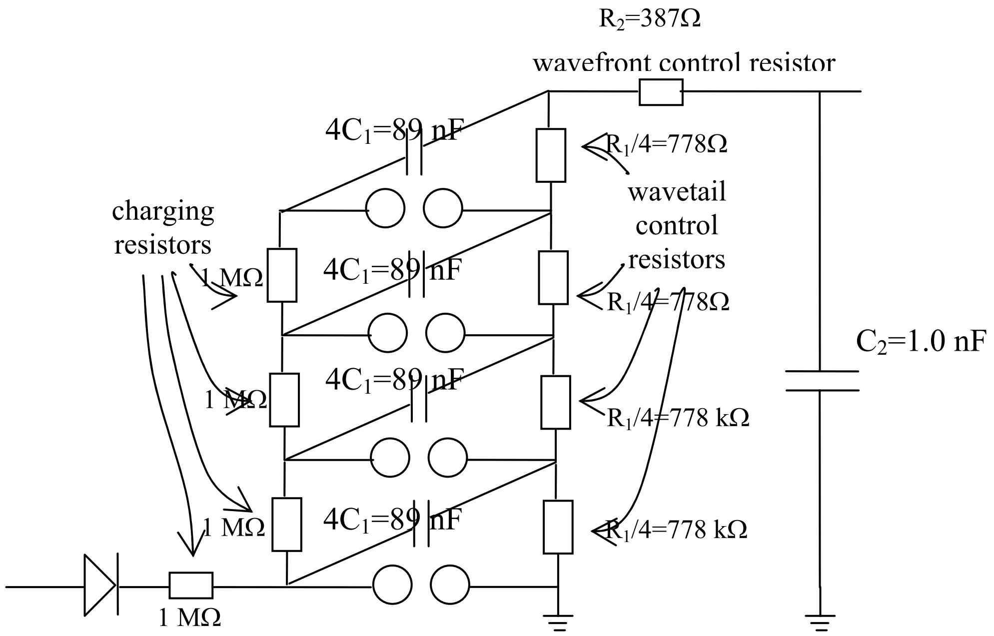
wavetail time $= 0.693\, R_1 C_1/\eta$, i.e. $50 \times 10^{-6} = 0.693\, R_1 \times 0.0222 \times 10^{-6} /0.957$

giving R_1 = 3.10 kΩ, giving 4 resistors each of value 3.10/4 = 778 Ω for the 4 stages

also $t_t = 0.693/\alpha$ giving $\alpha = 0.693/50 = 0.014\ (\mu s)^{-1}$ **[3 marks]**

The charging resistors may be selected to be much larger than any of the calculated values (say 1 MΩ each)

(e)



[3 marks]

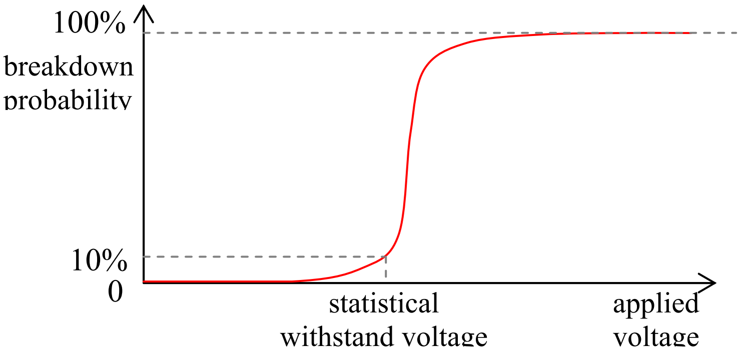
(f)

therefore overall waveform is

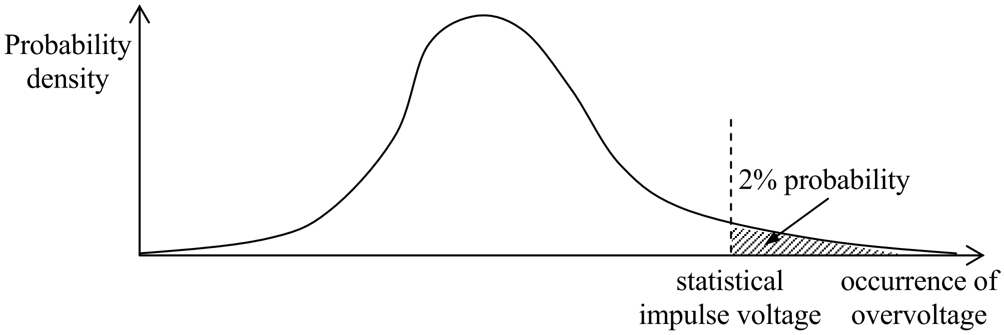
$287\ (e^{-0.014t} - e^{-2.70\, t})$ kV with t in μs **[1 mark]**

4 (a) The aim of statistical method is to quantify the risk of failure of insulation through numerical analysis of the statistical nature of the overvoltage magnitudes and of electrical withstand strength of insulation.

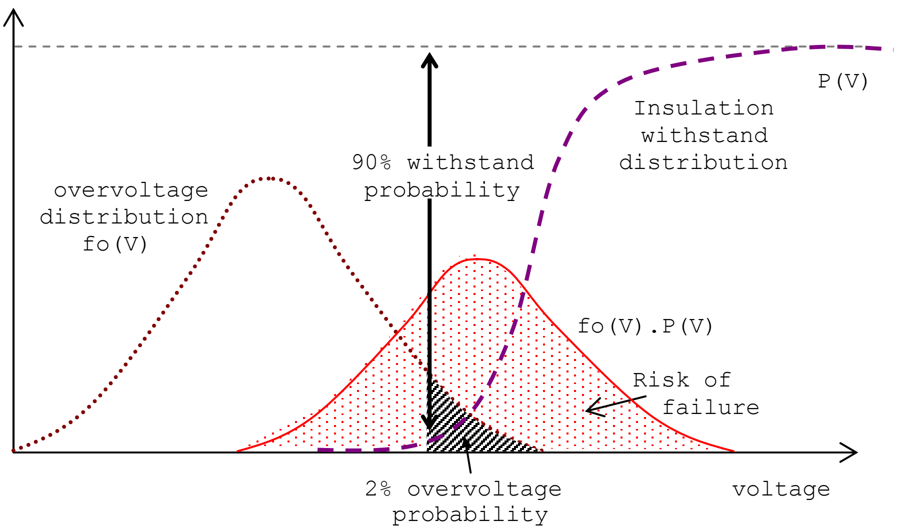
Statistical Impulse Withstand Voltage is the peak value of a switching or lightning impulse test voltage at which insulation exhibits, under the specified conditions, a 90% probability of withstand or a 10% probability of breakdown.



Statistical Impulse Voltage is the switching or lightning overvoltage applied to equipment as a result of an event of one specific type on the system (line energising, reclosing, fault occurrence, lightning discharge, etc), the peak value of which has a 2% probability of being exceeded.



The risk of failure of the insulation is dependant on the integral of the product of the overvoltage density function $f_0(V)$ and the probability of insulation failure P(V). Thus the risk of flashover per switching operation is equal to the area under the curve $\int f_0(V)*P(V)*dV$.



Since a suitable insulation cannot be found such that the withstand distribution does not overlap with the overvoltage distribution, in the statistical method of analysis, the insulation is selected such that the 2% overvoltage probability coincides with the 90% withstand probability as shown. **[2 marks]**

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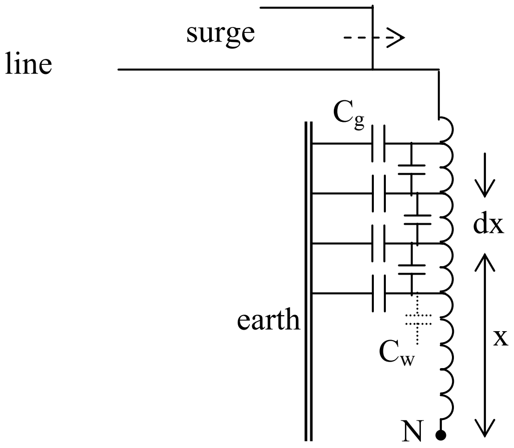
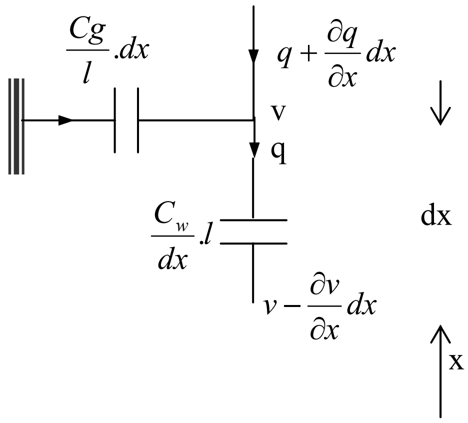
(b) the capacitance distribution dictates the behaviour during initial period, as shown





consider an elemental length dx in a winding of length *l*.

The effective capacitance of the series and shunt parts are given as

The charge balance is given as $\frac{C_w}{dx}.l$ and $\frac{Cg}{l}.dx$

Thus, $\frac{\partial q}{\partial x}.dx = \frac{C_g}{l}.dx.v$, $\quad \frac{\partial v}{\partial x}.dx.\frac{C_w}{dx}.l = q$

$$\therefore \quad \frac{\partial q}{\partial x} = \frac{C_g}{l}.v, \qquad C_w.l.\frac{\partial v}{\partial x} = q$$

$$\therefore \quad \frac{\partial^2 v}{\partial x^2} - \frac{C_g}{C_w}.\frac{1}{l^2}.v = 0$$

This has a solution of the form

$$\therefore \quad v = A\cosh\frac{a}{l}x + B\sinh\frac{a}{l}x$$

where $a = \sqrt{\frac{C_g}{C_w}}$ **[4 marks]**

at $x = l$, $v = V$; so that $v = A\cosh\frac{a}{l}x + B\sinh\frac{a}{l}x$

(c) since the neutral is solidly earthed

at $x = 0$, $v = 0$,

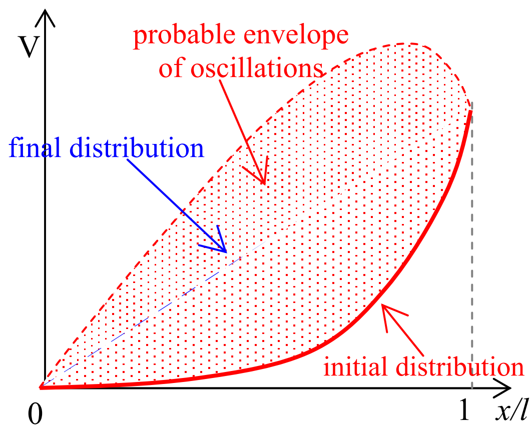
so that $A.\cosh 0 + B.\sinh 0 = 0$

$$\therefore \quad A = 0, \quad \text{giving} \quad B = \frac{V}{\sinh a},$$

$$\therefore \quad v = \frac{V\sinh\frac{a}{l}.x}{\sinh a}$$

if $C_g = 5$ nF and $C_w = 200$ pF

$$a = \sqrt{\frac{C_g}{C_w}} = \sqrt{(5\times10^{-9}/200\times10^{-12})} = 5$$



Since neutral is earthed, final distribution is linear

Oscillations will occur in an envelope equally displaced above and below the final distribution as shown.

$$\therefore v = \frac{100}{\sinh 5}\sinh\frac{5}{l} = 1.348\sinh 5\frac{x}{l}$$

[4 marks]

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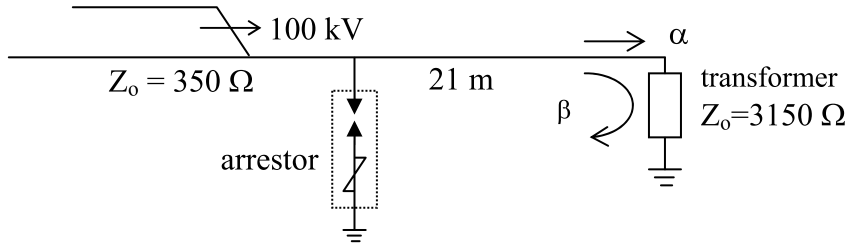
(d) If the separation is 20 m, travel time of line τ = 21/300 = 0.07 μs

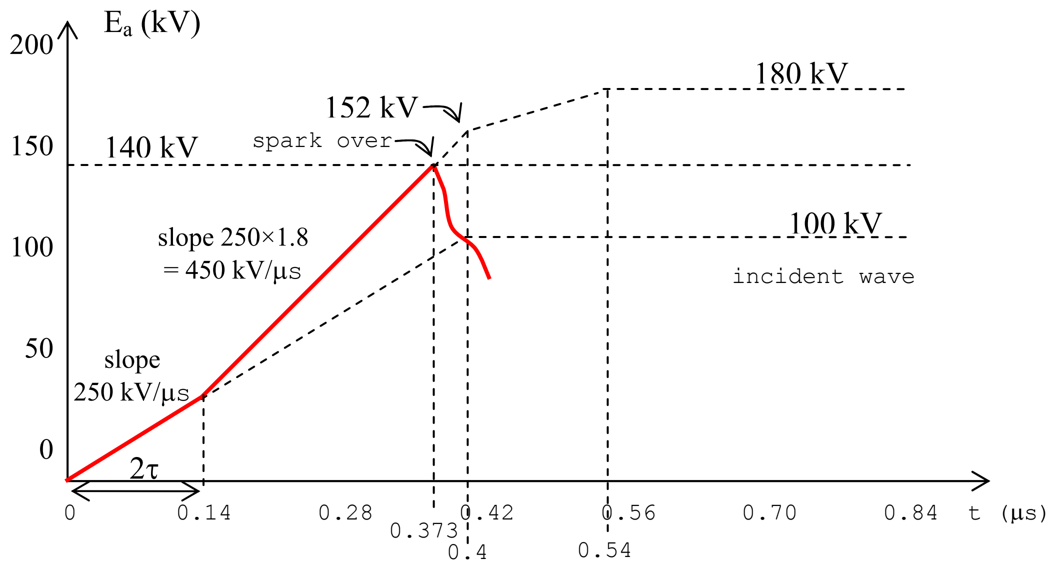
Transmission coefficient $\alpha = \dfrac{2 \times 3150}{3150+350} = 1.8$

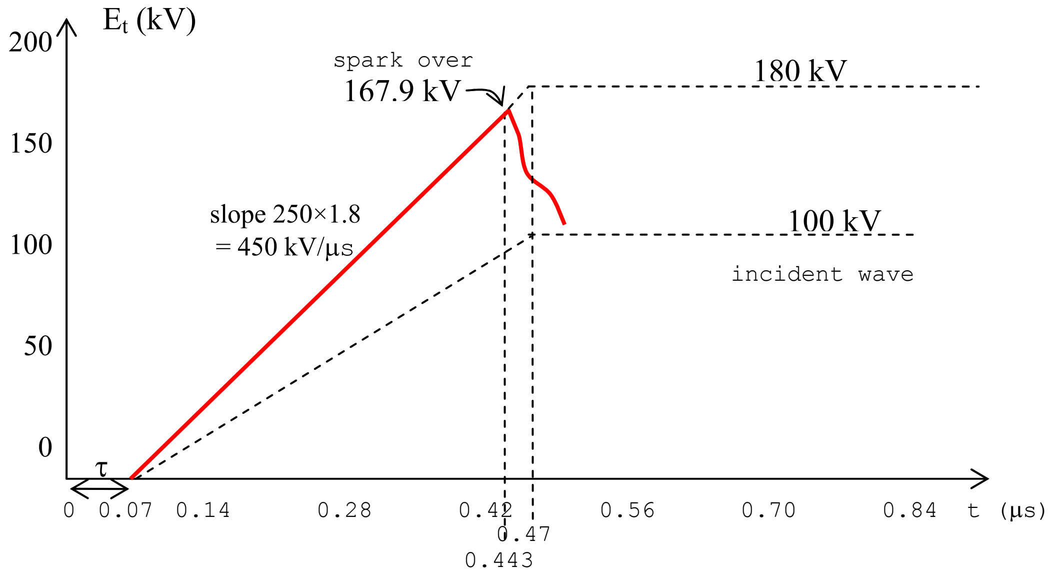
Reflection coefficient β = 1.8 - 1 = 0.8

wavefront time at 250 kV/μs = 100/250 = 0.4 μs



The voltage waveforms at the arrestor location and at the transformer location can be sketched as follows.





[3 marks]

at arrestor location

at time $2\tau = 0.14$ μs just before the reflection from transformer arrives back at arrestor

$E_a = 250\times 0.14 = 35$ kV

breakdown occurs at 140 kV, which is a rise of another 140 – 35 = 105 kV at a slope of 1.8×250

Thus breakdown occurs after a further 105/450 = 0.233 μs at 0.233 + 0.14 = **0.373 μs**

[2 marks]

This is felt at transformer location at a further 0.07 μs later at 0.373 + 0.07 = 0.443 μs

Thus maximum voltage to which transformer will rise will be 450 × (0.443 – 0.07) = **167.9 kV**

[2 marks]

alternate for obtaining maximum value

Prove equation and apply

$$E_t = E_a + \beta \frac{\partial v}{\partial t} \times \frac{2l}{300} = 140 + 0.8 \times 250 \times \frac{2\times 21}{300} = 167.9 \text{ kV}$$

maximum possible transmitted surge of $(1+\beta)E = 180$ kV is not exceeded and hence the above is the maximum surge.