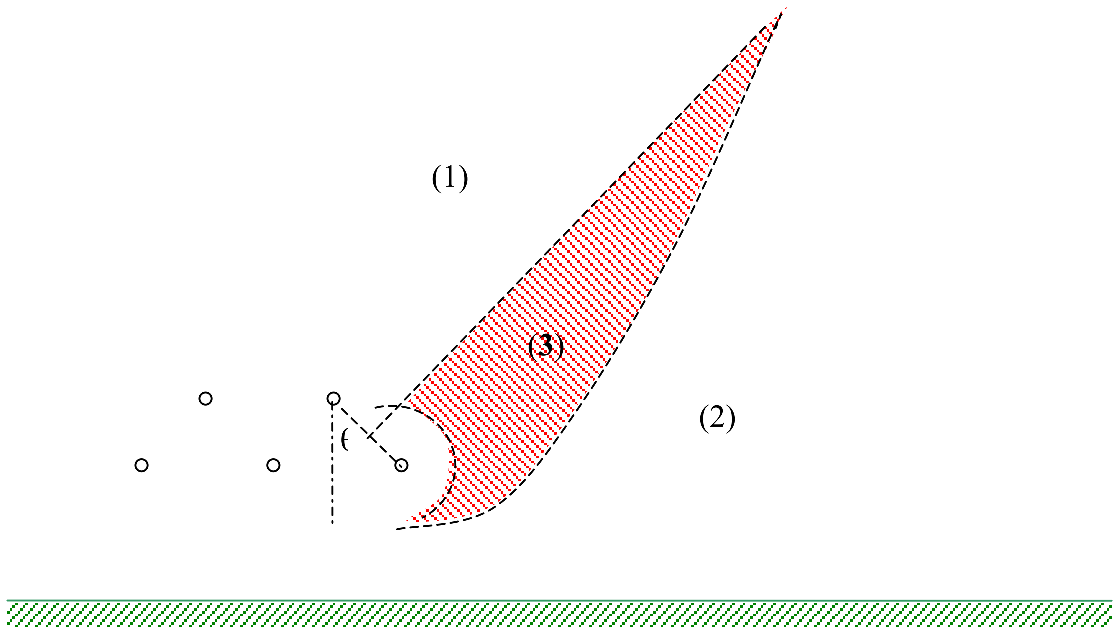
EE 402 – Insulation Co-ordination – Answers

1. (a)



[1 marks]

Figure shows the earth wire coverage of a transmission line against direct strikes.

The region (1) represents the region in which lightning will most likely strike the earth wire and thus provide protection against direct strikes. The locus of the lower boundary of this region is approximately defined by the perpendicular bisector of the line joining the phase conductor (the outermost for a horizontal arrangement and the uppermost in the case of a vertical arrangement) and the earth wire.

The region (2) represents the region in which lightning avoids both the overhead conductor as well as the earth wire but strikes some nearby object. The region has the upper boundary defined approximately by a parabolic locus. This locus is taken as equidistant from both the earth plane as well as the phase conductor. {This assumption is not exactly true as the phase conductor is a better attractor of lightning due to its sharper configuration).

Depending on the strength of the charge on the leader core, lightning will be initiated at a distance away from the object struck. Thus if the leader core could approach very close to the phase conductor before it discharges, then that particular stroke will be weak. This defines a minimum region within which lightning strikes terminating on the line does not do any damage. This region thus has a circular locus around the conductor, which is not be considered in risk evaluation.

The region (3) is the balance region, demarcated by the locus of region (1), the locus of region (2) and the circular locus where the stroke is too weak to cause damage. In this region (3) the lightning stroke is most likely to terminate on the phase conductor. The area (3) is thus a measure of the efficiency of the earth-wire protection. The smaller this region is the better the shielding provided by the overhead earth wires. For the same semi-vertical shielding angle θ, the taller the tower the lesser the efficiency of protection provided by the earth wire. Further if the semi-vertical angle of shielding is reduced, the area (3) reduces giving better protection. Thus to obtain the same degree of protection, taller towers require smaller protection angles. **[2 marks]**

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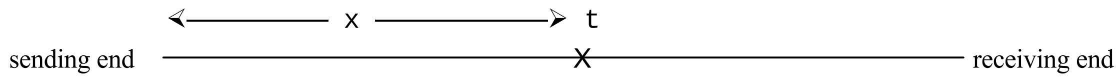
(b) The transient behaviour of a transmission line is governed by the partial differential equation $a^2 \frac{\partial^2 v}{\partial x^2} = \frac{\partial^2 v}{\partial t^2}$ (show brief derivation) (where $a^2 = 1/l.c$) **[2 marks]**

This is satisfied by the expression $v = f(x-at) + F(x+at)$



Consider the function f(x-*a*t)

at the point (x_o, t_o), function has value f (x_o - a t_o)

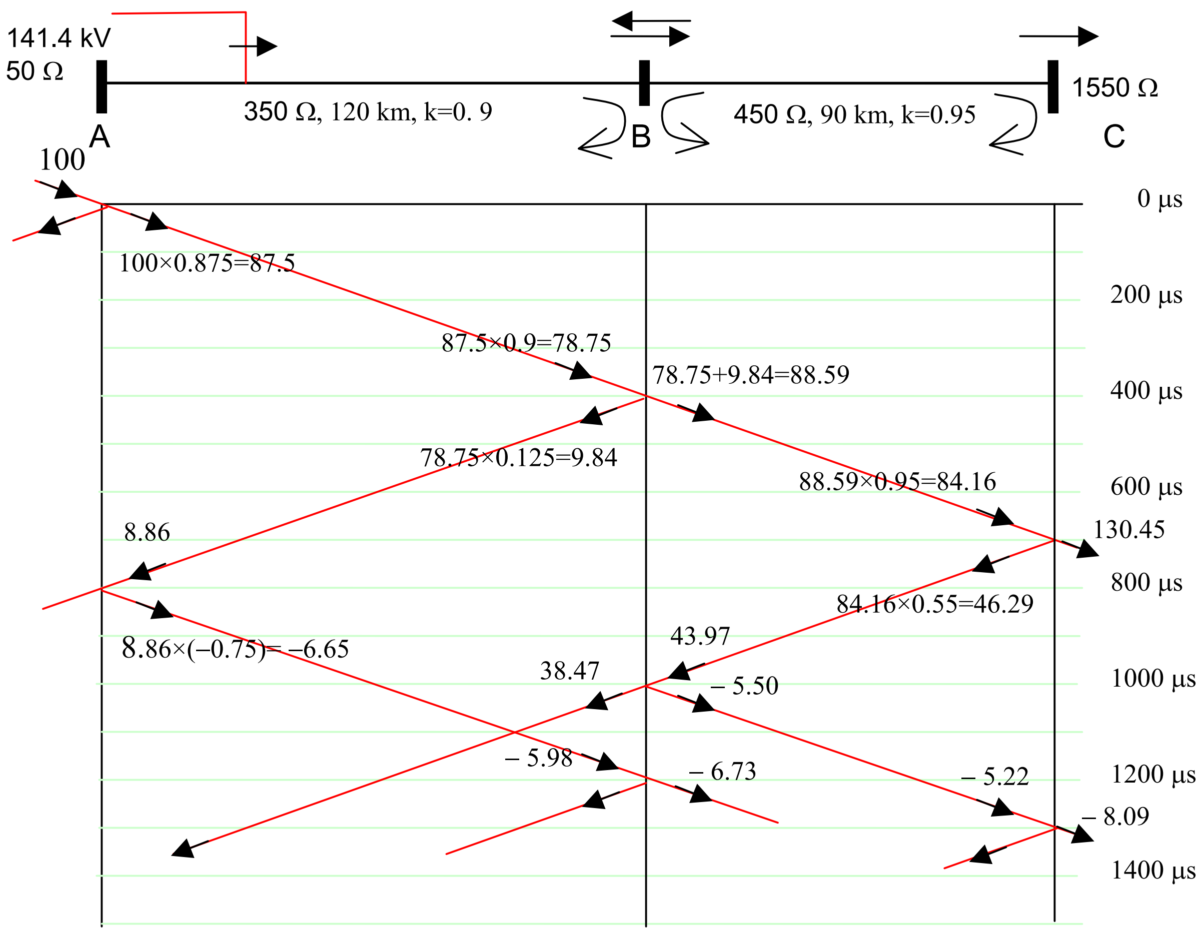
if it moves forward at constant velocity *a*, then after time t, it would have travelled a distance *a.t* so that its co-ordinates would be (x_o+ *a*t, t_o+t)

at the point (x_o+ *a*t, t_o+t), function has value f (x_o+ *a*t - *a* (t_o+t)) or f (x_o - *a* t_o).

Thus the function f(x-*a*t) remains constant when travelling forward at velocity *a*. Thus it represents a forward travelling wave. Similarly, F(x+at) represents a reverse travelling wave.

Thus the surge on a transmission line can be represented by a forward travelling wave and a reverse travelling wave **[2 marks]**

(c) For ac surge, soon after peak waveform, equivalent to step of magnitude √2×100 = 141.4 kV



reflection coefficient at B, for surge from AB $\frac{450-350}{450+350} = 0.125$

and for surge from CA $\frac{350-450}{350+450} = -0.125$

transmission coefficient at B = 1 + 0.125 = 1.125 and 1 – 0.125 = 0.875

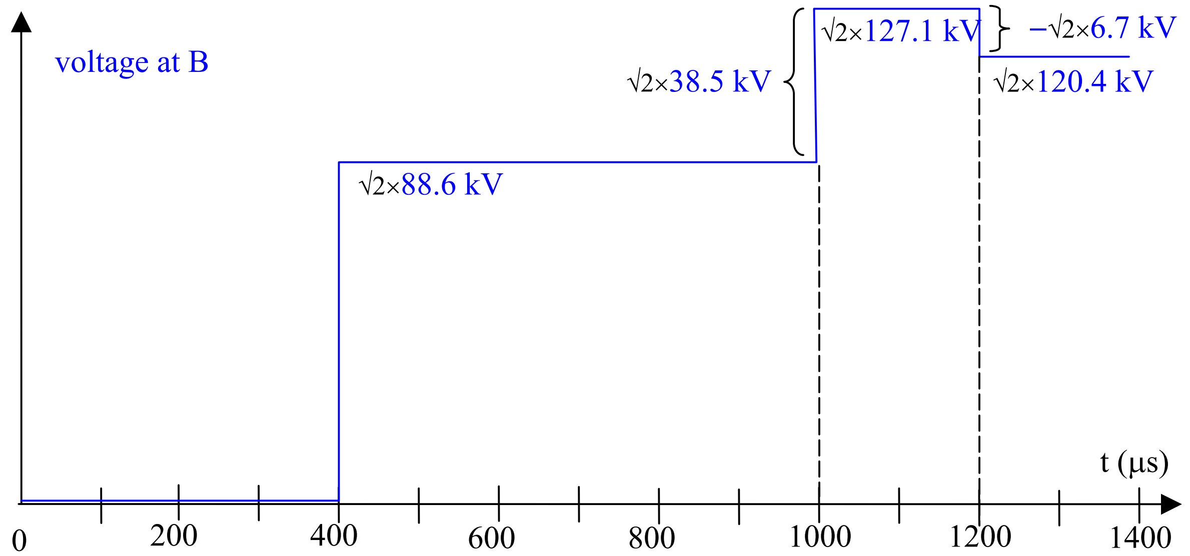
reflection coefficient at C, $\frac{1550-450}{1550+450} = 0.55$

transmission coefficient at C = 1+ 0.55 = 1.55

travel time of AB = 120/300 = 400 μs

travel time of BC = 90/300 = 300 μs **[8 marks]**

(d) Thus the waveforms of the voltages are



[3 marks]

2. (a) Derivation of

$$\frac{\partial^2 e}{\partial x^2} = l.c\frac{\partial^2 e}{\partial t^2},\quad a^2\frac{\partial^2 e}{\partial x^2} = \frac{\partial^2 e}{\partial t^2},\quad a = \frac{1}{\sqrt{l.c}}$$

$$e = f(x - a t) \text{ for forward wave}$$

$$l\frac{\partial e}{\partial t} = -\frac{\partial e}{\partial x} = -f'(x - a\,t)$$

$$\therefore i = \frac{1}{a\,l} f(x - at) = \frac{1}{a\,l}.e = \sqrt{\frac{c}{l}}.e$$

$$\text{i.e. } e = \sqrt{\frac{1}{c}}.i = Z_0.i \quad \text{where } Z_0 = \sqrt{\frac{1}{c}}$$

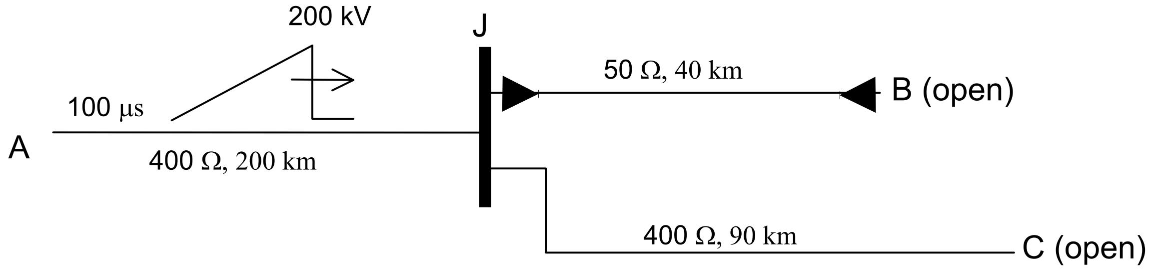
$$l = \frac{\mu_0}{2\pi}\log_e\frac{d}{r}\ \text{H/m},\qquad c = \frac{2\pi\,\varepsilon_0\,\varepsilon_r}{\log_e\frac{d}{r}}\ \text{F/m}$$

$$Z_0 = \sqrt{\frac{l}{c}} = \sqrt{\frac{\mu_0}{\varepsilon_0}\left(\frac{\log_e d/r}{2}\right)^2},\quad a = \frac{1}{\sqrt{lc}} = \frac{1}{\sqrt{\mu_0\,\varepsilon_0\varepsilon_r}} = \frac{velocity\ of\ light}{\sqrt{\varepsilon_r}}$$

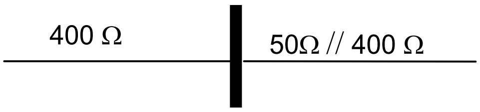
energy stored in surge = $\frac{1}{2} c\, v^2 + \frac{1}{2} l\, i^2 = \frac{1}{2} c\, v^2 + \frac{1}{2} c\, v^2 = c\, v^2$

[3 marks]

(b)



Derivation of the equivalence of the above circuit as



effective outgoing =50//400 = 44.44 Ω

reflection coefficient = $\dfrac{44.44-400}{44.44+400}$ = – 0.800 **[6 marks]**

(c)

travel time of AJ = 200/3×10^{-5} = 666$^2/_3$ μs

travel time of BJ = 40/2×10^{-5} = 200 μs

travel time of CJ = 90/3×10^{-5} = 300 μs

for surge along AJ at J,

transmitted surge into JB and JC at t=0 is (1-0.8)×200 = 40 kV

reflection coefficients at B and C (both open) are 1.0

voltage at B at t=200 μs is 2×40 = 80 kV

surge arriving at J from BJ at time 400 μs is 40 kV

for this surge, effective outgoing = 400//400 = 200 Ω,

and reflection coefficient = $\dfrac{200-50}{200+50}=0.6$

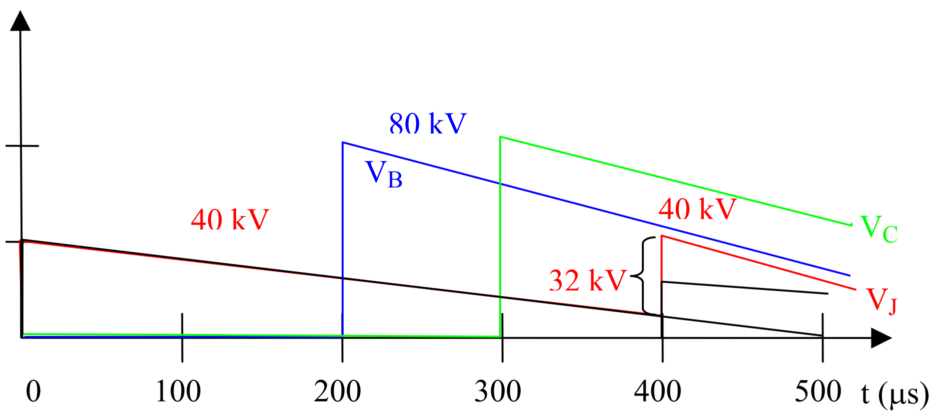
surge reflected back to JB = 20×0.6 = 12 kV

surge transmitted to JA and JC = 40×1.6 = 64 kV

voltage at C at t=300 μs is 2×40 = 80 kV

reflected surge arrives back at D at time 600 μs

No more surges would arrive at J during the first 500 μs **[2 marks]**



[2 marks]

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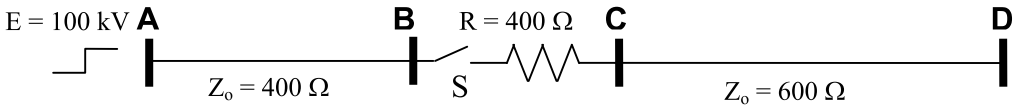
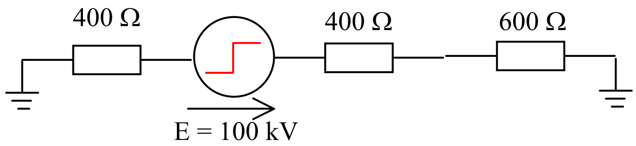
(d)



Prior to closure of switch, voltages are 100 kV at A and B, and 0 kV at C and D. Voltage across S = 100 kV. Since the relation between the surge voltage and surge current can be represented by $V = Z_o I$, the closure of the switch can be represented and changes can be obtained from the equivalent circuit

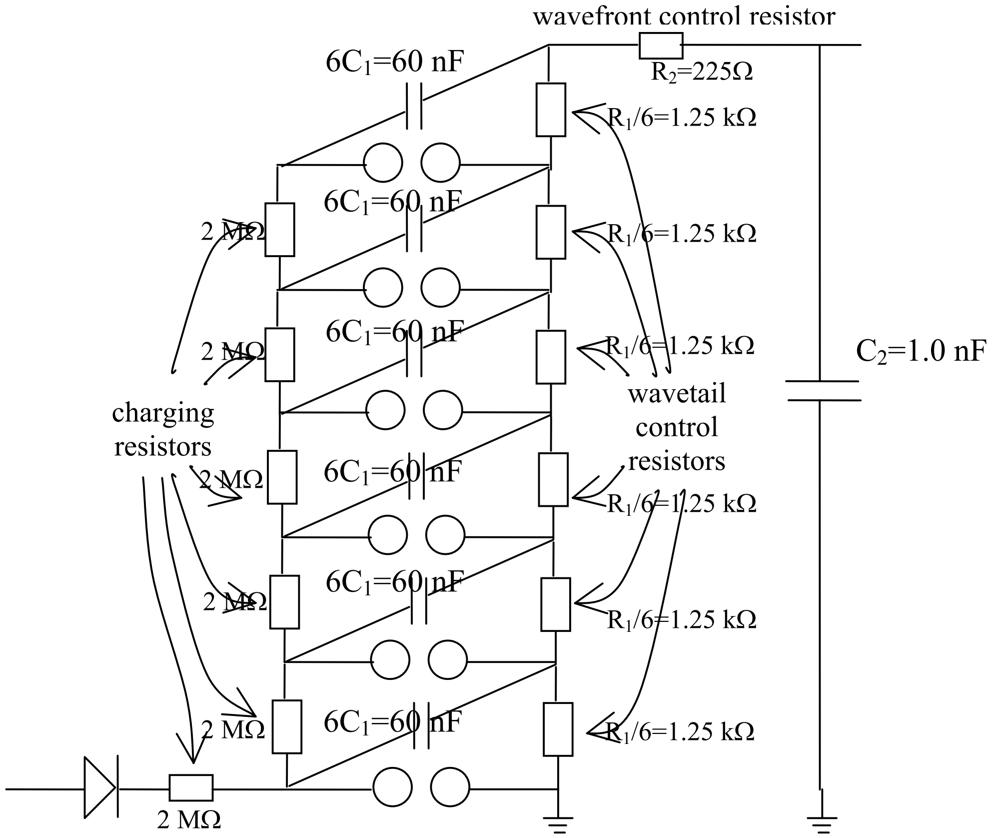


Changes in voltage across the resistances are 100*400/1400 = – 28.57 kV, 28.57 kV and 42.86 kV

Therefore the magnitudes of surges transmitted on to BA and CD are – 28.57 kV and 42.86 kV respectively.

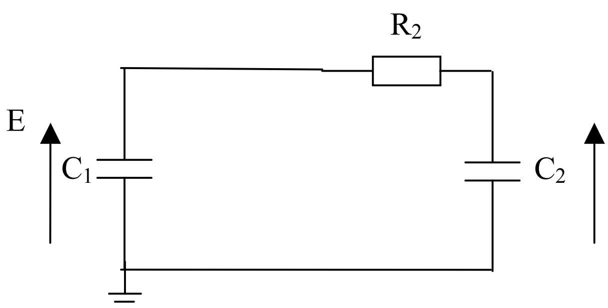
[4 marks]

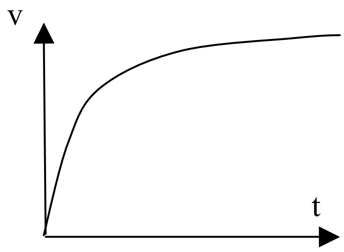
3 (a)



[4 marks]

(b) During wavefront, since $R_1 >> R_2$, the approximate charging circuit is





peak input voltage to impulse generator E = 6×50 = 300 kV

$V_m = \eta E$

voltage efficiency = $\eta = \dfrac{C_1}{C_1 + C_2}$ = 10/(10+1) = 0.909 = 90.9% **[2 marks]**

(c) peak output voltage = 300×0.909 = 272.7 kV

energy = $\frac{1}{2} C_1 V^2 = \frac{1}{2} \times 10\times10^{-9} (272.7\times10^3)^2 = 371.9$ J **[1 marks]**

(d) also, charging time constant $1/\beta = R_2 .(C_1//C_2) = R_2 C_1C_2/(C_1+C_2) = \eta R_2 C_2$

and an expression $v = V_{max}\left(1 - e^{-\beta t}\right)$

therefore $\beta = 1/(0.909\times225\times1\times10^{-9}) = 4.889\times10^{-6}$ s^{-1} or 4.889 $(\mu s)^{-1}$ **[1 marks]**

defining wavefront based on 30% to 90% and extrapolation

$0.3 V_m = V_m (1- e^{-\beta t1})$ giving $0.7 = e^{-\beta t1}$

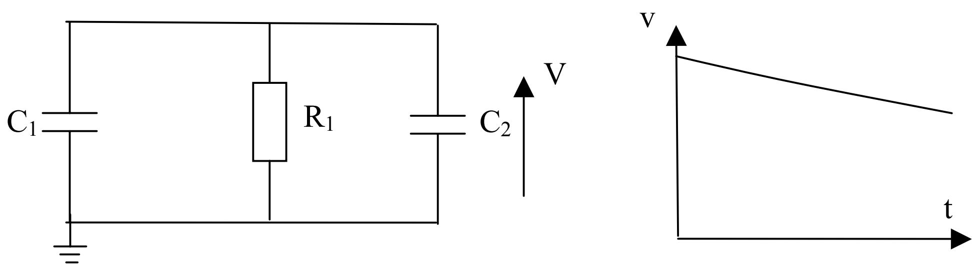
$0.9 V_m = V_m (1- e^{-\beta t2})$ giving $0.1 = e^{-\beta t2}$

therefore, $7 = e^{\beta(t2-t1)}$ giving $t_2 - t_1 = (\ln 7)/\beta = \eta R_2 C_2 1.946$

wavefront time = $(t_2 - t_1)/(0.9 - 0.3) = 3.243 \eta R_2 C_2$

$= 3.243\times0.909\times225\times1\times10^{-3} = 0.663$ μs **[3 marks]**

Similarly, during wavetail, since $R_2 << R_1$, the approximate charging circuit is



giving a discharging time constant $1/\alpha = R_1 .(C_1+C_2) = R_1 C_1/\eta$

therefore $\alpha = 0.909/7500\times10\times10^{-3} = 0.012$ $(\mu s)^{-1}$ **[1 marks]**

and an expression $v = V_{max} e^{-\alpha t}$

at wavetail $0.5 V_m = V_m e^{-\alpha t}$ giving $\alpha t_t = \ln (2)$

therefore wavetail time $t_t = 0.693/\alpha = 0.693 R_1 C_1/\eta$

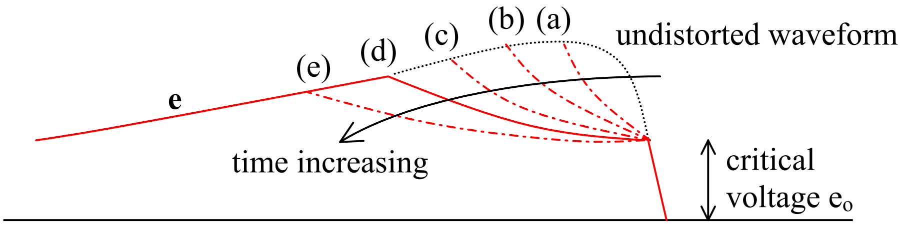
$= 0.693\times7500\times10\times10^{-3}/0.909 = 57.2$ μs **[1 marks]**

(e) therefore overall waveform is

$272.7 (e^{-0.012t} - e^{-4.889t})$ kV with t in μs **[2 marks]**

Brief explanation with suitable diagrams, why a trigatron gap is normally used in the first stage of a multi-stage impulse generator. **[2 marks]**

4



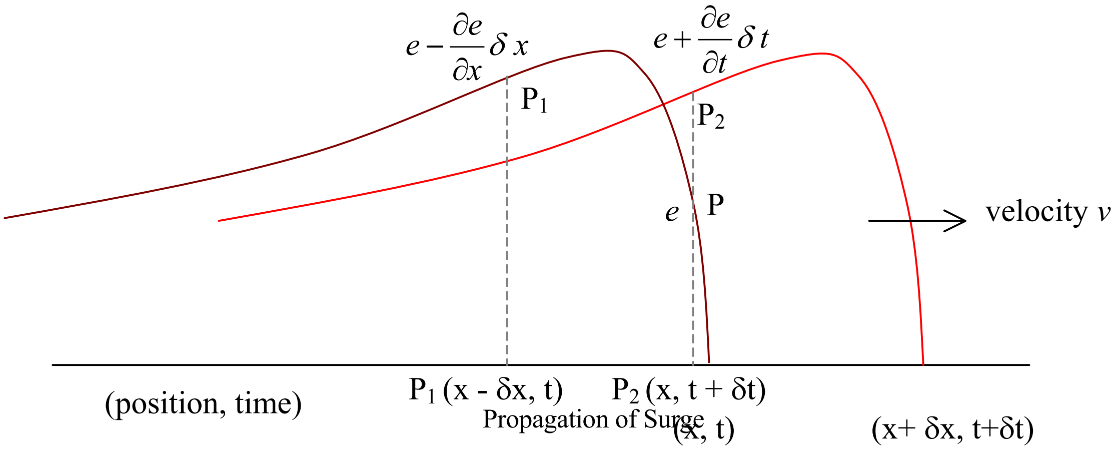
(a) When a surge voltage wave traveling on an overhead line causes an electric field around it exceeding the critical stress of air, corona will be formed. This extracts energy from the surge and waveform distortion occurs. Corona thus reduces the steepness of the wavefront above the critical voltage, as the surge travels down the line.

Energy associated with a surge waveform = $\frac{1}{2} C e^2 + \frac{1}{2} L i^2$

But the surge voltage **e** is related to the surge current **i** by the equation

$$i = \frac{e}{Z_0} = e\sqrt{\frac{C}{L}}, \quad i.e. \quad \tfrac{1}{2} L i^2 = \tfrac{1}{2} C e^2$$

So that the total wave energy = $C e^2$



Propagation of Surge

Let the voltage at a point P at position **x** be **e** at time **t**.

Then voltage at point P_1 just behind P would be $e - \frac{\partial e}{\partial x}\delta x$ at time t, or $e - \frac{\partial e}{\partial x}.v.\delta t.$

If the voltage is above corona inception, it would not remain at this value but would attain a value $e + \frac{\partial e}{\partial t}\delta t$ at P at time t+Δt , when the surge at P_1 moves forward to P_2.

[Note: $\frac{\partial e}{\partial x}, \frac{\partial e}{\partial t}$ would in fact be negative quantities on the wavefront.]

Thus corona causes a depression in the voltage from $(e - v\frac{\partial e}{\partial x}\delta t)$ to $(e + \frac{\partial e}{\partial t}\delta t)$, with a corresponding loss of energy of $C\left[(e - v\frac{\partial e}{\partial x}\delta t)^2 - (e + \frac{\partial e}{\partial t}\delta t)^2\right]$ or $-2Ce\left[v\frac{\partial e}{\partial x} + \frac{\partial e}{\partial t}\right]\delta t$.

The energy to create a corona field is proportional to the square of the excess voltage. i.e. $k(e - e_0)^2$.

Thus the energy required to change the voltage from e to $(e + \frac{\partial e}{\partial t}\delta t)$ is given by

$k\left[(e + \frac{\partial e}{\partial t}\delta t - e_0)^2 - (e - e_0)^2\right]$ or $2k(e - e_0)\frac{\partial e}{\partial t}\delta t$.

The loss of energy causing distortion must be equal to the change in energy required. Thus

$$-2Ce\left[v\frac{\partial e}{\partial x} + \frac{\partial e}{\partial t}\right]\delta t = 2k(e - e_0)\frac{\partial e}{\partial t}.\delta t$$

Rearranging and simplifying gives the equation $v\frac{\partial e}{\partial x} = -\left[1 + \frac{k}{C}.\frac{(e - e_0)}{e}\right]\frac{\partial e}{\partial t}$

Wave propagation under ideal conditions is written in the form $v\frac{\partial e}{\partial x} = -\frac{\partial e}{\partial t}$

Thus we see that the wave velocity has decreased below the normal propagation velocity, and that the wave velocity of an increment of voltage at **e** has a magnitude given by $v_e = \dfrac{v}{1 + \dfrac{k}{c}\left(\dfrac{e - e_0}{e}\right)}$

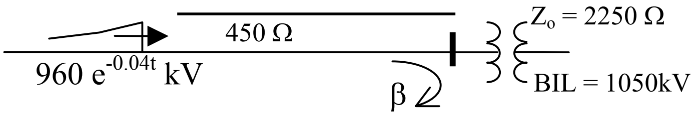
Thus the time of travel for an element at **e** when it travels a distance **x** is given by

$$t = \frac{x}{v_e} = \frac{x}{v}\left[1 + \frac{k}{C}\left[\frac{e - e_o}{e}\right]\right], \; i.e. \; \left[\frac{x}{v_e} - \frac{x}{v}\right] = \frac{x}{v}.\frac{k}{C}\left[\frac{e - e_o}{e}\right]$$

$\left(\dfrac{x}{v_e} - \dfrac{x}{v}\right)$ is the time lag Δt corresponding to the voltage element at **e**.

Thus $\dfrac{\Delta t}{x} = \dfrac{k}{v.C}\left[1 - \dfrac{e_0}{e}\right]$ μs/m **[12 marks]**

(b) $\beta = \dfrac{2250 - 450}{2250 + 450} = 0.6667$,



$\alpha = 1.6667$

for a BIL of 1050 kV and an insulation margin of 15%,

maximum permissible voltage = 1050 × 85/100 = 892.5 kV

Since there is a transmission coefficient of $\alpha = 1.6667$, the maximum permissible incident voltage must be reduced by this factor.

Thus maximum permissible incident surge = 892.5/1.6667 = 535.5 kV

Thus corona must reduce the incoming surge to 535.5 kV.

i.e. $535.5 = 960\, e^{-0.04t}$ which gives a delay time = 25×ln(960/535.5) = 14.59 μs

substitution in the equation gives

$$\frac{14.59}{x} = 0.01\left[1 - \frac{240}{535.5}\right] \mu s/m$$

x =2644 m or 2.64 km **[6 marks]**