University of Moratuwa, Sri Lanka

B. Sc. Engineering Honours Degree Course, Semester 2 Examination

EE2092 – THEORY OF ELECTRICITY

Short Answers

Question 1

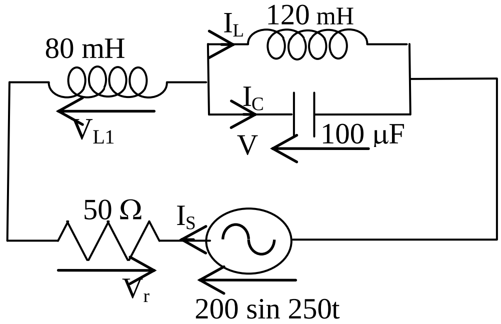


Figure Q1

(a) Write down the differential equations governing the behaviour of the circuit.

$200 \sin 250t = 50\, i_s + 0.08\, di_s/dt + v$

$i_s = i_L + i_C$

$v = 0.120\, di_s/dt = 10^4 \int i_C\, dt$ **[2 marks]**

(b) Calculate the total impedance of the circuit in ohm.

$$Z = 50 + j\,0.08{\times}250 + j\,0.12{\times}250//(10^4/j250) = 50 + j20 + \frac{j30\times(-j40)}{j30 - j40}$$

$=50 + j140\ \Omega$ (or $148.66\angle 70.35^0\,\Omega$) **[2 marks]**

(c) Calculate the current I_S supplied from the source.

$$I_s = \frac{\frac{200}{\sqrt{2}}\angle 0^0}{148.66\angle 70.35^0} = 0.951\angle -70.35^0\,A$$ **[1 mark]**

(d) Determine the currents I_L and I_C.

$$I_L = \frac{-j40}{j30 - j40} I_s = 4I_s = 3..805\angle -70.35^0\,A$$

$$I_C = \frac{j30}{j30 - j40} I_s = -3I_s = 2.854\angle 109.65^0\,A$$ **[2 marks]**

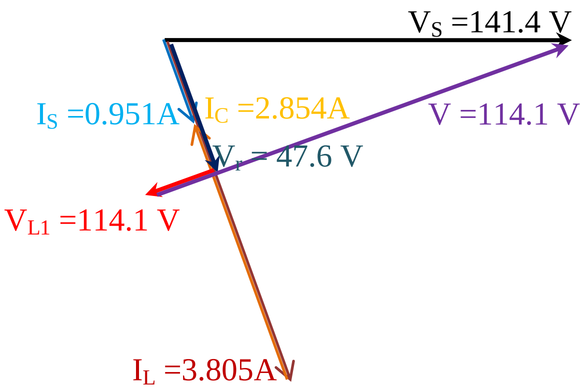
(e) Determine the voltages V_r, V_{L1} and V.

$V_r = 50\, I_s = 47.57\angle{-70.35^0}\ V$

$V_{L1} = j250{\times}0.08\, I_s = 19.02\angle 19.65^0\ V$

$V = j30\times I_L = 114.1\angle{-70.35^0}\ V$ **[2 marks]**

(f) Sketch a phasor diagram showing all the voltages and currents in the circuit.



[3 marks]

(g) If the supply frequency is allowed to vary, determine the frequency at which series resonance would occur.

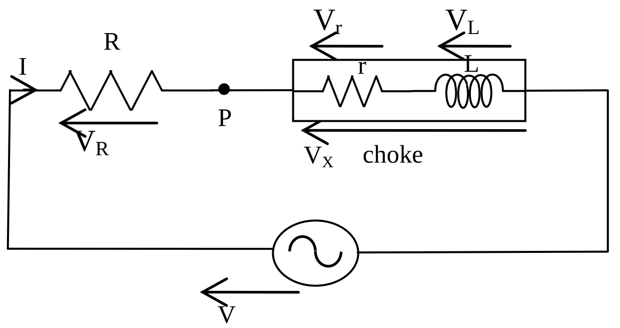
$$j\omega\times 0.08+\frac{j\,0.12\omega\times\left(\dfrac{10^4}{j\omega}\right)}{j\,0.12\omega+\dfrac{10^4}{j\omega}}=0$$

$j\omega 0.08\times j\omega 0.12 + 10^4\times 0.08 + 0.12\times 10^4 = 0$

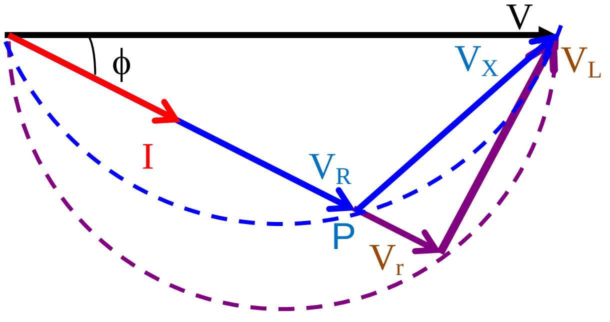
$0.0096\ \omega^2 = 0.2\times 10^4$

$\omega = 456.4$ *rad/s (or 72.64 Hz)* **[2 marks]**

Question 2



(a) A practical choke (inductance L and series resistance r), connected in series with a pure resistance R, is supplied from a constant voltage source (V,ω). If the resistance R is allowed to vary, sketch the locus of the voltage of the intersection P between the choke and the resistor.



[3 marks]

(b) One of the readings in Q2(a) corresponds to V = 250V at 50 Hz, current I = 10A lagging voltage by 30^0, the voltage magnitudes across the resistor V_R = 150V and choke V_X = 141.6V. Complete the phasor diagram and calculate the inductance L and the resistance r of the choke.

Phasor diagram is completed as in above diagram

$V_R + V_r = V\cos\phi = 250\cos 30^0 = 216.5\ V$

$V_R = 150\ V,\ \therefore V_r = 216.5 - 150 = 66.5\ V$

$r = 66.5/10 = 6.65\ \Omega$

$V_L = V\sin\phi = 250\sin 30^0 = 125\ V$

$L = 125/(10\times 2\pi\times 50) = 39.79\ mH$ **[3 marks]**

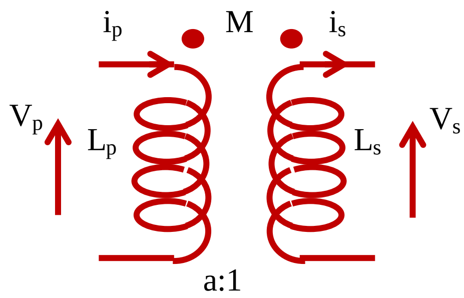
(c) Derive the non-coupled equivalent circuit of a two winding transformer.

Consider the coupled circuit shown

From Kirchoff's voltage law

$$V_p = L_p\, p\, i_p - M\, p\, i_s$$

and $$V_s = -(L_s\, p\, i_s - M\, p\, i_p)$$



For near ideal transformers

$$|V_p| : |V_s| \approx a$$

$$|i_p| : |i_s| \approx 1/a$$

Thus

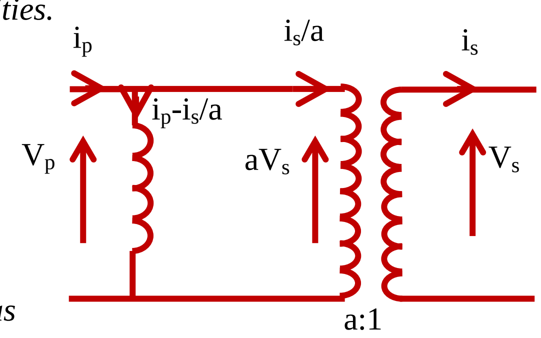
V_p *and* aV_s *are comparable quantities*

i_p *and* i_s/a *are comparable quantities.*

Using these comparable variables

$$V_p = L_p\, p\, i_p - aM\, p\, i_s/a$$

and $$aV_s = -(a^2 L_s\, p\, i_s/a - aM\, p\, i_p)$$



If $\boldsymbol{i_p}$ *and* $\boldsymbol{i_s/a}$ *are unequal,*

Leakage current = $\boldsymbol{i_p - i_s/a}$

re-formulated in terms of this leakage current as

$$V_p = L_p\, p\, i_p - aM\, p\, i_s/a + aM\, p\, i_p - aM\, p\, i_p$$

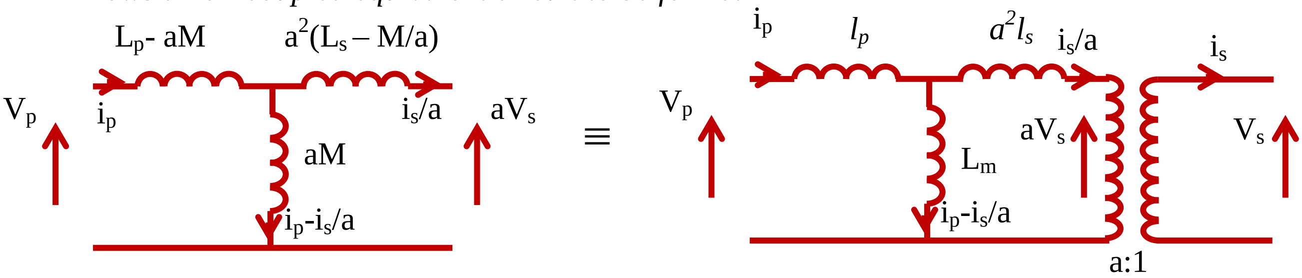
$$= (L_p - aM)\, p\, i_p + aM\, p\,.(i_p - i_s/a)$$

and $$aV_s = -(a^2 L_s\, p\, i_s/a - aM\, p\, i_p) + aM\, p\, i_s/a - aM\, p\, i_s/a$$

$$= -a^2\left(L_s - \frac{M}{a}\right) p\, i_s/a + aM\, p.(i_p - i_s/a)$$

Equations are expressed either in terms of i_p *and* $i_p - i_s/a$ *or in terms of* i_s/a *and* $i_p - i_s/a$.

Allows a non-coupled equivalent circuit to be formed



[2 marks]

A 80/20kV transformer has a primary winding inductance of 16H, and a coupling coefficient of 0.95. Using any standard equations derive the values of the inductive impedances of all the elements of the non-coupled equivalent circuit.

$a = 80/20 = 4$

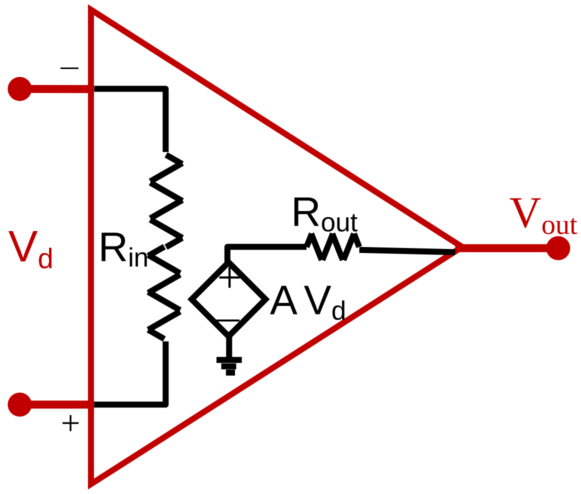
$L_p = 16\ H,\ L_s = L_p/a^2 = 16/4^2 = 1\ H$

$M = k\ \sqrt{(L_p\, L_s)} = 0.95 \times \sqrt{(16 \times 1)} = 3.8\ H$

$l_p = L_p - a\,M = 16 - 4 \times 3.8 = 0.8\ H$

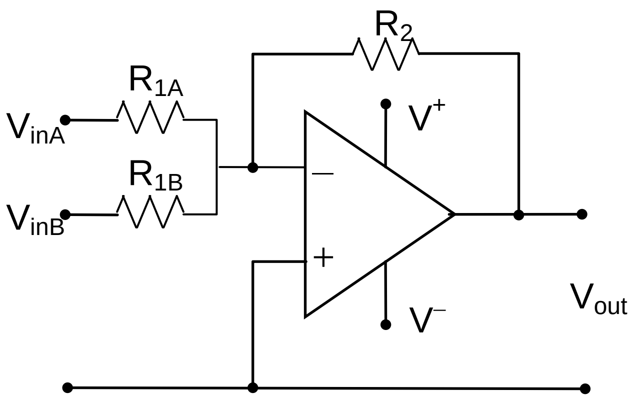
$l_s = L_s - M/a = 1 - 3.8/4 = 0.05\ H$ **[2 marks]**

(d) Sketch the equivalent circuit of an operational amplifier and show that the voltage difference of the input terminals may be taken as a virtual zero.



Since the gain A of an operational amplifier is very large (10^5 to 10^8), for any finite V_{out}, V_d becomes extremely small, and when the lower terminal of the input is earthed, the upper terminal becomes a virtual earth for all practical purposes. **[2 marks]**

(e) It is required to construct a summing amplifier to add 75% of input A voltage and 25% of input B voltage. If the resistance connected across the operational amplifier is 1 kΩ, use a diagram to show what should be the input resistances to be connected for the two inputs?



$$V_{out} = -R_2\left(\frac{V_{inA}}{R_{1A}} + \frac{V_{inB}}{R_{1B}}\right)$$

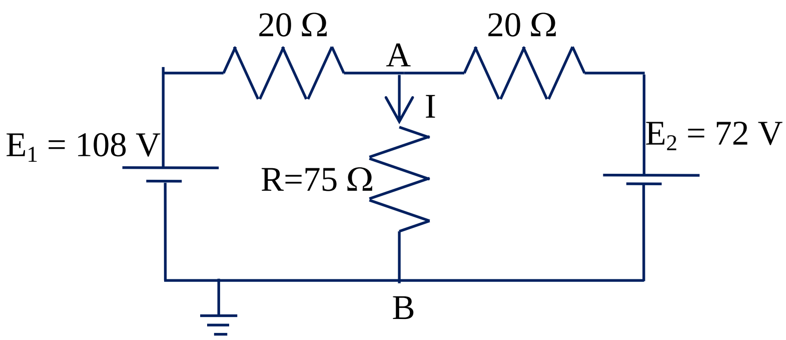
$= -(0.75\ V_{inA} + 0.25\ V_{inA})$

$R_2 = 1\ k\Omega$

$\therefore\ R_2/R_{1A} = 0.75$, and $R_2/R_{1B} = 0.25$,

i.e. $R_{1A} = 1000/0.75 = 1.33\ k\Omega$ and $R_{1B} = 1000/0.25 = 4\ k\Omega$ **[2 marks]**

Question 3



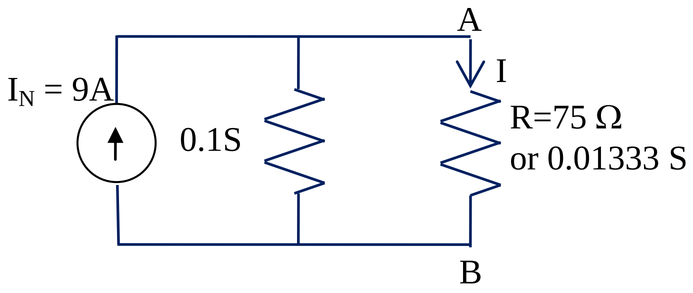
(a) For the circuit shown in figure Q3ab, obtain the Norton's equivalent circuit across AB and hence obtain the current I.

Short-circuiting AB, it is clearly seen that the short circuit current across AB is

$I_{SC\,AB}$ *= 108/20 + 72/20 = 9 A, i.e.* $I_N = 9\ A$

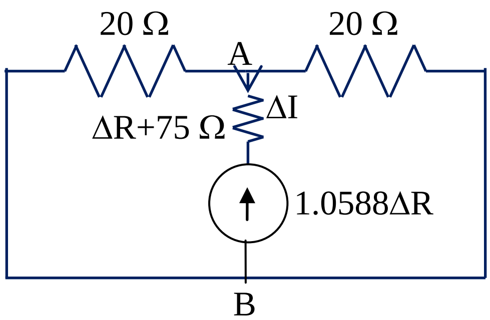
Equivalent Norton's admittance across AB is the parallel of 20Ω and 20Ω.

i.e. $Y_N = 0.05 + 0.05 = 0.10\ S$



Current I =9×0.013333/(0.1 + 0.01333) = 1.0588 A **[3 marks]**

(b) Using compensation theorem, determine the resistance to which R must be changed if the desired current I in figure Q3ab is 1.0A.



Using compensation theorem, required change in current =1 – 1 .0588 = – 0 .0588A

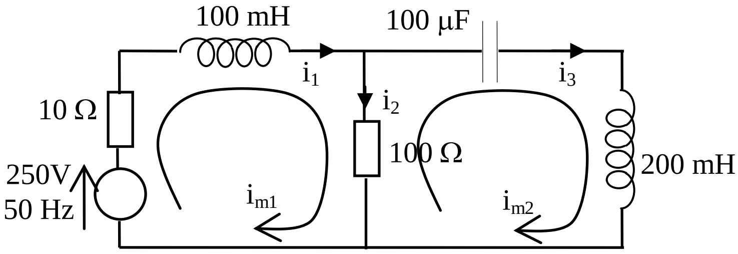
Using this together with the circuit,

$$\Delta I = -0.0588\ A = \frac{1.0588 \times \Delta R}{75 + \Delta R + 10}\ 1.0588$$

Giving $\Delta R = 85 \times 0.0588 = 5.0\ \Omega$

$\therefore$ *required value of R = 75+5 = 80 Ω* **[2 marks]**

(c) For the circuit shown in figure Q3c, write down the branch-mesh incidence matrix and the branch impedance matrix.



100 mH $\rightarrow 100\times10^{-3}\times100\pi = j\ 31.416\ \Omega$

200 mH $\rightarrow 200\times10^{-3}\times100\pi = j\ 62.832\ \Omega$

100 μF $\rightarrow 1/(j100\times10^{-6}\times100\pi) = -j31.831\ \Omega$

$$\text{Branch mesh incidence matrix} = \begin{bmatrix} 1 & 0 \\ 1 & -1 \\ 0 & 1 \end{bmatrix}$$

$$\text{Branch impedance matrix} = \begin{bmatrix} 10+j31.416 & 0 & 0 \\ 0 & 100 & 0 \\ 0 & 0 & j62.832-j31.831 \end{bmatrix}$$

[3 marks]

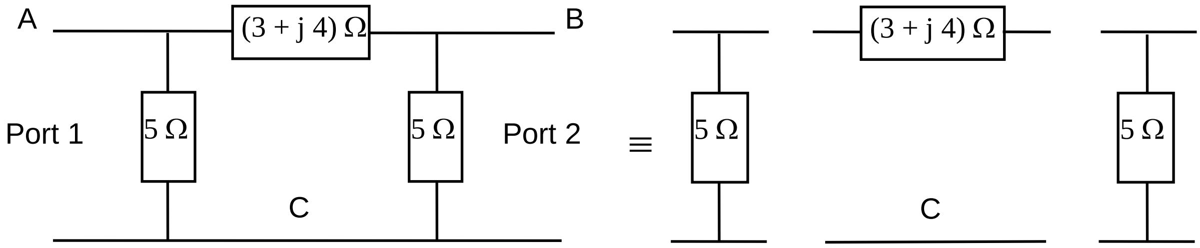
Hence determine the mesh impedance matrix.

$$[Z_m] = \begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 10+j31.416 & 0 & 0 \\ 0 & 100 & 0 \\ 0 & 0 & j62.832-j31.831 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & -1 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 110+j31.416 & -100 \\ -100 & 100+j31.00 \end{bmatrix}$$

[2 marks]

(d) Determine the ABDC matrix of the two-port network shown in figure Q3de.



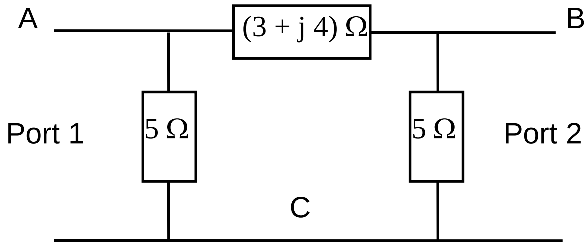
ABCD matrix can be obtained by the product of the individual ABCD matrices

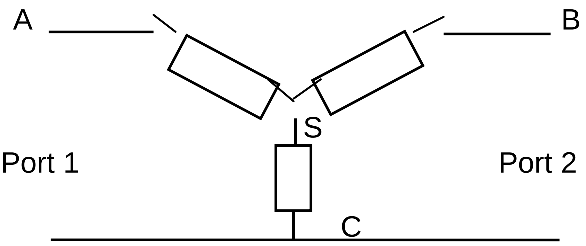
$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0.2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3+j4 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0.2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0.2 & 1 \end{bmatrix} \begin{bmatrix} 1.6+j0.8 & 3+j4 \\ 0.2 & 1 \end{bmatrix}$$

$$i.e. \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1.6+j0.8 & 3+j4 \\ 0.52+j0.16 & 1.6+j0.8 \end{bmatrix}$$

[2 marks]

(e) Convert the circuit shown in figure Q3de to an equivalent star-connected network between nodes A, B and C and redraw the circuit.





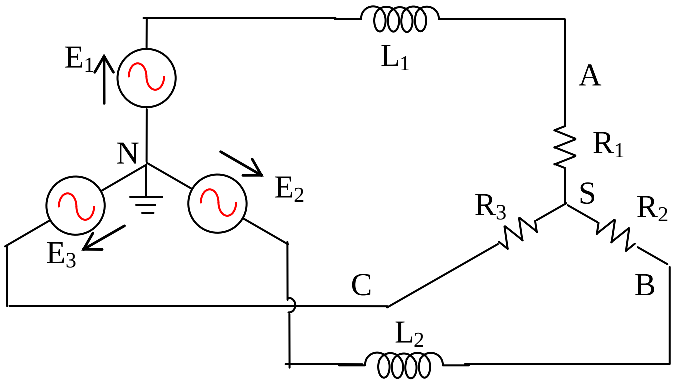
$Z_{AS} = Z_{AB} \times Z_{AC} / (Z_{AB} + Z_{BC} + Z_{CA}) = (3+j4) \times 5/(3+j4+5+5) = 1.838\angle 36.0^0 \Omega$

$Z_{BS} = Z_{BA} \times Z_{BC} / (Z_{AB} + Z_{BC} + Z_{CA}) = (3+j4) \times 5/(3+j4+5+5) = 1.838\angle 36.0^0 \Omega$

$Z_{CS} = Z_{CB} \times Z_{CA} / (Z_{AB} + Z_{BC} + Z_{CA}) = 5 \times 5/(3+j4+5+5) = 1.838\angle 17.1^0 \Omega$ **[2 marks]**

Question 4

(a) The supply shown in figure Q4(a) is a balanced 3phase, 400V, 50 Hz source. If $L_1 = L_2 = 100$ mH, and $R_1 = R_2 = R_3 = 100\ \Omega$, determine the voltage of the star point S.



$L_1 = L_2 = 100\ mH \rightarrow 100 \times 10^{-3} \times 100\pi = j\ 31.416\ \Omega$

$$V_S = V_{SN} = \frac{\sum Y \cdot V}{\sum Y} = \frac{\frac{230\angle 0^0}{100 + j31.416} + \frac{230\angle -120^0}{100} + \frac{230\angle 120^0}{100 + j31.416}}{\frac{1}{100 + j31.416} + \frac{1}{100} + \frac{1}{100 + j31.416}}$$

$= 23.95\angle -155.98^0$ V

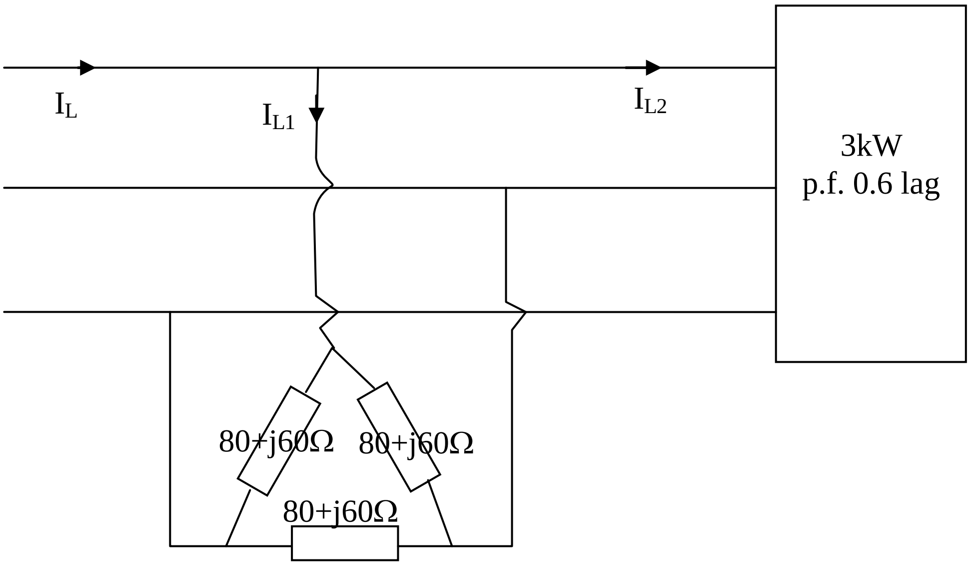
$$I_A = \frac{230\angle 0^0 - 23.95\angle -155.98^0}{100 + j31.416} = 2.41\angle -15.22^0 V$$ **[4 marks]**

Hence determine the potential at A

$V_A = V_S + 100\ I_A = 23.95\angle -155.98^0 + 100 \times 2.41\angle -15.22^0$

$= 223.3\angle -19.1^0$ V **[2 marks]**

(b) A balanced 400 V, 3-phase, 50 Hz supply feeds (i) a 3-phase delta-connected balanced load consisting of arms of value (80+j60) Ω each and (ii) a three phase motor load of 3 kW at a power factor of 0.6 lag, Determine the line current, power factor and the active power at the supply.



$$I_{L1} = \sqrt{3} \times \frac{400}{80 + j60} = 6.928\angle -36.9^0\,A$$

$$I_{L2} = \frac{3000}{\sqrt{3} \times 400 \times 0.6} = 7.217\angle -53.1^0\,A$$

$I_L = I_{L1} + I_{L2} = 6.928\angle\text{-}36.9^0 + 7.217\angle\text{-}53.1^0 = 14.0\angle\text{-}45.16^0$

Power factor = 0.705 lag

Active Power = $\sqrt{3}\times400\times14.0\times0.705 = 6839\ W$ **[3 marks]**

Determine also the rating of the delta connected capacitor bank required to improve the overall power factor to 0.95 lagging.

Reactive Power = $\sqrt{3}\times400\times14.0\times\sin 45.15^0 = 6827$ *var*

New power factor = $0.95 \rightarrow 18.19^0$

New reactive Power = $6839\times\tan 18.19^0 = 2248$ *var*

Rating of capacitor bank = 6827 – 2248 = 4579 var

Rating of each capacitor = 4579/3 = 1526 var **[2 marks]**

[or $100\times\pi\times C\times400^2 = 1526 \rightarrow 30.36\ \mu F$ *– not necessary]*

(c) The phase components of the currents in an unbalanced system A-B-C is given as $I_A = 20\angle -30^0$ A, $I_B = 15\angle 60^0$ A and $I_C = 20\angle -120^0$ A. Determine the sequence components of the currents.

$$\begin{bmatrix} I_{A0} \\ I_{A1} \\ I_{A2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}\begin{bmatrix} 20\angle -30^0 \\ 15\angle 60^0 \\ 20\angle -120^0 \end{bmatrix}$$

$I_{A0} = 1/3(20\angle\text{-}30^0 + 15\angle 60^0 + 20\angle\text{-}120^0) = 6.87\angle\text{-}44.03^0 A$

$I_{A1} = 1/3(20\angle\text{-}30^0 + 15\angle 180^0 + 20\angle 120^0) = 3.54\angle 136.4^0 A$

$I_{A2} = 1/3(20\angle\text{-}30^0 + 15\angle 300^0 + 20\angle 0^0) = 16.79\angle\text{-}27.16^0 A$ **[2 marks]**

If the corresponding sequence components of the voltages are $V_{A0} = 50\angle{-30^0}$ V, $V_{A1} = 200\angle 0^0$ V, $V_{A2} = 50\angle{-30^0}$ V, determine the total active power. *[1 mark]*

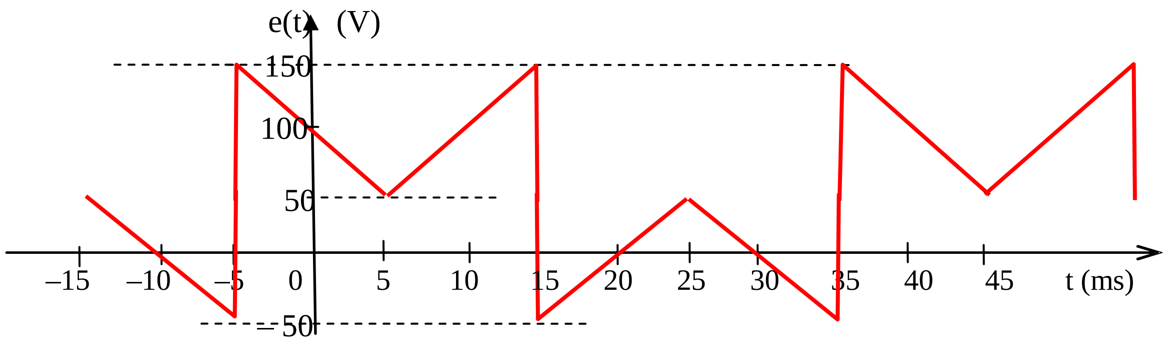
$P = 3\, V_{A0}\, I_{A0} \cos\phi_0 + 3\, V_{A1}\, I_{A1} \cos\phi_1 + 3\, V_{A2}\, I_{A2} \cos\phi_2$

$P = 3(50\times 6.87 \cos 14.03^0 + 200\times 3.54 \cos 136.4^0 + 50\times 16.79 \cos 2.84^0$

$= 1979\ W$

Question 5

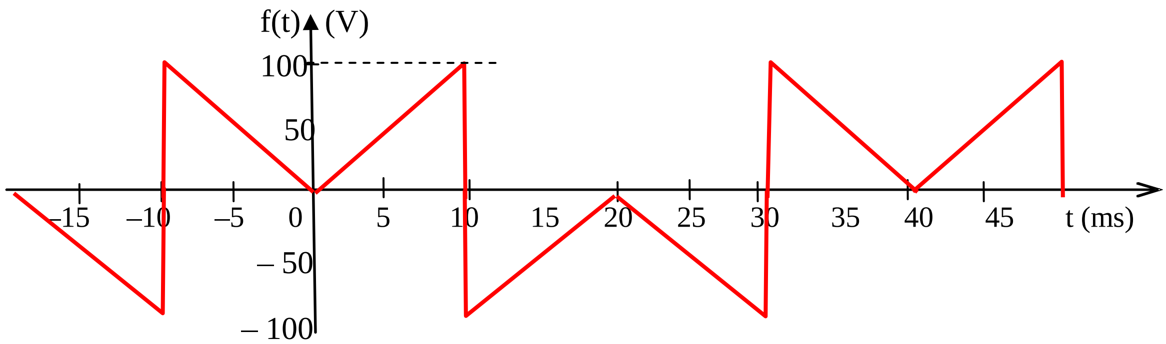
(a) For the voltage waveform e(t) shown in figure Q5a, write down any simplifications that can be used to determine the Fourier series and determine the first 4 significant terms of the Fourier series.



Shift waveform downwards by 50 V and left by 5ms (becomes even and gives a cosine series) [or right by 5ms - becomes odd and gives a sine series)

New waveform f(t), period T = 40 ms

$F(t) = e(t - 0.005) - 50$ **[2 marks]**



Waveform f(t) has even symmetry and half-wave symmetry with mean value zero.

Thus for f(t), the Fourier coefficients are given as

$A_o = 0$ *(since mean value = 0)*, $B_n = 0$ *(since even function)*,

$A_n = 0$ *for even n (since half wave symmetry)*

$$f(t) = 10^4 t \qquad 0 \le t \le 0.01$$

Thus for odd n,

$$A_n = 4\times\frac{2}{T}\int_0^{\frac{T}{4}} f(t).\cos n\omega_o t dt = \frac{8}{0.04}\int_0^{0.01} 10^4 t\cdot\cos n50\pi t.dt$$

$$A_n = 2\times10^6\left[\left.\frac{t\cdot\sin n50\pi t}{n\cdot 50\pi}\right|_0^{0.01} - \int_0^{0.01}\frac{\sin n50\pi t}{n\cdot 50\pi}\right] = 40\times10^3\left[\frac{0.01\times\sin\frac{n\pi}{2}}{n\cdot\pi} + \left.\frac{\cos n50\pi t}{n\cdot\pi}\right|_0^{0.01}\right]$$

$$= \frac{40\times10^3}{n\pi}\left[0.01\times\sin\frac{n\pi}{2} - \frac{1}{50n\pi}\right] = \frac{400}{n\pi}\sin\frac{n\pi}{2} - \frac{800}{(n\pi)^2} \quad \textit{for odd } n$$

[3 marks]

$$A_1 = \frac{400}{\pi}\sin\frac{\pi}{2} - \frac{800}{\pi^2} = \frac{400}{\pi} - \frac{800}{\pi^2} = 46.27$$

$$A_3 = \frac{400}{3\pi}\sin\frac{3\pi}{2} - \frac{800}{9\pi^2} = -\frac{400}{3\pi} - \frac{800}{9\pi^2} = -51.44$$

$$A_5 = \frac{400}{5\pi}\sin\frac{5\pi}{2} - \frac{800}{25\pi^2} = \frac{80}{\pi} - \frac{32}{\pi^2} = 22.22$$

$$A_7 = \frac{400}{7\pi}\sin\frac{7\pi}{2} - \frac{800}{49\pi^2} = -\frac{400}{7\pi} - \frac{800}{49\pi^2} = -19.84$$ **[1 mark]**

Thus the Fourier series of f(t) is

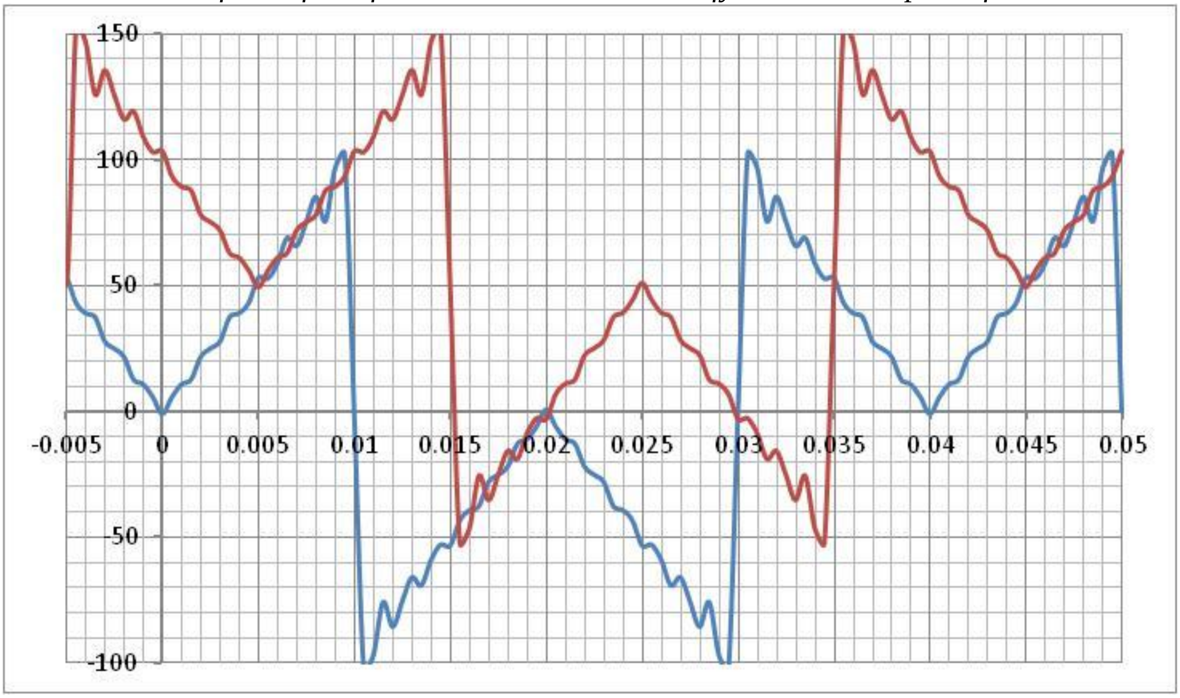
f(t) = 46.27cos50πt – 51.44 cos 150πt + 22.22 cos 250πt –19.84 cos350πt + …

and the Fourier series of the original waveform is

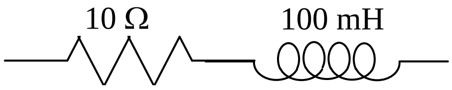
e(t) = 50 + e(t+0.005)

e(t) =50 + 46.27cos50π(t+0.005) – 51.44 cos 150π(t+0.005) + 22.22 cos 250π(t+0.005) –19.84 cos350π(t+0.005) + … **[1 mark]**

Reconstruction of waveform from Fourier series to verify answer. Not part of answer.



(b) If a current i(t) = 1 + 10 sin 100t + 3 sin (300t+π/3) + 2 sin(500t – π/6) is passed through a series combination of a resistor R = 10 Ω and an inductor L = 100 mH, determine the Fourier series of the resulting voltage v(t).



$V(t) = R.i(t) + L.di(t)/dt$

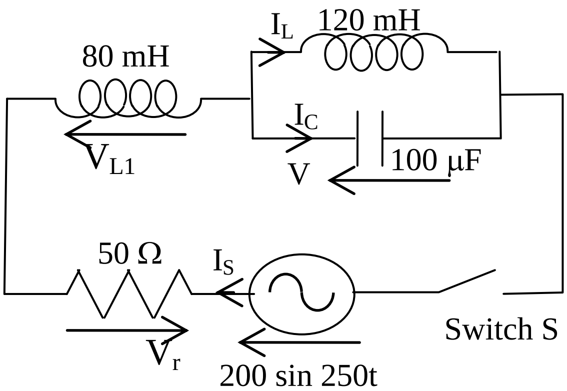
$= 10\times(1 + 10\sin 100t + 3\sin(300t+\pi/3) + 2\sin(500t - \pi/6))$

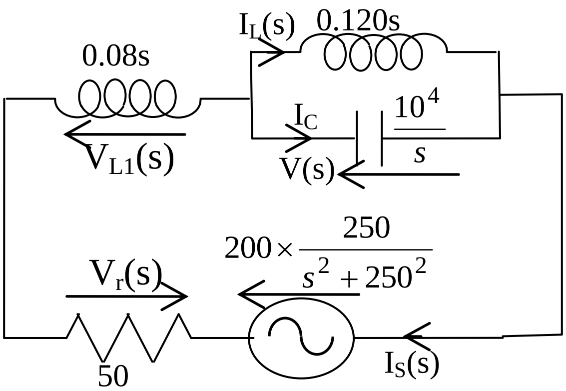
$+ 0.1\times(10\times100\times\cos 100t + 3\times300\times\cos(300t+\pi/3) + 2\times500\times\cos(500t - \pi/6))$

$= 10 + 141.4\sin(100t+\pi/4) + 94.8\sin(300t+\pi/3+71.56^0) + 101\sin(500t - \pi/6)$

[2 marks]

(c) Figure Q5c shows a circuit which is switched at time t = 0. Draw the corresponding Laplace transformed circuit





[1 mark]

Write an expression for the current $I_S(s)$ in the Laplace domain

$$I(s) = \frac{200\times\frac{250}{s^2+250^2}}{50 + 0.08s + \frac{0.12s\times\frac{10^4}{s}}{0.12s + \frac{10^4}{s}}} = \frac{200\times250\times(0.12s^2+10^4)}{(s^2+250^2)\left[(50+0.08s)(0.12s^2+10^4)+0.12\times10^4\right]}$$

[2 marks]

Hence show how the corresponding current $i_S(t)$ may be determined.

$$I(s) = \frac{As+B}{s^2+250^2} + \frac{C}{s+\alpha} + \frac{D}{s+\beta} + \frac{E}{s+\gamma}$$

$$i(t) = A\cos 250t + \frac{B}{250}\sin 250t + C\cdot e^{-\alpha t} + D\cdot e^{-\beta t} + E\cdot e^{-\gamma t}$$

[2 marks]

[END OF QUESTION PAPER]