University of Moratuwa, Sri Lanka
Faculty of Engineering
Department of Electrical Engineering
B. Sc. Engineering Honours Degree Course
Semester 2 Examination

EE2092 – THEORY OF ELECTRICITY

Time Allowed: 2 Hours May 2012

Additional Material

Graph Paper will be provided if required.
A table of Laplace transforms is provided on the other side of this page.

Instructions to Candidates

This paper contains 5 questions in 6 pages, including the cover page.
Answer All Questions.
This examination accounts for 70% of the module assessment.
Each question carries a total 14 marks. Maximum marks allocated for each part of a question is indicated in square brackets at the end of the part.
Total allocation for the paper is 70 marks.
This is a closed book examination and only authorised calculators will be permitted.

Technical Data:

Permeability of free space μ_0 $= 4\pi \times 10^{-7}$ H/m
Permittivity of free space ε_0 $= 8.854 \times 10^{-12}$ F/m
Velocity of light in free space $= 2.998 \times 10^{8}$ m/s

Table of Laplace Transforms of common causal functions *f(t)*

$f(t)$	$F(s) = \mathrm{L}[f(t)]$
Unit impulse – δt	1
Unit step – $U(t)$	$\frac{1}{s}$
t	$\frac{1}{s^2}$
t^n	$\frac{n!}{s^{n+1}}$
e^{-at}	$\frac{1}{(s+a)}$
$1-e^{-at}$	$\frac{a}{s(s+a)}$
te^{-at}	$\frac{1}{(s+a)^2}$
$t^n e^{-at}$	$\frac{n!}{(s+a)^{n+1}}$
$e^{-at}-e^{-bt}$	$\frac{b-a}{(s+a)(s+b)}$
$\sin(\omega t)$	$\frac{\omega}{(s^2+\omega^2)}$
$\sin(\omega t+\phi)$	$\frac{\omega\cos(\phi)+s\sin(\phi)}{(s^2+\omega^2)}$
$t\sin(\omega t)$	$\frac{2\omega s}{(s^2+\omega^2)^2}$
$\cos(\omega t)$	$\frac{s}{(s^2+\omega^2)}$
$\cos(\omega t+\phi)$	$\frac{s\cos(\phi)-\omega\sin(\phi)}{(s^2+\omega^2)}$
$t\cos(\omega t)$	$\frac{s^2-\omega^2}{(s^2+\omega^2)^2}$
$e^{-at}\sin(\omega t)$	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{-at}\cos(\omega t)$	$\frac{s+a}{(s+a)^2+\omega^2}$
$\sinh(\omega t)$	$\frac{\omega}{(s^2-\omega^2)}$
$\cosh(\omega t)$	$\frac{s}{(s^2-\omega^2)}$

Question 1

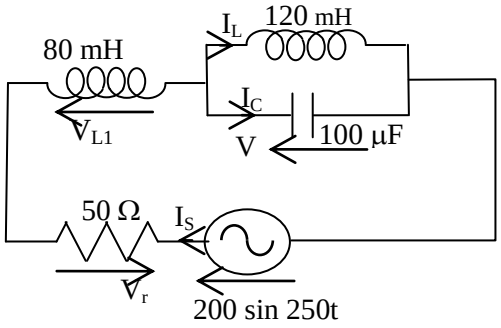

Figure Q1

For the circuit shown in figure Q1,

(a) Write down the differential equations governing the behaviour of the circuit. *[2 marks]*

(b) Calculate the total impedance of the circuit in ohm. *[2 marks]*

(c) Calculate the current I_S supplied from the source. *[1 mark]*

(d) Determine the currents I_L and I_C. *[2 marks]*

(e) Determine the voltages V_r, V_{L1} and V. *[2 marks]*

(f) Sketch a phasor diagram showing all the voltages and currents in the circuit. *[3 marks]*

(g) If the supply frequency is allowed to vary, determine the frequency at which series resonance would occur. *[2 marks]*

Question 2

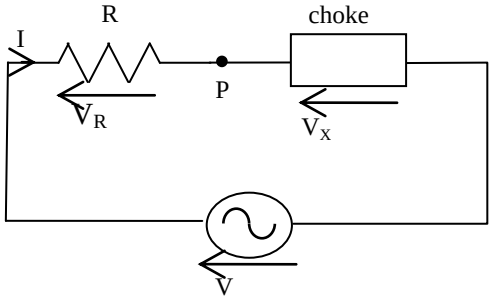

Figure Q2ab

(a) A practical choke (inductance L and series resistance r), connected in series with a pure resistance R, is supplied from a constant voltage source (V,ω). If the resistance R is allowed to vary, sketch the locus of the voltage of the intersection P between the choke and the resistor. *[3 marks]*

(b) One of the readings in Q2(a) corresponds to V = 250V at 50 Hz, current I = 10A lagging voltage by 30^0, the voltage magnitudes across the resistor V_R = 150V and choke V_X = 141.6V. Complete the phasor diagram and calculate the inductance L and the resistance r of the choke. *[3 marks]*

(c) Derive the non-coupled equivalent circuit of a two winding transformer. *[2 marks]*
A 80/20kV transformer has a primary winding inductance of 16H, and a coupling coefficient of 0.95. Using any standard equations derive the values of the inductive impedances of all the elements of the non-coupled equivalent circuit, *[2 marks]*

(d) Sketch the equivalent circuit of an operational amplifier and show that the voltage difference of the input terminals may be taken as a virtual zero. *[2 marks]*

(e) It is required to construct a summing amplifier to add 75% of input A voltage and 25% of input B voltage. If the resistance connected across the operational amplifier is 1 kΩ, use a diagram to show what should be the input resistances to be connected for the two inputs? *[2 marks]*

Question 3

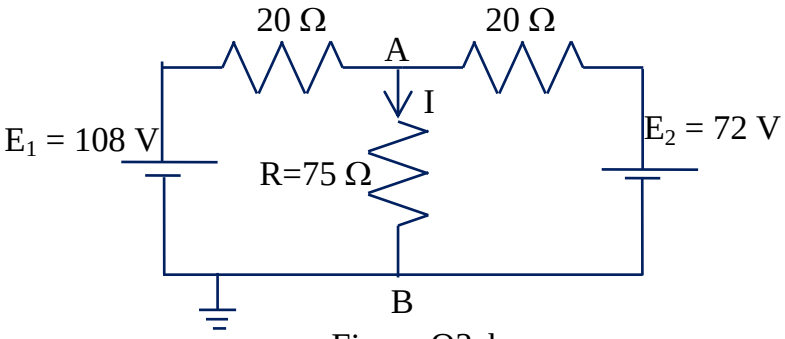

Figure Q3ab

(a) For the circuit shown in figure Q3ab, obtain the Norton's equivalent circuit across AB and hence obtain the current I. *[3 marks]*

(b) Using compensation theorem, determine the resistance to which R must be changed if the desired current I in figure Q3ab is 1.0A. *[2 marks]*

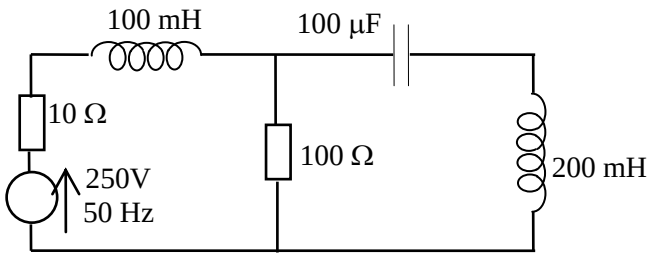

Figure Q3c

(c) For the circuit shown in figure Q3c, write down the branch-mesh incidence matrix and the branch impedance matrix. *[3 marks]*

Hence determine the mesh impedance matrix. *[2 marks]*

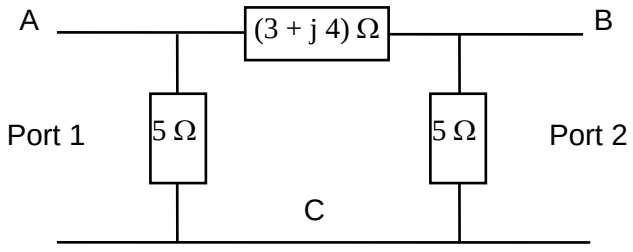

Figure Q3de

(d) Determine the ABDC matrix of the two-port network shown in figure Q3de. *[2 marks]*

(e) Convert the circuit shown in figure Q3de to an equivalent star-connected network between nodes A, B and C and redraw the circuit. *[2 marks]*

Question 4

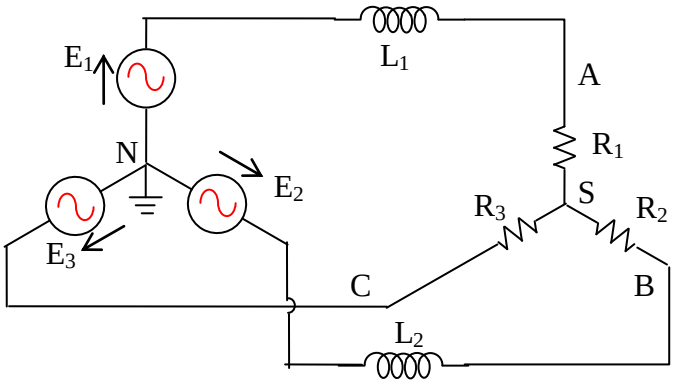

Figure Q4(a)

(a) The supply shown in figure Q4(a) is a balanced 3phase, 400V, 50 Hz source. If $L_1 = L_2 = 100$ mH, and $R_1 = R_2 = R_3 = 100\ \Omega$, determine the voltage of the star point S. *[4 marks]*

Hence determine the potential at A. *[2 marks]*

(b) A balanced 400 V, 3-phase, 50 Hz supply feeds (i) a 3-phase delta-connected balanced load consisting of arms of value (80+j60) Ω each and (ii) a three phase motor load of 3 kW at a power factor of 0.6 lag, Determine the line current, power factor and the active power at the supply. *[3 marks]*

Determine also the rating of the delta connected capacitor bank required to improve the overall power factor to 0.95 lagging. *[2 marks]*

(c) The phase components of the currents in an unbalanced system A-B-C is given as $I_A = 20\angle{-30^0}$ A, $I_B = 15\angle{60^0}$ A and $I_C = 20\angle{-120^0}$ A. Determine the sequence components of the currents. *[2 marks]*

If the corresponding sequence components of the voltages are $V_{A0} = 50\angle{-30^0}$ V, $V_{A1} = 200\angle{0^0}$ V, $V_{A0} = 50\angle{-30^0}$ V, determine the total active power. *[1 mark]*

Question 5

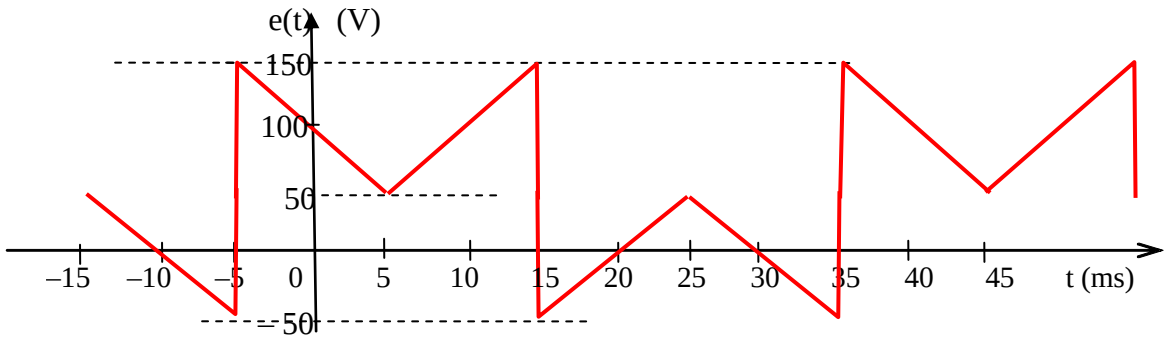

Figure Q5a

(a) For the voltage waveform e(t) shown in figure Q5a, write down any simplifications that can be used to determine the Fourier series and determine the first 4 significant terms of the Fourier series. *[7 marks]*

(b) If a current i(t) = 1 + 10 sin 100t + 3 sin (300t+π/3) + 2 sin(500t – π/6) is passed through a series combination of a resistor R = 10 Ω and an inductor L = 100 mH, determine the Fourier series of the resulting voltage v(t). *[2 mark]*

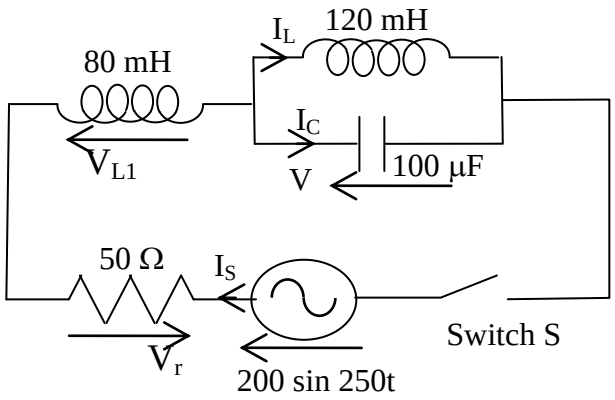

Figure Q5c

(c) Figure Q5c shows a circuit which is switched at time t = 0. Draw the corresponding Laplace transformed circuit *[1 mark]*

Write an expression for the current $I_s(s)$ in the Laplace domain *[2 marks]*

Hence show how the corresponding current $i_s(t)$ may be determined. *[2 marks]*

[END OF QUESTION PAPER]