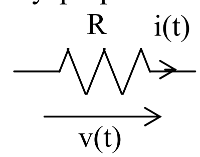
Network Theorems - *J. R. Lucas*

The fundamental laws that govern electric circuits are the Ohm's Law and the Kirchoff's Laws.

Ohm's Law

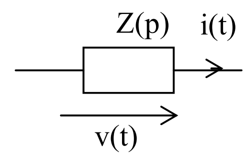
Ohm's Law states that the voltage *v(t)* across a resistor R is directly proportional to the current *i(t)* flowing through it.



$$v(t) \propto i(t)$$

or $$v(t) = R \cdot i(t)$$

This general statement of Ohm's Law can be extended to cover inductances and capacitors as well under alternating current conditions and transient conditions. This is then known as the Generalised Ohm's Law. This may be stated as



$v(t) = Z(p) \cdot i(t)$, where $p = d/dt$ = differential operator

Z(p) is known as the impedance function of the circuit, and the above equation is the differential equation governing the behaviour of the circuit.

For a resistor, $Z(p) = R$

For an inductor $Z(p) = L\,p$

For a capacitor, $Z(p) = \dfrac{1}{C\,p}$

In the particular case of alternating current, $p = j\omega$ so that the equation governing circuit behaviour may be written as

$V = Z(j\omega) \cdot I$, and

For a resistor, $Z(j\omega) = R$

For an inductor $Z(j\omega) = j\omega L$

For a capacitor, $Z(j\omega) = \dfrac{1}{j\omega C}$

We cannot analyse electric circuits using Ohm's Law only. We also need the Kirchoff's current law and the Kirchoff's voltage law.

Kirchoff's Current Law

Kirchoff's current law is based on the principle of conservation of charge. This requires that the algebraic sum of the charges within a system cannot change. Thus the total rate of change of charge must add up to zero. Rate of change of charge is current.

This gives us our basic Kirchoff's current law as the algebraic sum of the currents meeting at a point is zero.

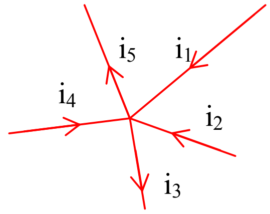
i.e. at a node, $\Sigma I_r = 0$, where I_r are the currents in the branches meeting at the node

This is also sometimes stated as the sum of the currents entering a node is equal to the sum of the current leaving the node.

The theorem is applicable not only to a node, but to a closed system.

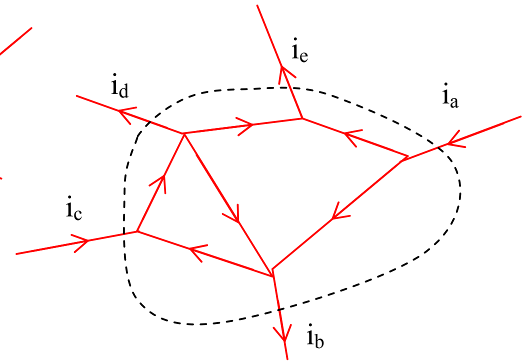
$i_1 + i_2 - i_3 + i_4 - i_5 = 0$

$i_1 + i_2 + i_4 = i_3 + i_5$



Also for the closed boundary,

$i_a - i_b + i_c - i_d - i_e = 0$



Kirchoff's Voltage Law

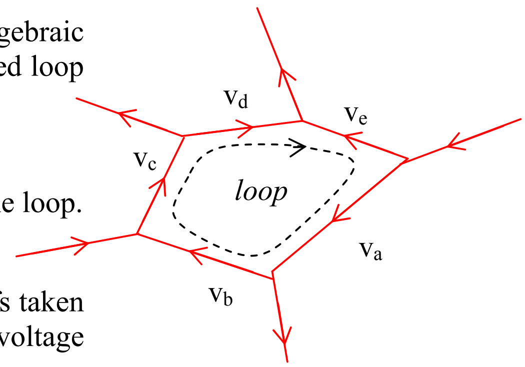
Kirchoff's voltage law is based on the principle of conservation of energy. This requires that the total work done in taking a unit positive charge around a closed path and ending up at the original point is zero.

This gives us our basic Kirchoff's law as the algebraic sum of the potential differences taken round a closed loop is zero.

i.e. around a loop, $\Sigma V_r = 0$,

where V_r are the voltages across the branches in the loop.

$v_a + v_b + v_c + v_d - v_e = 0$

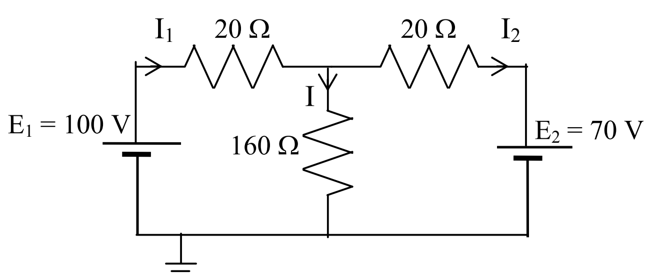


This is also sometimes stated as the sum of the emfs taken around a closed loop is equal to the sum of the voltage drops around the loop.

Although all circuits could besolved using only Ohm's Law and Kirchoff's laws, the calculations would be tedious. Various network theorems have been formulated to simplify these calculations.

Example 1

For the purposes of understanding the principle of the Ohm's Law and the Kirchoff's Laws and their applicability, we will consider only a resistive circuit. However it must be remembered that the laws are applicable to alternating currents as well.



For the circuit shown in the figure, let us use Ohm's Law and Kirchoff's Laws to solve for the current I in the 160 Ω resistor.

Using Kirchoff's current law

$I = I_1 - I_2$

Using Kirchoff's voltage law

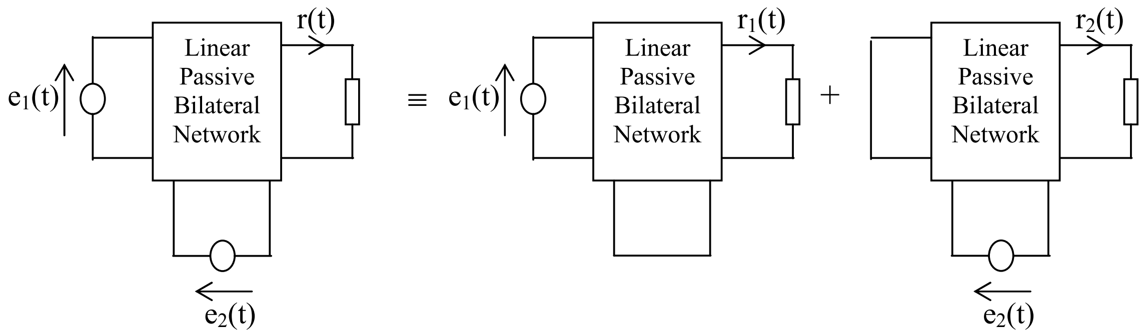
$100 = 20\, I_1 + 160\, (I_1 - I_2) \quad \Rightarrow \quad 10 = 18\, I_1 - 16\, I_2$

$-70 = 20\, I_2 - 160\, (I_1 - I_2) \quad \Rightarrow \quad 7 = 16\, I_1 - 18\, I_2$

which has the solution $I_1 = 1$ A, $I_2 = 0.5$ A and the unknown current I = 0.5A.

Superposition Theorem

The superposition theorem tells us that if a network comprises of more than one source, the resulting currents and voltages in the network can be determined by taking each source independently and superposing the results.



If an excitation $e_1(t)$ alone gives a response $r_1(t)$,

and an excitation $e_2(t)$ alone gives a response $r_2(t)$,

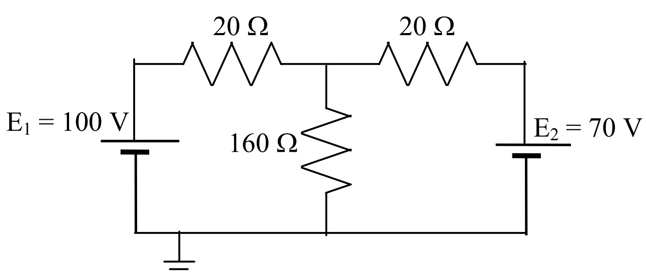
then, by superposition theorem, if the excitation $e_1(t)$ and the excitation $e_2(t)$ together would give a response $r(t) = r_1(t) + r_2(t)$

The superposition theorem can even be stated in a more general manner, where the superposition occurs with scaling.

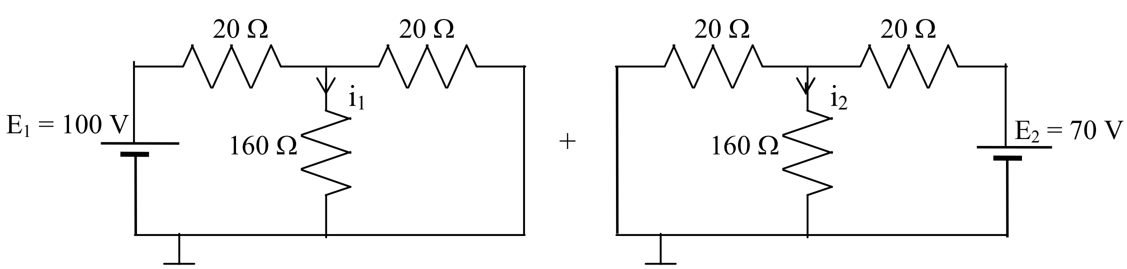
Thus an excitation of $k_1\, e_1(t)$ and an excitation of $k_2\, e_2(t)$ occuring together would give a response of $k_1\, r_1(t) + k_2\, r_2(t)$.

Example 2

Let us solve the same problem as earlier, but using Superposition theorem.



Solution



for circuit 1, source current $= \dfrac{100}{20 + 160 // 20} = \dfrac{100}{20 + \dfrac{160 \times 20}{180}} = \dfrac{100}{37.778} = 2.647\ A$

$\therefore i_1 = 2.647 \times \dfrac{20}{180} = 0.294$ A

Similarly for circuit 2, source current = $\dfrac{70}{20+160//20} = \dfrac{70}{20+\dfrac{160\times 20}{180}} = \dfrac{70}{37.778} = 1.853\,A$

$\therefore i_2 = 1.853\times\dfrac{20}{180} = 0.206$ A

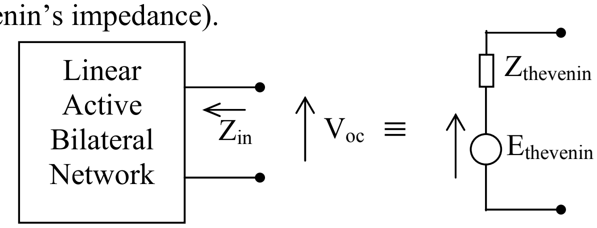
$\therefore$ unknown current $i = i_1 + i_2 = 0.294 + 0.206 = 0.500$ A

which is the same answer that we got from Kirchoff's Laws and Ohm's Law.

Thevenin's Theorem (or Helmholtz's Theorem)

The Thevenin's theorem, basically gives the equivalent voltage source corresponding to an active network.

If a linear, active, bilateral network is considered across one of its ports, then it can be replaced by an equivalent voltage source (Thevenin's voltage source) and an equivalent series impedance (Thevenin's impedance).



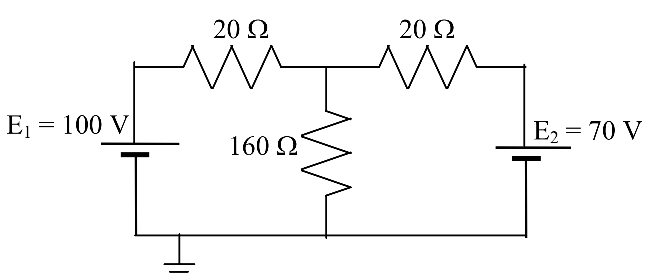
Since the two sides are identical, they must be true for all conditions. Thus if we compare the voltage across the port in each case under open circuit conditions, and measure the input impedance of the network with the sources removed (voltage sources short-circuited and current sources open-circuited), then

$E_{thevenin} = V_{oc}$, and

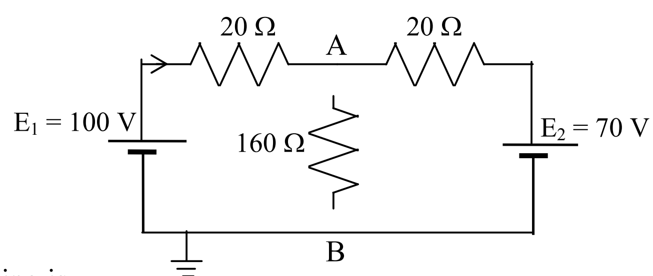
$Z_{thevenin} = Z_{in}$

Example 2

Let us again consider the same example to illustrate Thevenin's Theorem.



Solution



Since we wish to calculate the current in the 160 Ω resistor, let us find the Thevenin's equivalent circuit across the terminals after disconnecting (open circuiting) the 160 Ω resistor.

Under open circuit conditions, current flowing is

$= (100 - 70)/40 = 0.75$ A

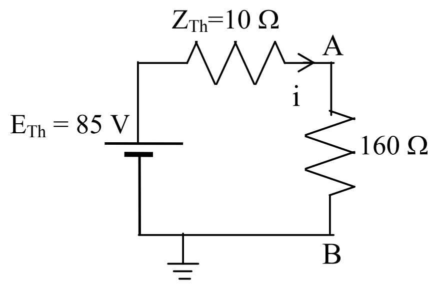
$\therefore \quad V_{oc,AB} = 100 - 0.75\times 20 = 85$ V

$\therefore$ $E_{Th} = 85$ V

The input impedance across AB (with sources removed) = 20//20 = 10 Ω.

$\therefore$ $Z_{Th} = 10$ Ω.

Therefore the Thevenin's equivalent circuit may be drawn with branch AB reintroduced as follows.



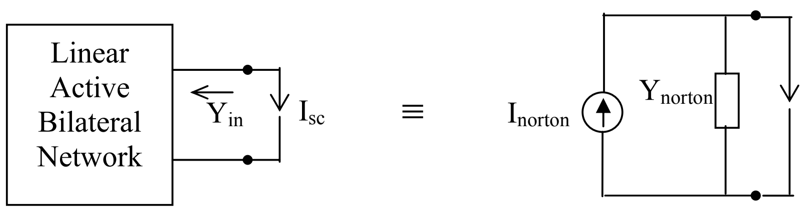
From the equivalent circuit, the unknown current i is determined as

$$i = \frac{85}{10 + 160} = 0.5\,A$$

which is the same result that was obtained from the earlier two methods.

Norton's Theorem

Norton's Theorem is the dual of Thevenin's theorem, and states that any linear, active, bilateral network, considered across one of its ports, can be replaced by an equivalent current source (Norton's current source) and an equivalent shunt admittance (Norton's Admittance).



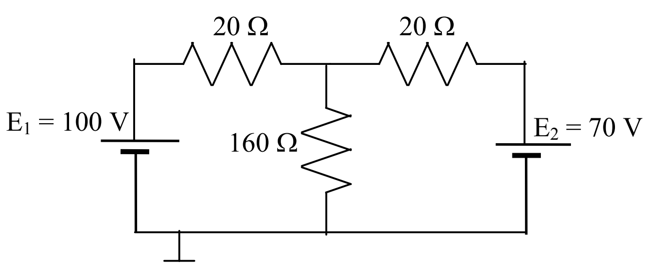
Since the two sides are identical, they must be true for all conditions. Thus if we compare the current through the port in each case under short circuit conditions, and measure the input admittance of the network with the sources removed (voltage sources short-circuited and current sources open-circuited), then

$$I_{norton} = I_{sc}\,,\quad \text{and}$$

$$Y_{norton} = Y_{in}$$

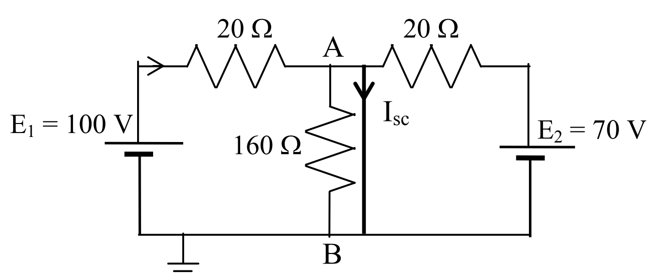
Example 3

Let us again consider the same example to illustrate Norton's Theorem.



Solution

Since we wish to calculate the current in the 160 Ω resistor, let us find the Norton's equivalent circuit across the terminals after short-circuiting the 160 Ω resistor.



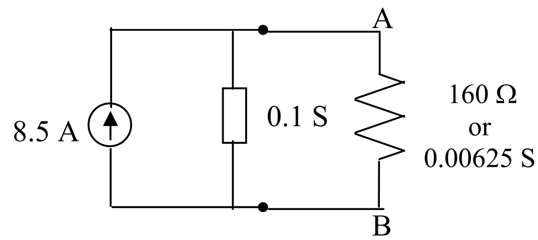
the short circuit current I_{sc} is given by

$I_{sc} = 100/20 + 70/20 = 8.5$ A

$\therefore \quad I_{norton} = 8.5$ A

Norton's admittance = 1/20 + 1/20 = 0.1 S

$\therefore$ Norton's equivalent circuit is



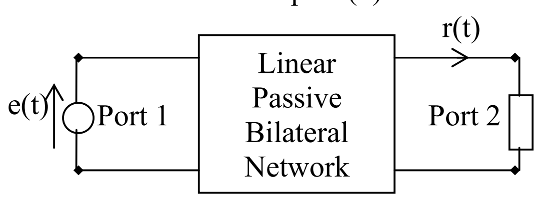
and the current in the unknown resistor is $8.5 \times \dfrac{0.00625}{0.1 + 0.00625} = 0.5\,A$

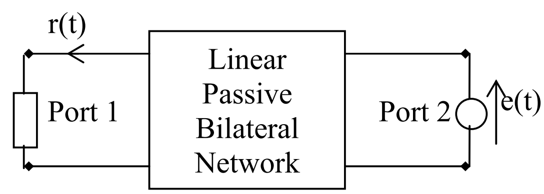
which is the same result as before.

Reciprocality Theorem

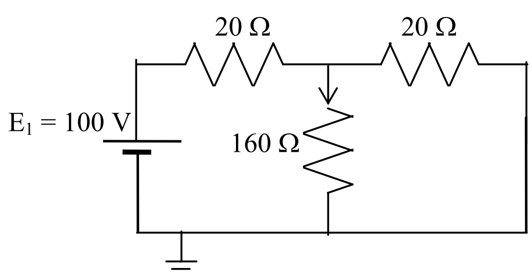
The reciprocality theorem tells us that in a linear passive bilateral network an excitation and the corresponding response may be interchanged.

In a two port network, if an excitation e(t) at port (1) produces a certain response r(t) at a port (2), then if the same excitation e(t) is applied instead to port (2), then the same response r(t) would occur at the other port (1).



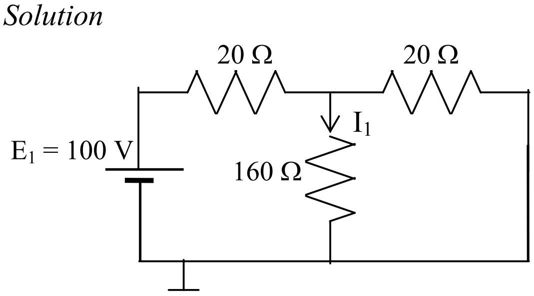


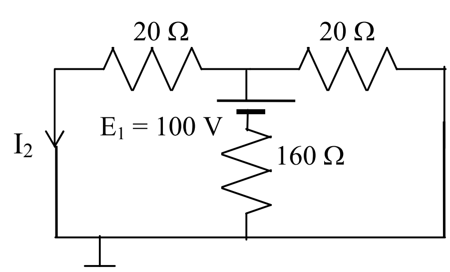
Example 4



Consider the earlier example, but with only one source. Determine the current in the 160 Ω branch. Now replace the 160 Ω resistor with the source in series with it and after short-circuit the source at the original location, find the current flowing at the original source location. Show that it verifies the Reciprocality theorem.

Solution





For the original circuit, current $I_1 = \dfrac{100}{20 + 160 // 20} \times \dfrac{20}{20 + 160} = \dfrac{2000}{37.778 \times 180} = 0.294\,A$

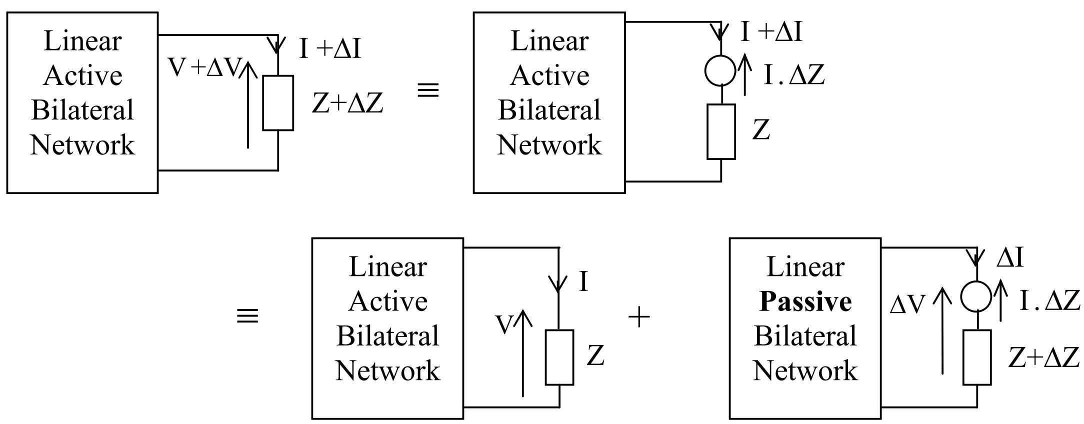
similarly for the new circuit, current $I_2 = \dfrac{100}{160 + 20//20} \times \dfrac{20}{20+20} = \dfrac{2000}{170 \times 40} = 0.294\,A$

It is seen that the identical current has appeared verifying the reciprocality theorem. The advantage of the theorem is when a circuit has already been analysed for one solution, it may be possible to find a corresponding solution without further work.

Compensation Theorem

In many circuits, after the circuit is analysed, it is realised that only a small change need to be made to a component to get a desired result. In such a case we would normally have to recalculate. The compensation theorem allows us to compensate properly for such changes without sacrificing accuracy.

In any linear bilateral active network, if any branch carrying a current I has its impedance Z changed by an amount ΔZ, the resulting changes that occur in the other branches are the same as those which would have been caused by the injection of a voltage source of (-) $I \,.\, \Delta Z$ in the modified branch.



Consider the voltage drop across the modified branch.

$$V+\Delta V = (Z+\Delta Z)(I+\Delta I) = Z \,.\, I + \Delta Z \,.\, I + (Z + \Delta Z) \,.\, \Delta I$$

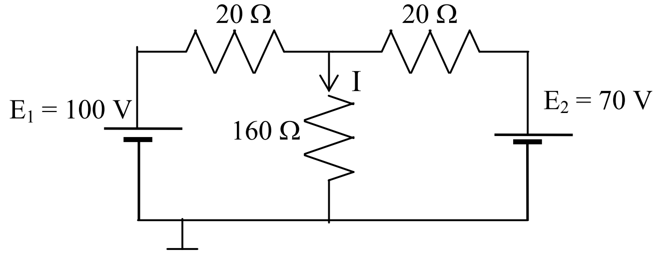
from the original network, $V = Z \,.\, I$

$$\therefore \quad \Delta V = \Delta Z \,.\, I + (Z + \Delta Z) \,.\, \Delta I$$

Since the value I is already known from the earlier analysis, and the change required in the impedance, ΔZ , is also known, $I \,.\Delta Z$ is a known fixed value of voltage and may thus be represented by a source of emf $I.\, \Delta Z$.

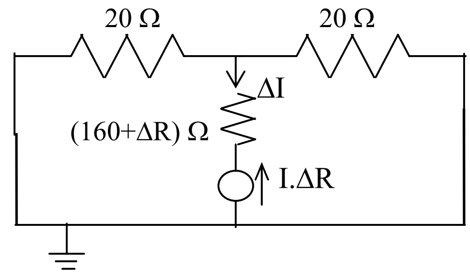
Using superposition theorem, we can easily see that the original sources in the active network give rise to the original current I, while the change corresponding to the emf $I.\, \Delta Z$ must produce the remaining changes in the network.

Example 5



From example 4, we saw that the current in the 160 Ω resistor is 0.5 A.

Let us say that we want to change the resistor by a quantity ΔR such that the current in the 160 Ω resistor is 0.600 A. Then the circuit for changes can be written as



$\Delta I = 0.6 - 0.5 = 0.1$ A

$I = 0.5$

$$\therefore \quad \Delta I = \frac{(-)0.5 \times \Delta R}{160 + \Delta R + 20 // 20}$$

$$\text{i.e.} \quad 0.1 = (-)\frac{0.5 \times \Delta R}{170 + \Delta R}$$

$\therefore\ 17 + 0.1\ \Delta R = (-)\ 0.5\ \Delta R$

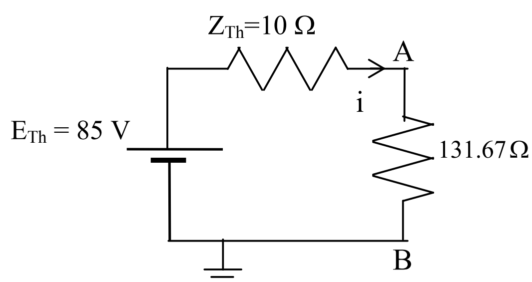
i.e. $\Delta R = (-)17/0.6 = (-)\ 28.333\ \Omega$

Therefore the required value of R = 160 – 28.333 = 131.67 Ω

This could have been calculated using Kirchoff's and Ohm's laws but would have been more complicated.

We can also check this answer with Thevenin's theorem as follows.

From Example 2, we had the Thevenin's circuit as shown, with the 160 Ω replaced by 131.67 Ω.



The current for this value can be quickly obtained as $i = \dfrac{85}{10 + 131.667} = 0.6\,A$

So you can also see that by knowing Thevenin's equivalent circuit for a given network, we can obtain solutions for many conditions with little additional calculations.

The same is true with Norton's theorem.

Maximum Power Transfer Theorem

As you are probably aware, a normal car battery is rated at 12 V and generally has an open circuit voltage of around 13.5 V. Similarly, if we take 7 pen-torch batteries, they too will have a terminal voltage of 7×1.5 = 13.5 V. However, you would also be aware, that if your car battery is dead, you cannot go to the nearest shop, buy 7 pen-torch batteries and start your car. Why is that ? Because the pen-torch batteries, although having the same open circuit voltage does not have the necessary power (or current capacity) and hence the required current could not be given. Or if stated in different terms, it has too high an internal resistance so that the voltage would drop without giving the necessary current.

This means that a given battery (or any other energy supply, such as the mains) can only give a limited amount of power to a load. The maximum power transfer theorem defines this power, and tells us the condition at which this occurs.

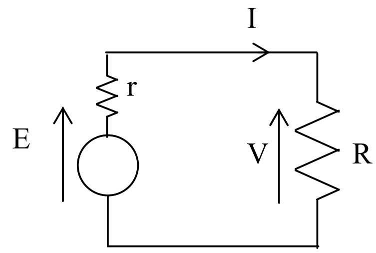
For example, if we consider the above battery, maximum voltage would be given when the current is zero, and maximum current would be given when the load is short-circuit (load voltage is zero). Under both these conditions, there is no power delivered to the load. Thus obviously in between these two extremes must be the point at which maximum power is delivered.

The Maximum Power Transfer theorem states that for maximum active power to be delivered to the load, load impedance must correspond to the conjugate of the source impedance (or in the case of direct quantities, be equal to the source impedance).

Let us analyse this, by first starting with the basic case of a resistive load being supplied from a source with only an internal resistance (this is the same as for d.c.)

Resistive Load supplied from a source with only an internal resistance

Consider a source with an internal emf of E and an internal resistance of r and a load of resistance R.



current I $= \dfrac{E}{R+r}$

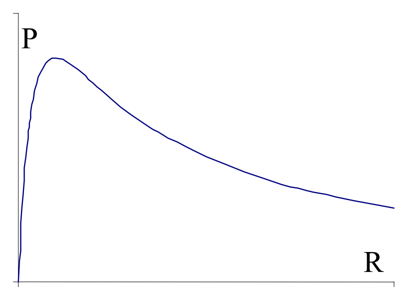
Load Power P = I^2. R $= \left(\dfrac{E}{R+r}\right)^2 \cdot R$

The source resistance is dependant purely on the source and is a constant, as is the source emf. Thus only the load resistance R is a variable.

To obtain maximum power transfer to the load, let us differentiate with respect to R.

$$\frac{dP}{dR} = \frac{E^2}{(R+r)^4} \cdot \left[(R+r)^2 \cdot 1 - R \cdot 2(R+r)\right] = 0 \text{ for maximum}$$

[Note: I said maximum, rather than maximum or minimum, because from physical considerations we know that there must a maximum power in the range. So we need not look at the second derivative to see whether it is maximum or minimum].



$\therefore$ $(R+r)^2 - 2R\,(R+r) = 0$

or $R + r - 2R = 0$

i.e. R = r for maximum power transfer.

value of maximum power = $P_{\max} = \left(\dfrac{E}{r+r}\right)^2 \cdot r = \dfrac{E^2}{4r}$

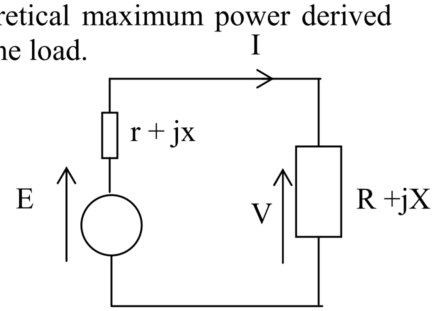
load voltage at maximum power = $\dfrac{E}{R+r}.R = \dfrac{E}{r+r}.r = \dfrac{E}{2}$

It is to be noted that when maximum power is being transferred, only half the applied voltage is available to the load, and the other half drops across the source. Also, under these conditions, half the power supplied is wasted as dissipation in the source.

Thus the useful maximum power will be less than the theoretical maximum power derived and will depend on the voltage required to be maintained at the load.

Load supplied from a source with an internal impedance

Consider a source with an internal emf of E and an internal impedance of z = (r + j x) and a load of impedance Z = R + jX.



current I $= \dfrac{E}{r + jx + R + jX} = \dfrac{E}{(r+R) + j(x+X)}$

magnitude of I = $\dfrac{E}{\sqrt{(r+R)^2 + (x+X)^2}}$

Load Power P = $|I|^2 . R = \dfrac{E^2}{(r+R)^2 + (x+X)^2} \cdot R$

Since there are two variables R and X, for maximum power $\dfrac{\partial P}{\partial R} = 0 \quad and \quad \dfrac{\partial P}{\partial X} = 0$

i.e. $\dfrac{E^2}{\left[(r+R)^2 + (x+X)^2\right]^2} \cdot \left[\left[(r+R)^2 + (x+X)^2\right] \cdot 1 - R\left[2(r+R)\right]\right] = 0$

and $\dfrac{E^2}{\left[(r+R)^2 + (x+X)^2\right]^2} \cdot \left[-R \cdot 2 \cdot (x+X)\right]$

The second equation gives $x + X = 0$ or $X = -x$

Substituting this in the first equation gives $(r+R)^2 - R \,.\, 2(r+R) = 0$

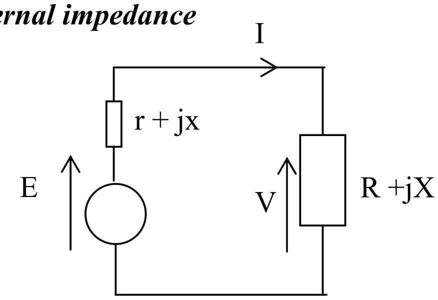
Since R cannot be negative, $r + R \neq 0$. $\therefore r + R - 2R = 0$

i.e. R = r

$\therefore$ $Z = R + j\,X = r - j\,x = z^*$

Therefore for maximum power transfer, the load impedance must be equal to the conjugate of the source impedance.

Load of fixed power factor supplied from a source with an internal impedance



Consider a source with an internal emf of E and an internal impedance of z = (r + j x) and a load of impedance Z = R + jX which has a given power factor k. [This situation is not uncommon, as for example if the load was an induction motor load, the power factor would have a fixed value such as 0.8 lag]

current I $= \dfrac{E}{r + jx + R + jX} = \dfrac{E}{(r+R) + j(x+X)}$

R and X are no longer independent but have the relationship *power factor f* = $\dfrac{R}{\sqrt{R^2 + X^2}}$

or $X = \sqrt{\dfrac{1}{f^2} - 1} \cdot R = k \,.\, R$

magnitude of I = $\dfrac{E}{\sqrt{(r+R)^2 + (x+X)^2}} = \dfrac{E}{\sqrt{(r+R)^2 + (x+k.R)^2}}$

Load Power P = $|I|^2 . R = \dfrac{E^2}{(r+R)^2 + (x+k.R)^2} \cdot R$

Since there is only one variable R, for maximum power $\frac{dP}{dR} = 0$

i.e. $$\frac{E^2}{\left[(r+R)^2 + (x+k.R)^2\right]^2} \cdot \left[\left[(r+R)^2 + (x+k.R)^2\right] \cdot 1 - R\left[2(r+R) + 2(x+k.R).k\right]\right] = 0$$

i.e. $(r+R)^2 + (x+k.R)^2 - 2R(r+R) - 2k.R(x+k.R) = 0$

i.e. $r^2 + \cancel{2\,r.R} + R^2 + x^2 + \cancel{2k.x.R} + k^2.R^2 \cancel{- 2\,R.r} - 2\,R^2 - \cancel{2k.R.x} - 2\,k^2.R^2 = 0$

i.e. $r^2 - R^2 + x^2 - k^2.R^2 = 0$

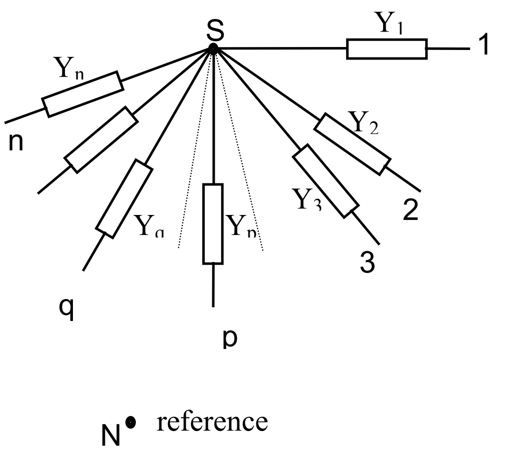
i.e. $R^2 + k^2R^2 = r^2 + x^2$. i.e. $R^2 + X^2 = r^2 + x^2$

i.e. $|Z| = |z|$

So even when the power factor of the load is different from that of the source, a condition that needs to be satisfied is that the magnitude of the load impedance must be equal to the magnitude of the source impedance.

Note: If limits are placed on the voltage, then maximum power will not always occur under the above condition, but at the limit of the voltage closest to the desired solution.

Millmann's Theorem



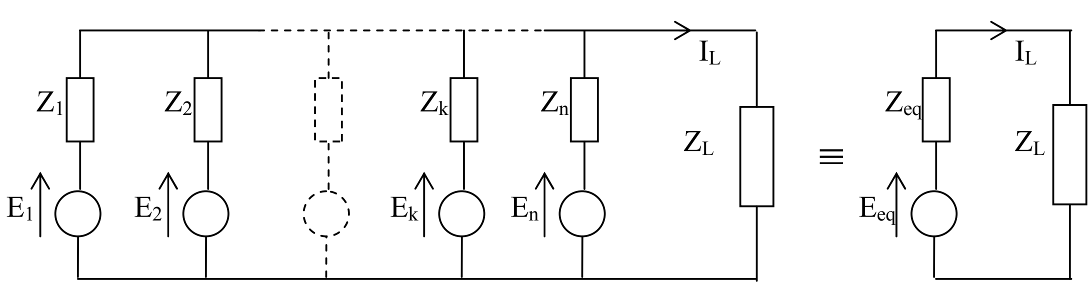
Consider a number of admittances Y_1, Y_2, Y_3 ….Y_p…. Y_q, …Y_n are connected together at a common point S. If the voltages of the free ends of the admittances with respect to a common reference N are known to be V_{1N}, V_{2N}, V_{3N} ….V_{pN}…. V_{qN}, …V_{nN}, then Millmann's theorem gives the voltage of the common point S with respect to the reference N as follows.

Applying Kirchoff's Current Law at node S $\sum_{p=1}^{n} I_p = 0$, $I_p = Y_p\,(V_{pN} - V_{SN})$

i.e. $$\sum_{p=1}^{n} Y_p (V_{pN} - V_{SN}) = 0,$$

i.e. $\sum_{p=1}^{n} Y_p V_{pN} = V_{SN} \sum_{p=1}^{n} Y_p$ so that $V_{SN} = \dfrac{\sum_{p=1}^{n} Y_p V_{pN}}{\sum_{p=1}^{n} Y_p}$

An extension of the Millmann theorem is the ***equivalent generator theorem***.

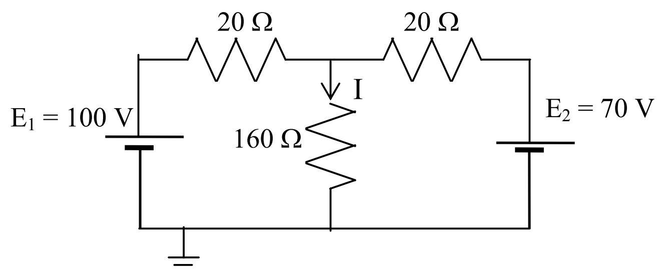


This theorem states that a system of voltage sources operating in parallel may be replaced by a single voltage source in series with an equivalent impedance given as follows (this is effectively the Thevenin's theorem applied to a number of generators in parallel).

$$E_{eq} = \frac{\sum_{k=1}^{n} E_k Y_k}{\sum_{k=1}^{n} Y_k}, \quad Y_{eq} = \sum_{k=1}^{n} Y_k$$

Example 6

The figure shown (also used in earlier examples) can be considered equivalent to two sources of 100 V and 70 V, with internal resistances 20 Ω each, feeding a load of 160 Ω. Using Millmann's theorem (or equivalent generator theorem) find the current I.



Solution

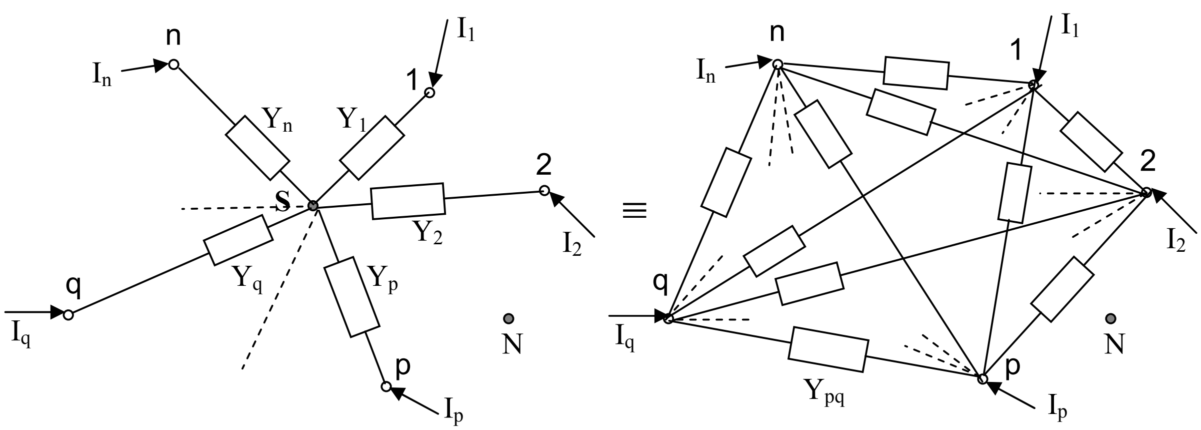
From Equivalent generator theorem, it has been shown that the equivalent generator has

$$E_{eq} = \frac{\frac{1}{20}100 + \frac{1}{20}70}{\frac{1}{20} + \frac{1}{20}} = 85 \text{ V} \quad \text{(same answer was obtained with Thevenin's Th}^{m}\text{)}$$

$$Z_{eq} = \frac{1}{\frac{1}{20} + \frac{1}{20}} = 10\,\Omega \quad \text{(again same answer was obtained with Thevenin's Th}^{m}\text{)}$$

Hence current $I = \frac{85}{10 + 160} = 0.5$ A

Rosen's Theorem (Nodal-Mesh Transformation Theorem)



Rosen's theorem tells us how we could find the mesh equivalent of a network where all the branches are connected to a single node. [In the mesh equivalent, all nodes are connected to each other and not to a common node as in the nodal network]. When the equivalent is obtained the external conditions are not affected as seen from the external currents in the above diagrams.

For the nodal network, from Millmann's theorem

$$V_{SN} = \frac{\sum_{p=1}^{n} Y_p V_{pN}}{\sum_{p=1}^{n} Y_p}, \text{ so that } \quad I_q = Y_q\,(V_{qN} - V_{SN}) = Y_q\,.V_{qN} - Y_q\,.\frac{\sum_{p=1}^{n} Y_p V_{pN}}{\sum_{p=1}^{n} Y_p}$$

$$I_q = \frac{Y_q V_{qN} \sum_{p=1}^{n} Y_p - \sum_{p=1}^{n} Y_p V_{pN}}{\sum_{p=1}^{n} Y_p} = \frac{Y_q \sum_{p=1}^{n} Y_p (V_{qN} - V_{pN})}{\sum_{p=1}^{n} Y_p}$$

For a definite summation, whether the variable p is used or the variable k is used makes no difference. Thus

$$I_q = \frac{Y_q \sum_{p=1}^{n} Y_p (V_{qN} - V_{pN})}{\sum_{k=1}^{n} Y_k} = \sum_{p=1}^{n} \left[\frac{Y_p Y_q}{\sum_{k=1}^{n} Y_k} \right] (V_q - V_p)$$

For the mesh network, from Kirchoff's current law, the current at any node is

$$I_q = \sum_{p=1}^{n} Y_{pq} (V_q - V_p)$$

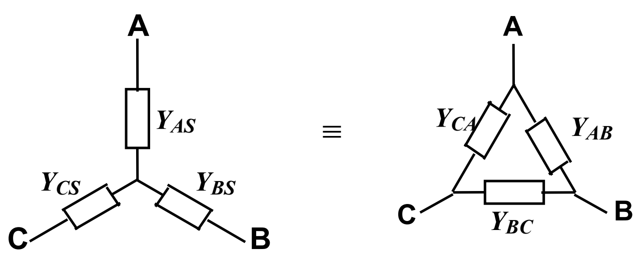
Comparing equations, it follows that a solution to equation is

$$Y_{pq} = \frac{Y_p Y_q}{\sum_{k=1}^{n} Y_k}$$ which is the statement of ***Rosen's theorem***

The converse of this theorem is in general not possible as there are generally more branches in the mesh network than in the nodal network.

However, in the case of the 3 node case, there are equal branches in both the nodal network (also known as star) and the mesh network (also known as delta).

Star-Delta Transformation



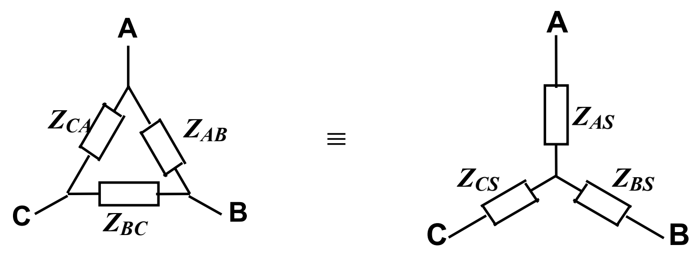
A *star connected* network of three admittances (or conductances) $\boldsymbol{Y_{AS}}$, $\boldsymbol{Y_{BS}}$, and $\boldsymbol{Y_{CS}}$ connected together at a common node S can be transformed into a *delta connected* network of three admittances $\boldsymbol{Y_{AB}}$, $\boldsymbol{Y_{BC}}$, and $\boldsymbol{Y_{CA}}$ using the following transformations. This has the same form as the general expression derived earlier.

$$\boldsymbol{Y_{AB} = \frac{Y_{AS}\,.Y_{BS}}{Y_{AS} + Y_{BS} + Y_{CS}},\; Y_{BC} = \frac{Y_{BS}\,.Y_{CS}}{Y_{AS} + Y_{BS} + Y_{CS}},\; Y_{CA} = \frac{Y_{CS}\,.Y_{AS}}{Y_{AS} + Y_{BS} + Y_{CS}}}$$

Note: You can observe that in each of the above expressions if we need to find a particular delta admittance element value, we have to multiply the two values of admittance at the nodes on either side in the original star-network and divide by the sum of the three admittances.

In the special case of three nodes, reverse transformation is also possible.

Delta-Star Transformation



A *delta connected* network of three impedances (or resistances) $\boldsymbol{Z_{AB}}$, $\boldsymbol{Z_{BC}}$, and $\boldsymbol{Z_{CA}}$ can be transformed into a *star connected* network of three impedances $\boldsymbol{Z_{AS}}$, $\boldsymbol{Z_{BS}}$, and $\boldsymbol{Z_{CS}}$ connected together at a common node S using the following transformations. [You will notice that I have used impedance here rather than admittance because then the form of the solution remains similar and easy to remember.]

$$\boldsymbol{Z_{AS} = \frac{Z_{AB}.Z_{CA}}{Z_{AB} + Z_{BC} + Z_{CA}}, Z_{BS} = \frac{Z_{AB}.Z_{BC}}{Z_{AB} + Z_{BC} + Z_{CA}}, Z_{CS} = \frac{Z_{CA}.Z_{BC}}{Z_{AB} + Z_{BC} + Z_{CA}}}$$

Note: You can observe that in each of the above expressions if we need to find a particular delta element value, we have to multiply the two impedance values on either side of node in the original star-network and divide by the sum of the three impedances.

Proof:

Impedance between A and C with zero current in B can be compared in the two networks as follows.

$$Z_{CA}//(Z_{AB} + Z_{BC}) \quad = \quad Z_{CS} + Z_{AS}$$

i.e. $$\frac{Z_{CA}(Z_{AB} + Z_{BC})}{Z_{CA} + Z_{AB} + Z_{BC}} = Z_{CS} + Z_{AS}$$

similarly $$\frac{Z_{AB}(Z_{BC} + Z_{CA})}{Z_{AB} + Z_{BC} + Z_{CA}} = Z_{AS} + Z_{BS}$$

and $$\frac{Z_{BC}(Z_{CA} + Z_{AB})}{Z_{BC} + Z_{CA} + Z_{AB}} = Z_{BS} + Z_{CS}$$

elimination of variables from the above equations gives the desired results.