University of Moratuwa, Sri Lanka
Faculty of Engineering
Department of Electrical Engineering
B. Sc. Engineering Honours Degree Course
Level 2 – Semester 2 Examination

EE201 – THEORY OF ELECTRICITY

Time Allowed: 3 Hours | March 2008

Additional Material

Graph Paper will be provided if required.
A table of Laplace transforms is provided on the other side of this page.

Instructions to Candidates

This paper contains 7 questions in 6 pages, including the cover page.
Answer All Questions.
This examination accounts for 70% of the module assessment.
Each question carries a total 10 marks. Maximum marks allocated for each part of a question is indicated in square brackets at the end of the part.
Total allocation for the paper is 70 marks.
This is a closed book examination and only authorised calculators will be permitted.

Technical Data:

Permeability of free space μ_0 $= 4\pi \times 10^{-7}$ H/m
Permittivity of free space ε_0 $= 8.854 \times 10^{-12}$ F/m
Velocity of light in free space $= 2.998 \times 10^{8}$ m/s

Table of Laplace Transforms of common causal functions *f(t)*

$f(t)$	$F(s) = \mathrm{L}[f(t)]$
Unit impulse – δt	1
Unit step – $U(t)$	$\frac{1}{s}$
t	$\frac{1}{s^2}$
t^n	$\frac{n!}{s^{n+1}}$
e^{-at}	$\frac{1}{(s+a)}$
$1 - e^{-at}$	$\frac{a}{s(s+a)}$
$t\,e^{-at}$	$\frac{1}{(s+a)^2}$
$t^n\,e^{-at}$	$\frac{n!}{(s+a)^{n+1}}$
$e^{-at} - e^{-bt}$	$\frac{b-a}{(s+a)(s+b)}$
$\sin(\omega t)$	$\frac{\omega}{(s^2+\omega^2)}$
$\sin(\omega t + \phi)$	$\frac{\omega\cos(\phi) + s\sin(\phi)}{(s^2+\omega^2)}$
$t\sin(\omega t)$	$\frac{2\omega s}{(s^2+\omega^2)^2}$
$\cos(\omega t)$	$\frac{s}{(s^2+\omega^2)}$
$\cos(\omega t + \phi)$	$\frac{s\cos(\phi) - \omega\sin(\phi)}{(s^2+\omega^2)}$
$t\cos(\omega t)$	$\frac{s^2-\omega^2}{(s^2+\omega^2)^2}$
$e^{-at}\sin(\omega t)$	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{-at}\cos(\omega t)$	$\frac{s+a}{(s+a)^2+\omega^2}$
$\sinh(\omega t)$	$\frac{\omega}{(s^2-\omega^2)}$
$\cosh(\omega t)$	$\frac{s}{(s^2-\omega^2)}$

Question 1

(a) A circuit consists of a series combination of (i) an inductance L, (ii) a resistance r, and (iii) a capacitor C and resistor R in parallel. Sketch the circuit diagram, taking v(t) as the input voltage. Write down the differential equation relating input voltage v(t) to the current $i_R(t)$ through the resistor R. *[4 marks]*

(b) If the input v(t) in question Q1(a) is a sinusoidal voltage E of angular frequency ω, determine the corresponding current in the resistor R. *[1 mark]*

(c) A circuit consisting of a resistance R, an inductance L and a variable capacitance C in series is supplied from an alternating supply of constant voltage E and constant angular frequency ω. The capacitance is adjusted until resonance is obtained at $C = C_o$.

(i) Sketch the circuit diagram and obtain an expression for the current in the circuit.

Under resonance conditions, determine (ii) the value of C_o, (iii) the value of the Q-factor Q, and (iv) the maximum value of current (I_m) in terms of E, R, L and ω. *[2 marks]*

The value of C is now adjusted in either direction to get the half-power points at C^- and C^+ respectively.

(v) Show that Q is given by $Q = \left|\frac{C}{C_0 - C}\right|$ at each of the half-power points. *[3 marks]*

Question 2

(a) Two inductances L_1 and L_2 are connected in series with mutual inductance M between them. Sketch the circuit and write down, from first principles, the differential equations relating the voltage across each of the inductances to the current. State any assumptions made in your derivations. *[1 mark]*

(b) By suitable integration, determine the total energy stored in the circuit of question 2(a) and determine an expression for the equivalent inductance of the combination. *[1 mark]*

(c) A 200 V a.c. source E of angular frequency 300 rad/s, has an internal impedance 1+j2 Ω. What would be the maximum active power that can be transferred to a motor load with power factor 0.8. What is the voltage across the motor under these conditions. *[5 marks]*

(d) A certain transformer has a core loss of 100 W when operating at a frequency of 50 Hz with laminations of thickness 0.5 mm. When the transformer is modified with laminations of thickness 0.4 mm and operated at 60 Hz, with the maximum flux density remaining unchanged, the total core loss is increased to 103.3 W. Determine the contributions of eddy current losses and hysteresis losses in the original transformer. *[3 marks]*

Question 3

The 2-port admittance matrix of a certain network is given as

$$\begin{bmatrix} 0.0385 - j0.182 & -0.0385 + j0.192 \\ -0.0385 + j0.192 & 0.0385 - j0.182 \end{bmatrix} S.$$

Determine

(a) the corresponding ABCD matrix *[3 marks]*

(b) load impedance Z_o which when connected at port 2 would give the input impedance at port 1 as Z_o *[2 marks]*

For the circuit shown in figure Q3,

(c) convert the dependent current source, with no direct shunt admittance, to equivalent dependent voltage sources. *[2 mark]*

(d) write down the Kirchoff's circuit equations *[1 marks]*

(e) determine the currents in all the branches *[2 marks]*

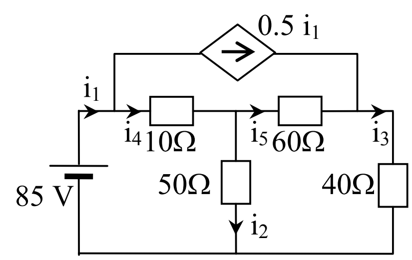

Figure Q3

Question 4

(a) If the circuit shown in figure Q4 has ideal sources, with $E_{AB} = 100\angle 0^o$V and $E_{CA} = 150\angle 90^o$V, determine the source voltage E_{BC}. *[1 mark]*

(b) If $Z_{AS} = 50\angle 0^o\ \Omega$, $Z_{BS} = 50\angle 30^o\ \Omega$ and $Z_{CS} = 50\angle 60^o\ \Omega$, selecting any suitable meshes, write down the mesh impedance matrix and the mesh voltage source. *[2 mark]*

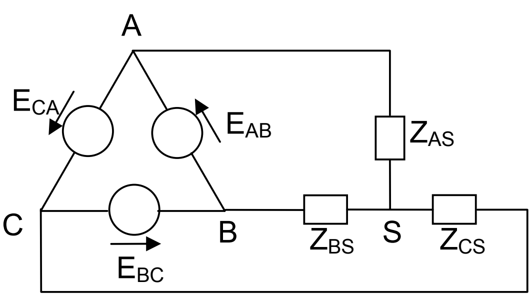

Figure Q4

(c) Determine the currents in the impedances Z_{AS} and Z_{CS}. *[3 marks]*

(d) Obtain the admittances Y_{AB} and Y_{CA} of the equivalent delta connected network corresponding to the star connected network Z_{AS}, Z_{BS} and Z_{CS}. *[2 marks]*

(e) Hence obtain the currents flowing in Z_{AB} and Z_{CA} in the converted circuit. *[1 marks]*

(f) Using the results of Q4(e) confirm the value of the current in Z_{AS}. *[1 mark]*

Question 5

A 3 phase, 50 Hz, 3-wire, balanced supply feeds a 1.5 kW, 0.6 lag, three phase motor at a voltage of 400 V.

Determine

(a) the line current supplied from the source, and *[1 mark]*

(b) the reactive power supplied. *[1 mark]*

If the distribution line supplying the motor has an impedance (l = 10 mH, r = 5 Ω) on each line, determine

(c) the voltage at the supply end of the line, *[2 marks]*

(d) the overall power factor at the source, and *[1 mark]*

(e) the capacitances that must be connected in delta across the combined load to improve the overall power factor to 0.95 lag, when the load voltage is maintained at 400 V. *[2 marks]*

For a system with phase sequence ABC, the currents flowing in the three phases, under fault conditions, are found to be $I_A = 10\angle 0^o$A, $I_B = 10\angle -120^o$A and $I_C = 50\angle 90^o$A.

(f) Determine the sequence components of the currents in all three phases. *[3 marks]*

Question 6

The current waveform of a certain power electronic circuit has the waveform shown in figure Q6.

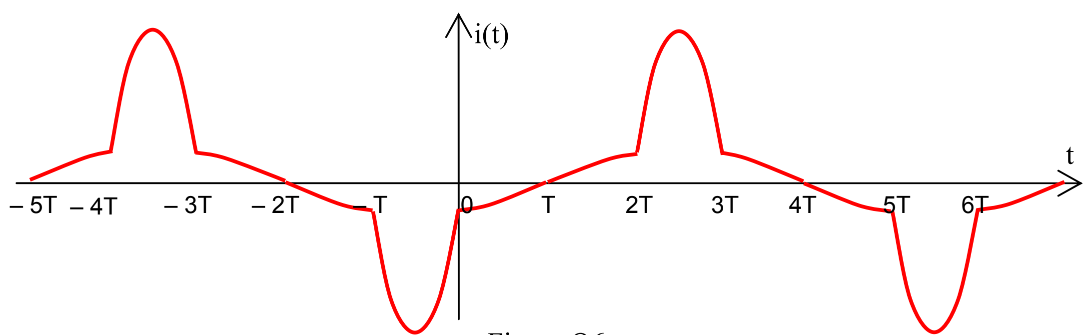

Figure Q6

A section of the waveform may be approximated as follows:

$i(t) = -\cos \omega_o t \qquad\qquad 0 \le t \le 2T$

$i(t) = -\cos \omega_o t + 4 \sin 3\omega_o t \qquad 2T \le t \le 3T$

The rest of the waveform can be deduced from symmetry.

Determine the

(a) period of the waveform and the value of ω_o in terms of T *[1 mark]*

(b) Fourier series of the current to 3 significant frequency terms *[7 mark]*

If the current i(t) is drawn from a source 100 sin ω_ot, having an internal resistance of 1 Ω, determine

(c) an expression for the terminal voltage. *[2 marks]*

Question 7

The equivalent circuit of a high voltage impulse generator is shown in figure Q7.

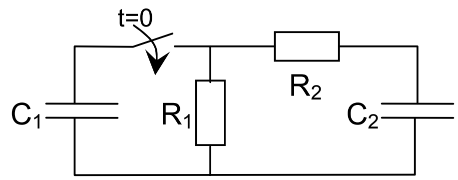

Figure Q7

(a) Obtain the Laplace Transform equivalent circuit, when the initial charge on the capacitor C_1 is q_o and there is no initial charge on the capacitor C_2. *[1 mark]*

(b) Write down an expression for the output voltage across the capacitor C_2 in the transform domain. *[3 marks]*

(c) If C_1 = 90 nF, C_2 = 10 nF, q_o = 0.0225 C, R_1 = 1 kΩ and R_2 = 100 Ω, obtain an expression for the output voltage in the time domain. *[3 marks]*

(d) Sketch the output voltage waveform. *[1 mark]*

(e) Determine the magnitude of the peak value of and the time at it occurs. *[2 marks]*

[END OF QUESTION PAPER]