BSc Engineering Degree Course, Level 2

EE201 – Theory of Electricity – *Professor J R Lucas*

Problems on Circuit Transients

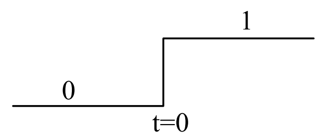
Introduction

Switching of a circuit from one state to another, either by a change in the source condition or by a change in the circuit condition, causes a transient state in the circuit. During this period the behaviour of the circuit is governed by a differential equation. The solution of such a differential equation has two parts, namely the *complementary function* which refers to the changes occurring during the *transient state* and the *particular solution* which refers to a possible solution, and in particular this solution may be taken from the *steady state* condition.

Unit Step h(t)

One of the commonest reasons for the occurrence of transients is the switching on or switching off of a supply. In such cases, the level of the supply just before and just after would represent two distinct levels, such as in a physical step. Such waveforms are known as step waveforms, and the basic form of this is the **unit step** which has a magnitude **unity** and starts at t=0.

$$h(t) = 0, \text{ for } t \le 0$$
$$= 1, \text{ for } t \ge 0$$

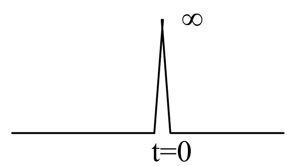


Unit Impulse δ(t)

Another of the common reasons for the occurrence of transients is an impulsive action, which has a very large magnitude for a negligible time. This is characterised by the **unit impulse** which has zero magnitude other than at time zero when it has infinite magnitude. Also the area enclosed by the waveform is unity.

$$\delta(t) = 0, \text{ for } t < 0$$
$$= \infty, \text{ for } t = 0$$
$$= 0, \text{ for } t > 0$$

$$\int_{-\infty}^{\infty} \delta(t).dt = 1$$



The unit impulse function also has the following properties.

$$\int_{-\infty}^{\infty} f(t) \cdot \delta(t)\,.\,\mathrm{dt} = \mathrm{f}(0)$$

and $$\int_{-\infty}^{\infty} f(t-\tau) \cdot \delta(t)\,.\,\mathrm{dt} = \mathrm{f}(\tau)$$

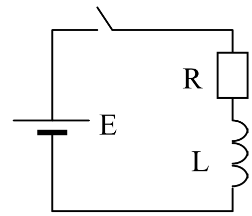
The unit impulse is the derivative of the unit step.

Both the unit step and the unit impulse can be used to define the natural behaviour of a network.

Problems

Ex 1 – Simple R L circuit supplied from a step voltage source (switch closed onto a battery)

If the switch is closed at time t=0, the voltage across the R L combination would be e(t) which is a step of magnitude E [or E.h(t)] and not a constant as is the supply voltage E.



$$e(t) = 0, \text{ for } t \le 0$$
$$= E, \text{ for } t \ge 0$$

Thus the differential equation governing the behaviour of the circuit would be

$$e(t) = R \,.\, i(t) + L.\frac{d\,i(t)}{d\,t} = R \,.\, i + L \,.\, p\, i$$

For complementary function, R + L p = 0, giving p = – R/L

A particular solution is the steady state solution, which gives the current under steady conditions as i(t) = E/R. [no voltage drop across inductor at steady state].

Therefore the solution can be written in the form $i(t) = \frac{E}{R} + A \cdot e^{-\frac{R}{L}t}$ where A is a constant.

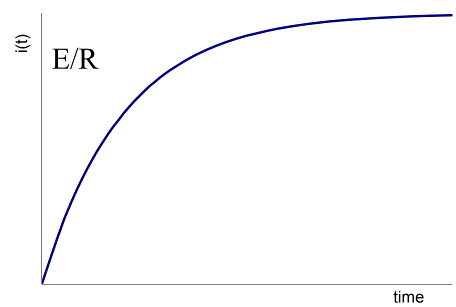
The constant A can be obtained from the initial conditions as follows.

At t=0, i(t) = 0 [since the current through an inductance cannot change suddenly]

$$\therefore\ 0 = \frac{E}{R} + A \cdot e^{-\frac{R}{L}\times 0} = \frac{E}{R} + A$$

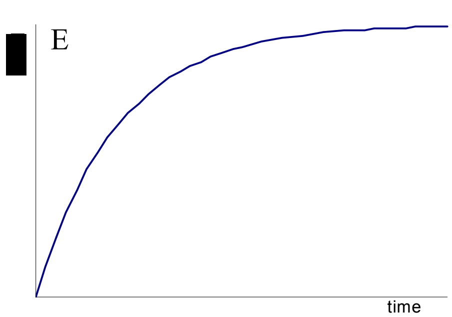
giving $A = -\frac{E}{R}$.

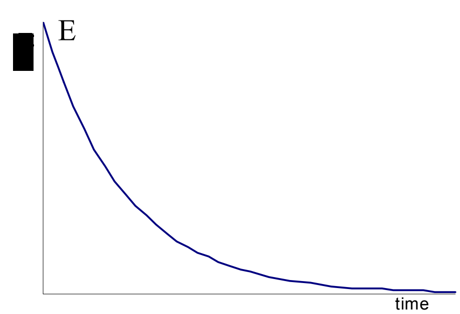
Therefore the solution is $i(t) = \frac{E}{R}\left(1 - e^{-\frac{R}{L}t}\right)$.



The voltage drops across each of the elements can now be obtained as

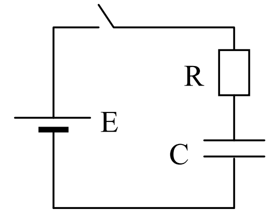
$$v_R(t) = R.i(t) = E\left(1 - e^{-\frac{R}{L}t}\right) \quad \text{and} \quad v_L(t) = L.\frac{d\,i(t)}{dt} = E.e^{-\frac{R}{L}t}$$





Ex 2 – Simple R C circuit supplied from a step voltage source (switch closed onto a battery)

The differential equation governing the behaviour of the circuit would be



$$e(t) \quad = \; R \,.\, i(t) + \frac{1}{C}\int i(t)\,dt = R.i(t) + \frac{1}{C.p}i(t)$$

For complementary function, $R + \frac{1}{C\,p} = 0$, giving p = – 1/RC

A particular solution is the steady state solution, which gives the current under steady conditions as i(t) = 0. [no current through capacitor at steady state].

Therefore the solution can be written in the form $i(t) = 0 + A \cdot e^{-\frac{1}{RC}t}$ where A is a constant.

A can be obtained from the initial conditions as follows.

At t=0, $v_C(t) = 0$ [since the voltage across a capacitor cannot change suddenly].

∴ $v_R(t) = E$ and $i(t) = E/R$

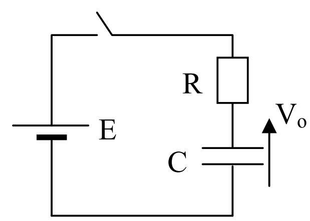
$\frac{E}{R} = A \cdot e^{-\frac{1}{RC}\times 0}$ giving A = E/R. Therefore the solution is $i(t) = \frac{E}{R}e^{-\frac{1}{RC}t}$.

The voltage drops across the elements can now be obtained using Ohm's law.

The sketches of voltage and current can be done as in the earlier example.

Ex 3 –R C circuit supplied from a step voltage source, but with C initially charged to V_o

The differential equation governing the behaviour of the circuit would be



$$e(t) \quad = \; R \,.\, i(t) + \frac{1}{C}\int i(t)\,dt + V_o = R.i + \frac{1}{C.p}i + V_o$$

The complementary function remains the same as in Ex2 except that the value of the constant will be different.

The particular solution would again be zero.

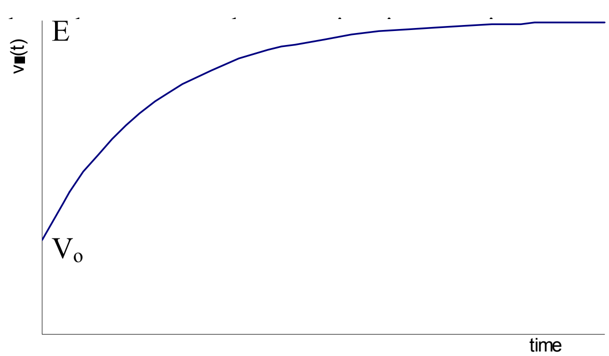
At t=0, $v_C(t) = V_o$ [since the voltage across a capacitor cannot change suddenly].

∴ $v_R(t) = E - V_o$ and $i(t) = (E - V_o)/R$

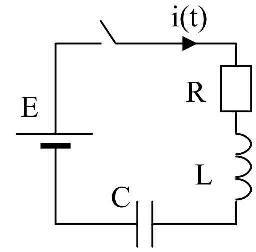
$$\therefore \frac{E - V_o}{R} = A \cdot e^{-\frac{1}{RC}\times 0} \quad \text{giving} \quad A = \frac{E - V_o}{R} \qquad \therefore i(t) = \frac{E - V_o}{R} \cdot e^{-\frac{1}{RC}t}$$

It can be shown using Ohm's law that

$$v_C(t) = E - \left(E - V_o\right) \cdot e^{-\frac{1}{RC}t}$$



Ex 4 –R L C circuit supplied from a step voltage source



$$e(t) = R.i(t) + L\frac{d\,i(t)}{d\,t} + \frac{1}{C}\int i(t)\,dt = R.i(t) + L.p.i(t) + \frac{1}{C.p}i(t)$$

$$p.e(t) = R.p.i(t) + L.p^2.i(t) + \frac{1}{C}i(t)$$

The complementary function is $R.p + L.p^2 + 1/C = 0$

The solution of this equation can be real or complex dependant on the values of components.

(a) R = 480 Ω, L = 0.4 H, C = 2.5 μF, E = 120 V

complementary function is $0.4\,p^2 + 480\,p + 4\times10^5 = 0$

i.e. $p^2 + 1200\,p + 10^6 = 0$ giving $p = -600 \pm \sqrt{600^2 - 10^6} = -600 \pm j800$

The particular solution in this case would be i(t)=0 at t=∞

giving a solution of the form $A.e^{-600t}.e^{j800t} + B.e^{-600t}.e^{-j800t}$, or $C.e^{-600t}.\cos(800t+\theta)$

Using initial conditions [2 are required here as there are 2 energy storing elements]

i(t) = 0 and $v_C(t)=0$ at t=0

$0 = C.e^{-600\times0}.\cos(800\times0+\theta)$ gives $\theta = \pm90^o$ as C cannot be zero[trivial solution]

giving the solution $i(t) = C.\,e^{-600t}.\sin 800t$

Also since i(0)=0, $v_R(0) = 0$. ∴ $v_L(0) = 120 = 0.4\times\frac{d\,i}{d\,t}$ giving $\frac{d\,i}{d\,t} = 300$ at t=0

$$\frac{d\,i}{d\,t} = C.\left[-600\times e^{-600\times0}.\sin 800\times0 + e^{-600\times0}.800\times\cos 800\times0\right] = 300$$

giving C = 300/800 = 0.375

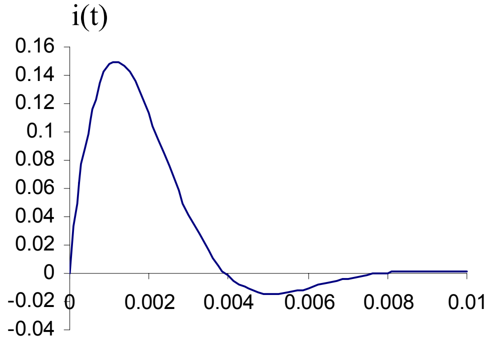
∴ $i(t) = 0.375\,e^{-600t}.\sin 800t$ A

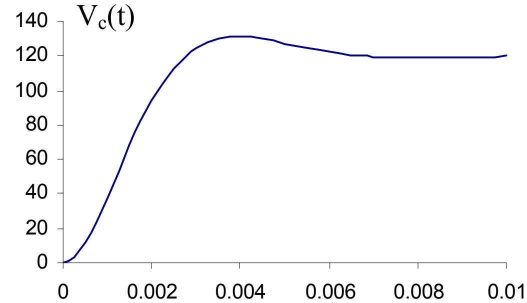
Using Ohm's law, the voltages across the elements are given as

$v_R(t) = 480\,i(t) = 180\,e^{-600t}.\sin 800t$ V

$v_L(t) = 0.4\,p.i(t) = -90\,e^{-600t}.\sin 800t + 120\,e^{-600t}.\cos 800t$ V

$v_C(t) = 120 - 90\,e^{-600t}.\sin 800t - 120\,e^{-600t}.\cos 800t$ V





(b) R = 800 Ω, L = 0.4 H, C = 2.5 μF, E = 120 V

complementary function is $0.4\,p^2 + 800\,p + 4\times10^5 = 0$

i.e. $p^2 + 2000\,p + 10^6 = 0$ giving $(p + 1000)^2 = 0$ or $p = -1000$ (repeated roots)

In this case the solution is of the form

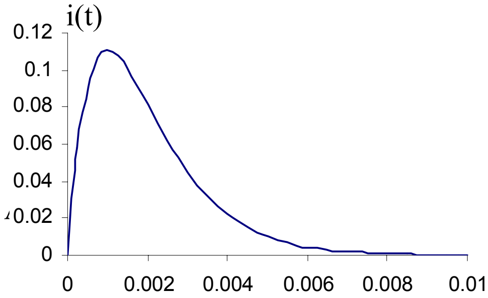
$i(t) = A\,t\,e^{-1000t} + 0$

At t=0, i(t) = 0, [automatically satisfied]

and di(t)/dt = 300

$\dfrac{di}{dt} = A\,e^{-1000\times0} - 1000A\times0.e^{-1000\times0} = 300$ giving

$\therefore\ i(t) = 300\,t\,e^{-1000t}$ A



(c) R = 1000 Ω, L = 0.4 H, C = 2.5 μF, E = 120 V

complementary function is $0.4\,p^2 + 1000\,p + 4\times10^5 = 0$

i.e. $p^2 + 2500\,p + 10^6 = 0$ giving

$(p + 500)(p + 2000) = 0$ or $p = -500$ or -2000

In this case the solution is of the form

$i(t) = A\,e^{-500t} + B\,e^{-2000t}$

At t=0, i(t) = 0 = A + B

At t=0, $v_C(t) = 0$, gives di(t)/dt = 300.

i.e. $-500A - 2000B = 300$, gives A = 0.2 = -B

$\therefore\ i(t) = 0.2\,(e^{-500t} - e^{-2000t})$ A