# EE 201 - THEORY OF ELECTRICITY – Short Answers

## Level 2 Semester 1 Examination - September 2005

1. (a) When parallel resonance occurs, current becomes a minimum. It is easily seen that $\omega L = 1/\omega C$ to give an infinite impedance

$$L = \frac{1}{\omega^2 C} = \frac{1}{1000^2 * 10 \times 10^{-6}} = 0.1\,\text{H} = \underline{\underline{100\text{ mH}}}$$

Current at resonance = $\underline{\underline{0\text{ A}}}$

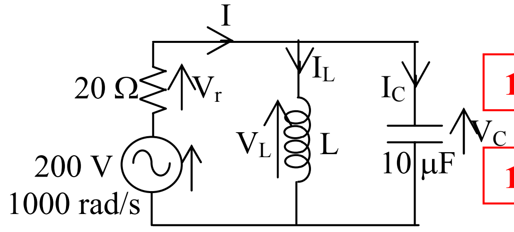


Figure Q1bc

**1 mark**

**1 mark**

(b) if L = 50 mH

$j\omega L = j\,1000 \times 50 \times 10^{-3} = j\,50\ \Omega$

$1/j\omega C = 1/j1000 \times 10 \times 10^{-6} = -j\,100\ \Omega$

impedance of L and C in parallel $= \dfrac{j50 \times (-j100)}{j50 - j100} = j100\,\Omega$

$\therefore$ total impedance of circuit $= 20 + j\,100\ \Omega = 101.98\angle 78.69^\circ\ \Omega$

$\therefore\ I = 200\angle 0^\circ / 101.98\angle 78.69^\circ = \underline{\underline{1.961\angle -78.69^\circ\text{ A}}}$

**1 mark**

$I_L = 1.961\angle -78.69^\circ \times (-j100)/(j50 - j100) = \underline{\underline{3.922\angle -78.69^\circ\text{ A}}}$

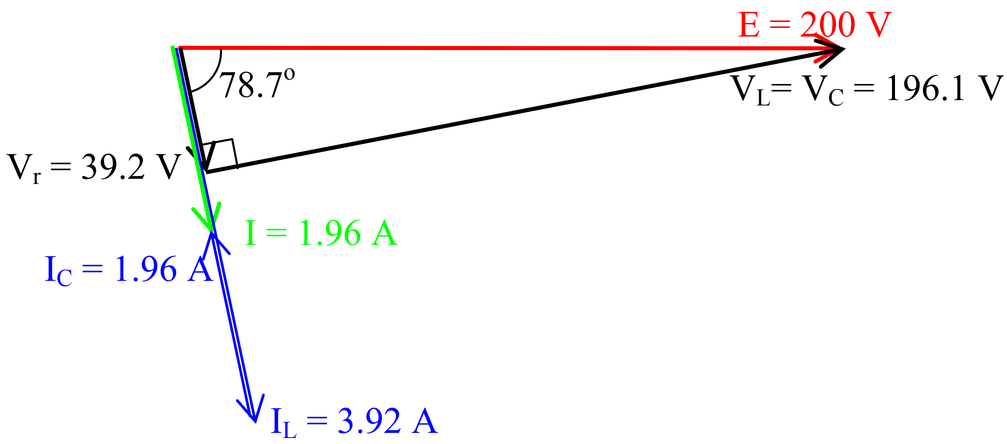
$I_C = 1.961\angle -78.69^\circ - 3.922\angle -78.69^\circ = -1.961\angle -78.69^\circ = \underline{\underline{1.961\angle 101.31^\circ\text{ A}}}$

$V_r = 20\,I = \underline{\underline{39.22\angle -78.69^\circ\text{ V}}}$

$V_L = V_C = j\omega L.I_L = j\,50 \times 3.922\angle -78.69^\circ = \underline{\underline{196.1\angle 11.31^\circ\text{ V}}}$

**2 marks**

(c)




**2 marks**

(d) *e(t) = 282.8 cos 1000 t* corresponds to an rms value of 100 V and phase angle of 0 as in section (b). Thus

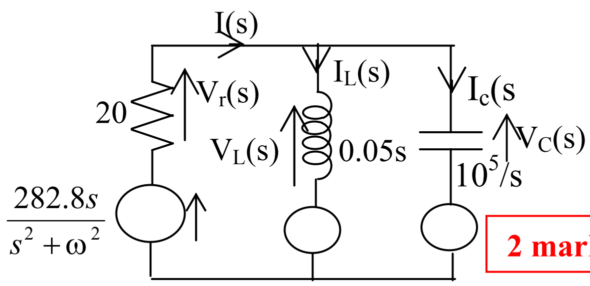
$i_L(t) = \sqrt{2} \times 3.922 \cos(1000t - 78.69^\circ)$,

$i_L(1\text{ms}) = \sqrt{2} \times 3.922 \cos(57.30^\circ - 78.69^\circ) = \underline{\underline{5.16\text{ A}}}$

$v_c(t) = \sqrt{2} \times 196.1 \cos(1000t + 11.31^\circ)$,

$v_c(1\text{ms}) = \sqrt{2} \times 196.1 \cos(57.30^\circ + 11.31^\circ) = \underline{\underline{101.1\text{ V}}}$

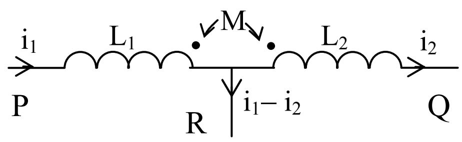
**2 marks**




**2 marks**

# EE 201 - THEORY OF ELECTRICITY – Short Answers

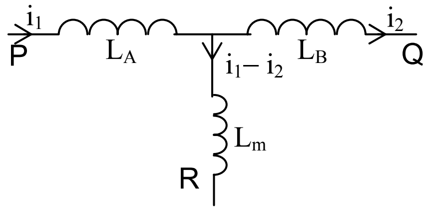
2. (a) Consider the two coupled coils on two arms PR and QR of a T-junction as shown with currents $i_1$ and $i_2$ flowing in them and a current $(i_1 - i_2)$ flowing in the common branch.




Since the coils are wound in opposition as indicated by the dots, applying Kirchoff's voltage law between PR, and then again between RQ,

$$V_{PR} = L_1\, p\, i_1 - M\, p\, i_2 \quad \text{and}$$

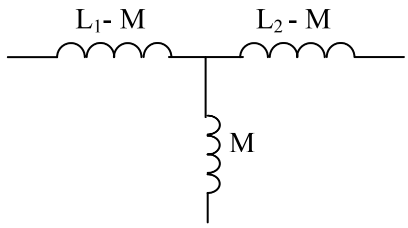
and $$V_{RQ} = L_2\, p\, i_2 - M\, p\, i_1$$




If a non-coupled equivalent circuit is to be obtained, voltage drops in a branch should only correspond to currents in it own branch. So re-writing

$$V_{PR} = (L_1 - M)\, p\, i_1 + M\, p\, (i_1 - i_2)$$

and $$V_{RQ} = (L_2 - M)\, p\, i_2 + M\, p\, (i_1 - i_2)$$




These two equations would be satisfied with

$L_A = L_1 - M$, $L_B = L_2 - M$, and $L_m = M$

This transformation will be valid, independent of what the directions marked for the currents in the diagrams.

**2 marks**

(b)

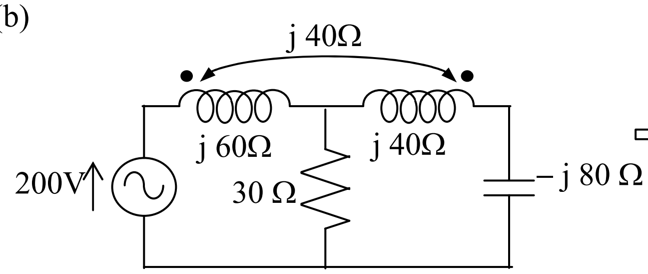
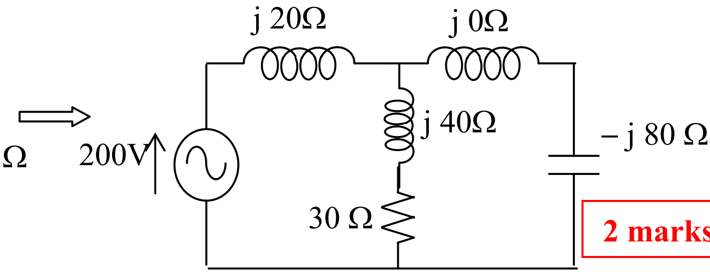



Figure Q2b

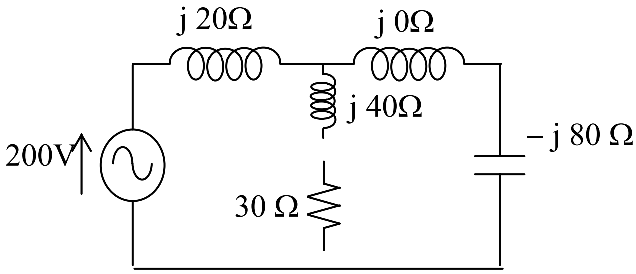



**2 marks**

(c) For Thevenin's equivalent circuit,

Thevenin's voltage across 30 Ω

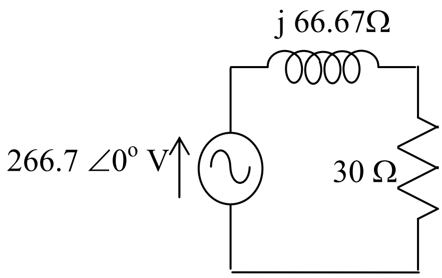
$$= 200 \times \frac{-j80}{j20 - j80} = 266.7\ \angle 0^{o}\ \text{V}$$




Thevenin's impedance = j40 + j20//(-j80)

$$= j40 + \frac{j20 \times (-j80)}{j20 - j80} = j66.67\ \Omega$$

Thevenin's equivalent circuit is




**3 marks**

(d) Current through the 30 Ω resistor

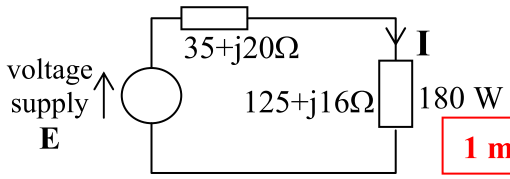
$= 266.7\angle 0^{o}/(30+j66.67) = 266.7\angle 0^{o}/73.11\angle 65.77^{o}$

$=$ 3.648 A

**2 marks**

## EE 201 - THEORY OF ELECTRICITY – Short Answers

3. (a) $I^2 R = P$




supply current $\mathbf{I} = \sqrt{\frac{180}{125}} = \underline{\underline{1.2 \text{ A}}}$

**1 mark**

circuit impedance z = (35+125) + j (20+16) = 160 + j 36 Ω

supply voltage $\mathbf{V} = (160 + j36) * 1.2 = 164\angle 12.68^0 * 1.2 = 196.8\angle 12.68^0 = \underline{\underline{196.8 \text{ V}}}$

**2 marks**

supply power factor = cos 12.68° = $\underline{\underline{0.976 \text{ lag}}}$

**1 mark**

(b) using the star-delta conversion theorem,

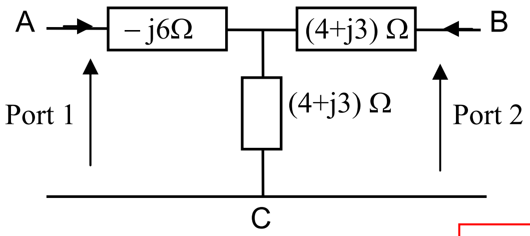



Figure Q3

$$Y_{AC} = \frac{Y_A \cdot Y_C}{Y_A + Y_B + Y_C} = \frac{\frac{1}{-j6} \times \frac{1}{(4+j3)}}{\frac{1}{-j6} + \frac{1}{(4+j3)} + \frac{1}{(4+j3)}}$$

$$Y_{AC} = \frac{1}{(4+j3) - j6 - j6} = \frac{1}{4 - j9} = 0.0412\text{+j } 0.0928,$$

**2 marks**

$Z_{AC} = 4 - j9\ \Omega = \underline{\underline{9.85\angle -66.0^o\ \Omega}}$

(c) The y-parameter matrix can be obtained from

$$\begin{bmatrix} I_A \\ I_B \end{bmatrix} = \begin{bmatrix} y_{AA} & y_{AB} \\ y_{BA} & y_{BB} \end{bmatrix} \begin{bmatrix} V_A \\ V_B \end{bmatrix},$$

$$y_{AA} = \left. \frac{I_A}{V_A} \right|_{V_B=0} = \frac{1}{-j6 + 2 + j1.5} = \frac{1}{2 - j4.5} = 0.203\angle +66.0^o = 0.082 + \text{j } 0.186$$

$$y_{BA} = \left. \frac{I_B}{V_A} \right|_{V_B=0} = \frac{I_B}{I_A} \times \left. \frac{I_A}{V_A} \right|_{V_B=0} = -\frac{1}{2} \times \frac{1}{2 - j4.5} = -\frac{1}{4 - j9} = 0.102\angle -114.0^o = -0.041 - \text{j } 0.093$$

$$y_{BB} = \left. \frac{I_B}{V_B} \right|_{V_A=0} = \frac{1}{4 + j3 + \frac{(4+j3)(-j6)}{(4+j3-j6)}} = \frac{1}{9.76 + j1.32} = 0.102\angle -7.70^0 = 0.101 - \text{j } 0.0137$$

$$y_{AB} = \left. \frac{I_A}{V_B} \right|_{V_A=0} = \frac{I_A}{I_B} \times \left. \frac{I_B}{V_B} \right|_{V_A=0} = -\frac{4 + j3}{4 + j3 - j6} \times \frac{1}{9.76 + j1.32} = -\frac{1}{4 - j9} = 0.102\angle -114.0^o$$

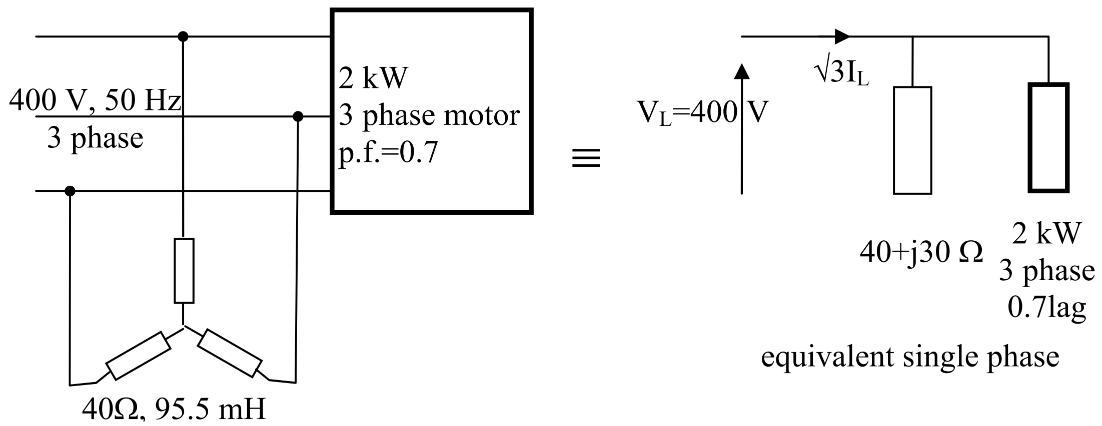
$$= -0.041 - \text{j } 0.093$$

$$\begin{bmatrix} y_{AA} & y_{AB} \\ y_{BA} & y_{BB} \end{bmatrix} = \underline{\underline{\begin{bmatrix} 0.203\angle 66^o & 0.102\angle -114^o \\ 0.102\angle -114^o & 0.102\angle -7.7^o \end{bmatrix}}} \text{ S}$$

**4 marks**

# EE 201 - THEORY OF ELECTRICITY – Short Answers

4. (a)




95.5mH → $95.5\times10^{-3}\times 2\pi\times50 = j30.00\Omega$, gives star connected impedance = 40 + j30 Ω

∴ $\sqrt{3}I_L$ for star connected load = 400/(40 + j30) = $8\angle{-36.87^o}$

and $\sqrt{3}I_L$ for motor load → 2000/400×0.7 = 7.143 at lagging (motor) p.f. of 0.7

→ $7.143\angle{-45.57^o}$

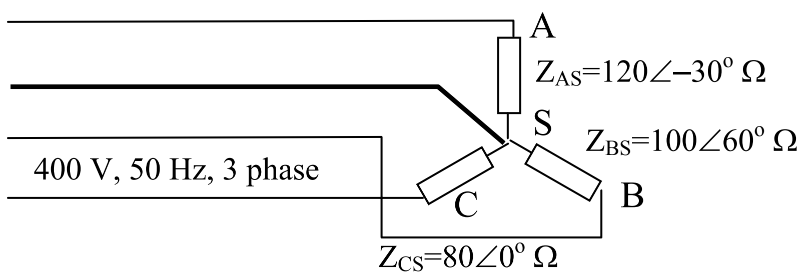
∴ total $\sqrt{3}I_L$ for load = $8\angle{-36.87^o} + 7.143\angle{-45.57^o}$ = 6.4 – j 4.8 + 5.00 – j 5.10

= 11.4 – j 9.9

∴ line current supplied from supply = (11.4 – j 9.9)/ √3 = $\underline{\underline{8.72\angle{-41.0^o}\ A}}$ **3 marks**

Power factor of supply = cos $41.0^o$ = $\underline{\underline{0.755\ \text{lag}}}$ **1 mark**

(b)




currents in the 3 phases are

$$I_A = \frac{230.9\angle 0^o}{120\angle -30^o} = \underline{\underline{1.924\angle 30^o\,A}},\quad I_B = \frac{230.9\angle -120^o}{10\angle 60^o} = \underline{\underline{2.309\angle -180^o\,A}},$$

$$I_C = \frac{230.9\angle 120^o}{80\angle 0^o} = \underline{\underline{2.886\angle 120^o\,A}}$$

**2 marks**

(c) sequence components of the currents are given by

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}\begin{bmatrix} 1.924\angle 30^o \\ 2.309\angle -180^o \\ 2.886\angle 120^o \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1.924\angle 30^o + 2.309\angle -180^o + 2.886\angle 120^o \\ 1.924\angle 30^o + 2.309\angle -60^o + 2.886\angle 0^o \\ 1.924\angle 30^o + 2.309\angle 60^o + 2.886\angle 240^o \end{bmatrix}$$

$$I_{a0} = \frac{1}{3}(1.666 + j0.962 - 2.309 - 1.443 + j2.500) = -0.695 + j1.154 = \underline{\underline{1.347\angle 121.1^o\,A}}$$

$$I_{a1} = \frac{1}{3}(1.666 + j0.962 + 1.154 - j2.000 + 2.886) = 1.902 - j0.346 = \underline{\underline{1.933\angle -10.3^o\,A}}$$

**3 marks**

$$I_{a2} = \frac{1}{3}(1.666 + j0.962 + 1.154 + j2.000 - 1.443 - j2.500) = 0.459 + j0.154 = \underline{\underline{0.484\angle 18.6^o\ A}}$$

(d) Since there is only a positive sequence voltage, there can be only power associated with positive sequence.

Power associated with zero sequence = 0 W

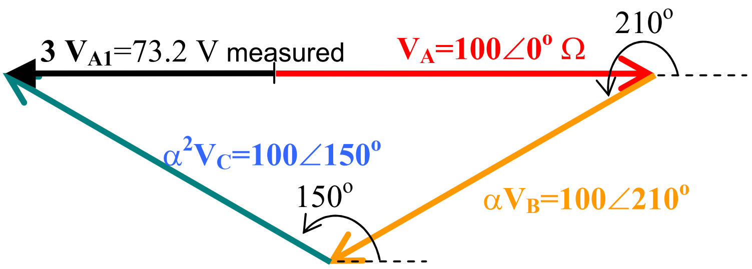
Power associated with positive sequence = 3×230.9×1.933×cos(-10.3°) = 1317 W

Power associated with negative sequence = 0 W

**1 marks**

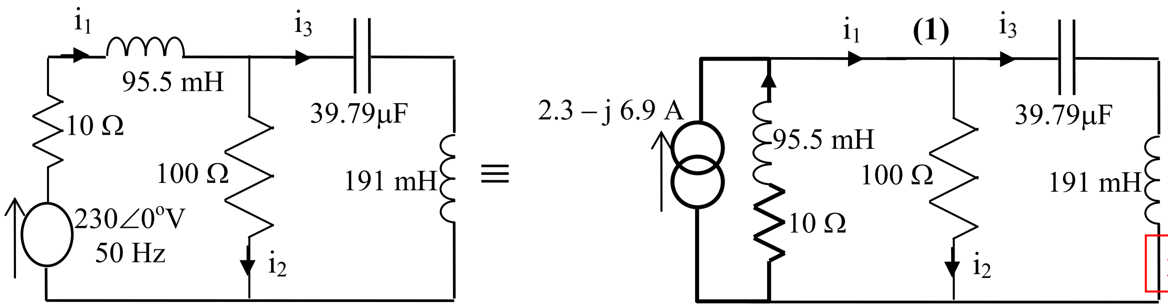
(e) $V_A = 100\angle 0^o$ V, $V_B = 100\angle 90^o$ V, $V_C = 100\angle\text{-}90^o$ V

$V_{A1} = (V_A + \alpha V_B + \alpha^2 V_C)/3 = (100\angle 0^o + 100\angle 210^o + 100\angle 150^o)/3$




**2 marks**

5.




**2 mark**

circuit can be drawn by taking the 10 Ω and the 95.5 mH inductor together in obtaining the equivalent current source. There is only one independent node.

Norton's current source = 230/(10+j30) = 2.3 – j 6.9 = 7.30∠–71.56°

95.5 mH → 2π × 50 × 95.5 × $10^{-3}$ → j 30.0

39.79 μF → 1/2π×50×39.79×$10^{-6}$ → – j 80.0

191 mH → 2π × 50 × 191 × $10^{-3}$ → j 60.0

node branch incidence matrix = $\underline{\underline{[-1 \quad 1 \quad 1]}}$

**2 marks**

nodal admittance matrix =

$$\left[\frac{1}{10+j30} + \frac{1}{100} + \frac{1}{-j80+j60}\right] = [0.01 - j0.03 + 0.01 + j0.05] = [0.02 + j0.02] = 0.02828\angle 45^o$$

**2 marks**

Voltage at node (1) = (2.3 – j6.9)/0.02828∠45° = 7.302∠-71.57°/0.02828∠45° = 258.2∠-116.6° V

current in branch (1) = 2.3– j 6.9–258.2∠-116.6° /(10+j30)

= 2.3 – j 6.9 – 258.2∠-116.6° /31.62∠71.57° = 2.3 – j 6.9 – 8.166∠-188.2°

= 2.3 – j 6.9 + 8.08 – j 1.16 = 10.38 – j 8.06 = $\underline{\underline{13.14\angle\text{-}37.8^o}}$ A

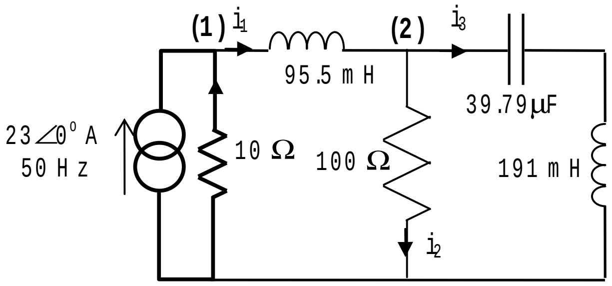
**2 marks**

current in branch (2) = $258.2\angle{-116.6^o}/100 = 2.58\angle{-116.6^o}$ A

current in branch (3) = $258.2\angle{-116.6^o}/(-j20) = 12.9\angle{-26.6^o}$ A

**2 marks**

alternate circuit by taking only 10 Ω for obtaining current source.




This is longer and has 2 independent nodes.

node branch incidence matrix is given by

$$\begin{bmatrix} -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 1 \end{bmatrix}$$

the nodal admittance matrix is given by

$$\begin{bmatrix} \frac{1}{10}+\frac{1}{j30} & -\frac{1}{j30} \\ -\frac{1}{j30} & \frac{1}{j30}+\frac{1}{100}+\frac{1}{-j20} \end{bmatrix} = \begin{bmatrix} 0.1-j0.0333 & j0.0333 \\ j0.0333 & 0.01+j0.0167 \end{bmatrix}$$

$$\begin{bmatrix} 23 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.1-j0.0333 & j0.0333 \\ j0.0333 & 0.01+j0.0167 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix},$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} 0.01+j0.0167 & -j0.0333 \\ -j0.0333 & 0.1-j0.0333 \end{bmatrix} \begin{bmatrix} 23 \\ 0 \end{bmatrix},$$

$\Delta = (0.1 - j\,0.0333)(0.01+j0.0167) - (j0.0333)^2 = 0.001+0.000555+0.00111+j\,0.001666 - j\,0.000333$

$= 0.0026666 + j\,0.0013333 = 0.00298\angle 26.57^o$

$V_1 = (0.01+j0.0167)\times 23 / 0.00298\angle 26.57^o = 0.0194\angle 59.04^o \times 23 / 0.00298\angle 26.57^o = 150.32\angle 32.4^o$

$V_2 = -j0.0333 \times 23 / 0.00298\angle 26.57^o = 258.2\angle{-116.6^o}$

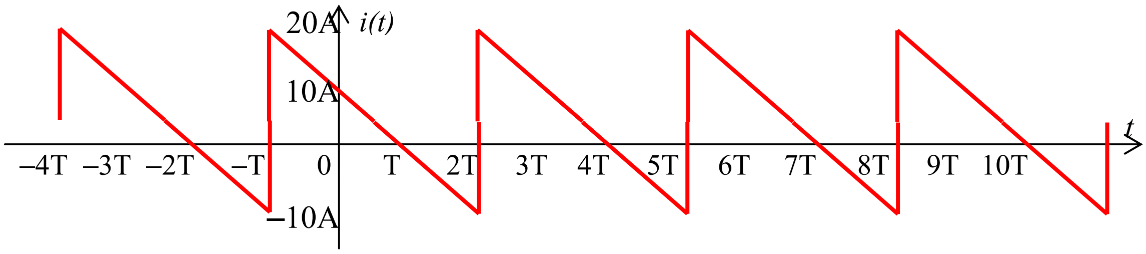
current in branch 1 = $(V_1 - V_2)/j30 = (126.9+j80.55 + 115.6 + j230.9)/j30 = 13.16\angle{-37.9^o}$ A

current in branch 2 = $V_2/100 = 2.58\angle{-116.6^o}$ A

current in branch 3 = $V_2/(-j20) = 12.9\angle{-26.6^o}$ A

Note: Current in branch 1 is not equal to $8.13\angle 171.8^o$ A as the current of the source of $23\angle 0^o$ A must be added as part of the branch 1.

6.




Period = 3T, ω = 2π/3T

(a) mean value of waveform = ½ (20 – 10) = 5 A

(b) average (rectified) value = ½(20×2T+10×T)/3T = 25/3 = 8.33 A

(c) r.m.s. value

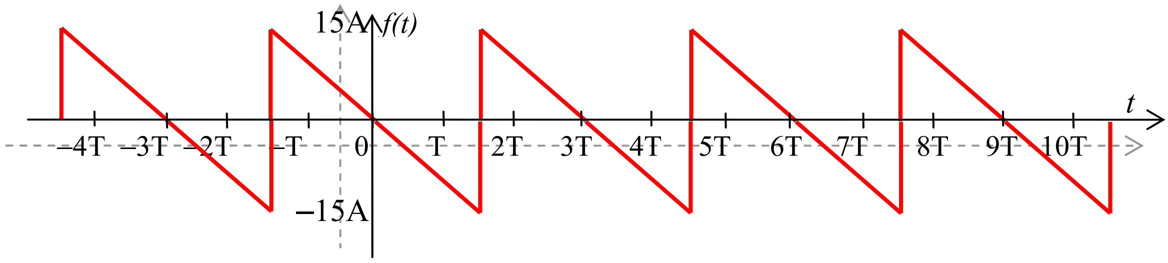
$$= \sqrt{\frac{1}{3T}\int_{-T}^{2T}\left(10-\frac{10}{T}\times t\right)^2\cdot dt} = \sqrt{\frac{1}{3T}\times\frac{1}{3}\left(10-\frac{10}{T}\times t\right)^3\times\left(-\frac{T}{10}\right)\Bigg|_{-T}^{2T}} = \sqrt{-\frac{1}{90}\left((-10)^3-20^3\right)\Bigg|_{-T}^{2T}}$$

$= \sqrt{100} = 10.0$ A

(d) form factor = 10.0/8.33 = 1.2

**2 marks**

(e) waveform may be shifted by the mean value on the y-axis and by an amount T/2 on the x-axis, as follows to obtain a more symmetrical waveform




where i(t) = 5 + f(t – T/2), period = 3T, 3ωT = 2π

$f(t) = A_0/2 + \sum A_n \cos n\omega t + B_n \sin n\omega t$

function f(t) has mean value = 0 → $A_0/2 = 0$

f(t) is odd → $A_n = 0$ for all n

f(t) does not have half-wave symmetry → even harmonics are present

$$\therefore B_n = 2\times\frac{2}{3T}\int_0^{3T/2} f(t).\sin n\omega t.dt = \frac{4}{3T}\left[\int_0^{3T/2}\left(-\frac{10t}{T}\right).\sin n\omega\, t.dt\right]$$

$$\text{i.e. } B_n = \frac{4}{3T}\times\left[\left(-\frac{10t}{T}\right).\frac{\cos n\omega\, t}{-n\omega}\Bigg|_0^{3T/2} - \int_0^{3T/2}\left(-\frac{10}{T}\right).\frac{\cos n\omega\, t}{-n\omega}.dt\right]$$

$$= \frac{4}{3T}\left[\left(\frac{15}{n\omega}\right)\cos\frac{3n\omega T}{2} - \left(\frac{10}{T}\right).\frac{\sin n\omega\, t}{(n\omega)^2}\Bigg|_0^{3T/2}\right] = \left[\frac{30}{n\pi}\cdot\cos n\pi - \frac{30}{(n\pi)^2}\cdot\sin n\pi\right]$$

$B_1 = (30/\pi).\cos\pi - 0 = -9.55$, $B_2 = (30/2\pi).\cos 2\pi - 0 = 4.77$,

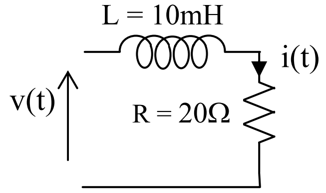
$B_3 = (30/3\pi).\cos 3\pi - 0 = -3.18$, $B_4 = (30/4\pi).\cos 4\pi - 0 = 2.39$, $B_5 = \ldots\ldots$

**4 marks**

∴ f(t) = - 9.55 sin (2π/3T)t + 4.77 sin (2×2π/3T)t – 3.18 sin (3×2π/3T)t +....

and $i(t) = 5 + f(t - \frac{T}{2}) = 5 - 9.55\sin\left(\frac{2\pi}{3T}t - \frac{\pi}{3}\right) + 4.77\sin\left(\frac{4\pi}{3T}t - \frac{2\pi}{3}\right) + 3.18\sin\left(\frac{2\pi}{T}t\right) + \ldots$ **2 marks**

T = 10 ms, $\omega = 2\pi/3T = 200\pi/3 = 209.4$ rad/s




(f) $V_n = I_n*[20 + jn(2\pi/0.01)\times 0.01] = I_n*[20 + j\,2n\pi/3]$

$|V_n| = I_n*\sqrt{[20^2 + 4\pi^2 n^2/9]}$, $\theta_n = \tan^{-1}[n\pi/10]$

$V_{dc} = 5*20 = 100$ V

$V_{1m} = -9.55*\sqrt{[20^2 + 4\pi^2/9]} = -192.0$

$\theta_1 = \tan^{-1}[\pi/30] = 6.0^o$

$V_{2m} = 4.77*\sqrt{[20^2 + 16\pi^2/9]} = 97.5$, $\theta_2 = \tan^{-1}[2\pi/30] = 11.83^o$

$V_{3m} = -3.18*\sqrt{[20^2 + 36\pi^2/9]} = -66.7$, $\theta_3 = \tan^{-1}[3\pi/30] = 17.44^o$

**2 marks**

and $v(t) = 100 - 192\sin\left(\frac{2\pi}{3T}t - 54.0^o\right) + 97.5\sin\left(\frac{4\pi}{3T}t - 108.2^o\right) + 66.7\sin\left(\frac{2\pi}{T}t + 17.4^o\right) + .$

OR using $v(t) = L\frac{di}{dt} + R\,i$

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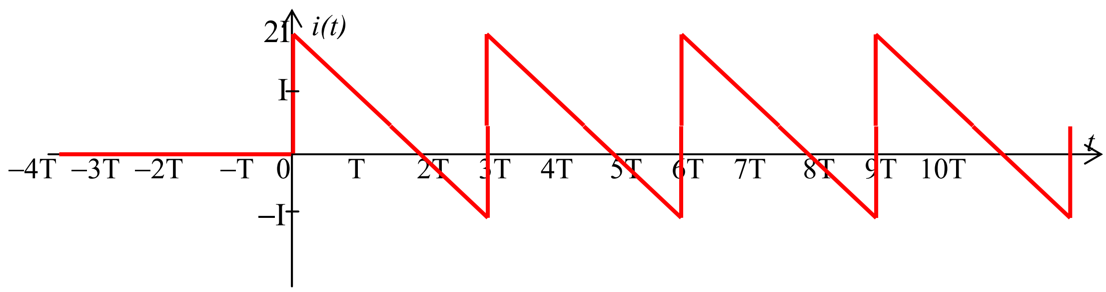
7\. (a) Laplace transform of the unit step L[h(t)] = $\int_0^\infty 1.e^{-st}dt = \left.\frac{e^{-st}}{-s}\right|_0^\infty = \frac{1}{s}$, **1 mark**

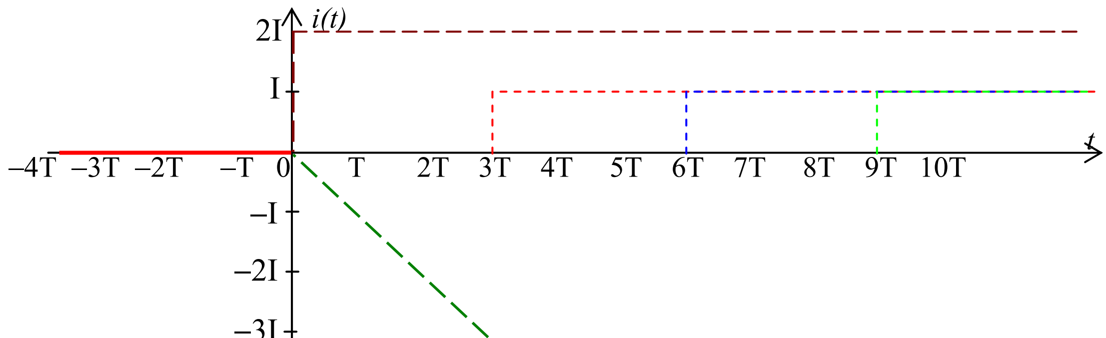
(b) for unit ramp L[r(t)] = $\int_0^\infty t.e^{-st}dt = t.\left.\frac{e^{-st}}{-s}\right|_0^\infty - \int_0^\infty \frac{e^{-st}}{-s}.dt = -\left.\frac{e^{-st}}{(-s)^2}\right|_0^\infty = \frac{1}{s^2}$ **1 mark**

(c) for shifted waveform L[f(t-a)] $= \int_0^\infty f(t-a).e^{-st}dt = \int_{-a}^\infty f(t).e^{-s(t+a)}d(t+a)$

$= \int_0^\infty f(t).e^{-st}.e^{-sa}dt = e^{-sa}.F(s)$ **1 mark**

(d) given waveform can be split into ramp and steps as follows



i.e. $i(t) = 2I.h(t) - \frac{I}{T}r(t) + I\,h(t-3T) + I\,h(t-6T) + I\,h(t-9T) + ....$

$$I(s) = \frac{2I}{s} - \frac{I}{s^2T} + \frac{I}{s}e^{-3sT} + \frac{I}{s}e^{-6sT} + \frac{I}{s}e^{-9sT} + ....$$

**5 marks**

Alternate Method

For a repetitive waveform, with period T

$$f(t) = f_1(t)\Big|_o^T + f_2(t)\Big|_T^{2T} + f_3(t)\Big|_{2T}^{3T} + f_4(t)\Big|_{3T}^{4T} + .....$$

$$= f_1(t)\Big|_o^T + f_2(t-T)\Big|_0^T + f_3(t-2T)\Big|_0^T + f_4(t-3T)\Big|_0^T + .....$$

$$F(s) = L\left[f_1(t)\Big|_o^T\right] + L\left[f_2(t-T)\Big|_0^T\right] + L\left[f_3(t-2T)\Big|_0^T\right] + L\left[f_4(t-3T)\Big|_0^T\right] + .....$$

$$= L\left[f_1(t)\Big|_o^T\right] + e^{-sT}L\left[f_1(t-T)\Big|_0^T\right] + e^{-2sT}L\left[f_1(t-2T)\Big|_0^T\right] + e^{-3sT}L\left[f_1(t-3T)\Big|_0^T\right] + .....$$

$$= \frac{1}{1-e^{-sT}}L\left[f_1(t)\Big|_0^T\right]$$

for the given waveform, considering one period only

$$F_1(s) = \int_0^{3T}\left(2I - \frac{I}{T}t\right)\cdot e^{-st}dt = \left(2I - \frac{I}{T}t\right)\cdot\frac{e^{-st}}{-s}\Bigg|_0^{3T} - \int_0^{3T}\left(-\frac{I}{T}\right)\cdot\frac{e^{-st}}{-s}dt$$

$$= -\frac{I\cdot e^{-3sT}}{-s} + \frac{2I}{s} + \frac{\left(e^{-3sT}-1\right)}{s^2T}$$

$$F(s) = \frac{1}{1-e^{-3sT}}\left[\frac{2I}{s} + \frac{I\cdot e^{-3sT}}{s} + \frac{\left(e^{-3sT}-1\right)}{s^2T}\right] = \frac{2I}{s} - \frac{I}{s^2T} + \frac{e^{-3sT}}{1-e^{-3sT}}\frac{I}{s}$$