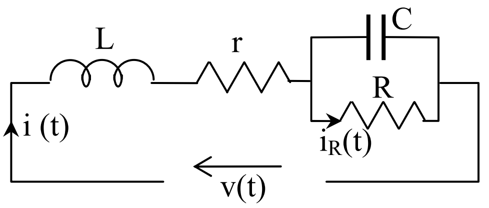
Level 2 Semester 2 Examination - March 2008

1 (a) For the circuit diagram shown on the right
Using differential operator as p = d/dt



$$i(t) = \frac{1}{Lp + r + \dfrac{R \times \dfrac{1}{Cp}}{R + \dfrac{1}{Cp}}} v(t) = \frac{v(t)}{Lp + r + \dfrac{R}{RCp + 1}}$$

$$= \frac{RCp + 1}{(Lp + r)(RCp + 1) + R} v(t)$$

Also, $i_R(t) = \dfrac{\dfrac{1}{Cp}}{\dfrac{1}{Cp} + R} i(t) = \dfrac{1}{(Lp + r)(RCp + 1) + R} v(t) = \dfrac{1}{LRCp^2 + (L + rRC)p + r + R} v(t)$

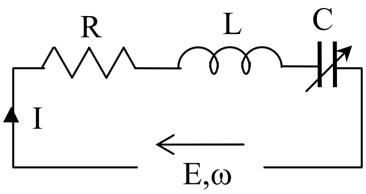
Giving the differential equation relationship as

$$v(t) = LRC\frac{d^2 i_R(t)}{dt^2} + (L + rRC)\frac{d\, i_R(t)}{dt} + (r + R)\, i_R(t)$$ [4 marks]

(b) For a sinusoidal voltage E the angular frequency ω, the differential operator p can be replaced by jω to give the expression for the current. Thus

$$I_R = \frac{E}{(r + R - LRC\omega^2) + j\omega(L + rRC)}$$ [1 mark]

(c) (i) For the circuit shown, $Z = R + j\omega L + \dfrac{1}{j\omega C}$, and



current $I = \dfrac{E}{R + j\omega L + \dfrac{1}{j\omega C}}$ [½ mark]

At resonance

(ii) $\omega = \dfrac{1}{\sqrt{LC_o}}$ giving $C_o = \dfrac{1}{L\omega^2}$ [½ mark]

(iii) $Q = \dfrac{L\omega}{R}$ [½ mark]

(iv) $I_{\max} = \dfrac{E}{R}$ [½ mark]

(v) at the half-power points, power is half and the current $|I| = \dfrac{I_{\max}}{\sqrt{2}} = \dfrac{E}{\sqrt{2}R}$

i.e. $\dfrac{E^2}{2R^2} = \dfrac{E^2}{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}$, giving $2R^2 = R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2$

or $R^2 = \left(\omega L - \dfrac{1}{\omega C}\right)^2$, *or* $\omega L - \dfrac{1}{\omega C} = \pm R$ giving $\dfrac{1}{\omega C} = \omega L \mp R$

i.e. $\frac{1}{C} = \omega^2 L \mp R\omega = \frac{1}{C_o} \mp \frac{1}{C_o Q}$ giving $\mp\frac{1}{C_o Q} = \frac{1}{C} - \frac{1}{C_o}$ or $\frac{1}{Q} = \left|\frac{C_o - C}{C}\right|$

$Q = \left|\frac{C}{C_o - C}\right|$ [3 marks]

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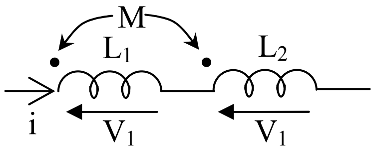
2 (a) Consider the case of the inductances aiding each other



$v_1 = L_1\frac{di}{dt} + M\frac{di}{dt},\quad v_2 = L_2\frac{di}{dt} + M\frac{di}{dt}$ [1 mark]

(for the opposing case, a negative sign would be associated with M)

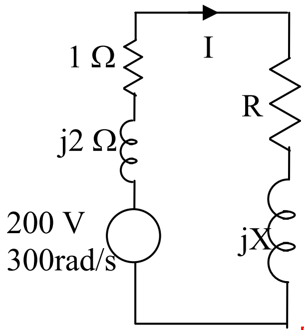
(b) Total energy W = ∫ $v_1.i_1.dt + v_2.i_2.dt = \int\left(L_1\frac{di}{dt} + M\frac{di}{dt}\right).i.dt + \left(L_2\frac{di}{dt} + M\frac{di}{dt}\right).i.dt$

$= \int (L_1.di + M.di).i.dt + (L_2.di + M.di).i.dt = \frac{1}{2}(L_1+L_2+2M)i^2$ [½ mark]

Energy stored in equivalent inductance is $\frac{1}{2} L_{eq} i^2$

∴ Equivalent inductance = $L_1 + L_2 + 2M$ [½ mark]

(c) For the circuit shown in the figure on right



Power factor of motor $= 0.8 = \frac{R}{\sqrt{R^2 + X^2}}$

so that $0.64R^2 + 0.64X^2 = R^2$

giving $X = 0.75\ R$ [½ mark]

Thus $I = \frac{200}{(R+1) + j(X+2)}$

and power delivered to load $P = |I|^2 R = \frac{200^2.R}{(R+1)^2 + (0.75R+2)^2}$ [½ mark]

for maximum power transfer, $\frac{dP}{dR} = 0$

giving $[(R+1)^2+(0.75\ R+2)^2].1 - R.[2(R+1)+2(0.75\ R+2)\times 0.75] = 0$

i.e. $R^2+2R+1 + (0.5625R^2+3R+4-2R^2 - 2R - 2\times 0.5625R^2 - 3R = 0$

so that $1.5625R^2 = 5$

i.e. R = 1.78885 Ω, X = 1.34164Ω [1½ marks]

∴ $P_{max} = \frac{200^2 \times 1.78885}{2.78885^2 + (0.75\times 1.78885 + 2)^2} = 3777$ W [1 mark]

$I = \frac{200}{\sqrt{2.78885^2 + (0.75\times 1.78885+2)^2}} = 45.95$ A [½ marks]

Alternate Solution: $P_{max} = 45.95^2 \times 1.78885 = 3777$ W.

voltage across load $= \sqrt{R^2 + X^2}.I = \sqrt{1.78885^2 + 1.34164^2} \times 45.95 = 102.75$ V [1 mark]

(d) Core loss $W = W_{hyst} + W_{eddy} = 100$ at 50 Hz, t = 0.5 mm

Hysteresis loss $W_{hyst} \propto B_m^{1.6}.f$, $W_{hyst} = k_h.f$ when B_m does not vary; and

Eddy current loss $W_{eddy} \propto B_m^{2}.f^2.t^2$, $W_{eddy} = k_e.f^2.t^2$ when B_m does not vary

also at 50 Hz, t = 0.5 mm $W = 100 = k_h \times 50 + k_e \times 50^2 \times 0.5^2$

and at 60 Hz, t = 0.4 mm $W = 103.3 = k_h \times 60 + k_e \times 60^2 \times 0.4^2$ [1 mark]

eliminating k_h gives

$1.2 \times 100 - 103.3 = 1.2 \times k_e \times 50^2 \times 0.5^2 - k_e \times 60^2 \times 0.4^2$

re-arranging to leave k_e in the most convenient form to obtain desired results,

$16.7 = k_e \times 50^2 \times 0.5^2 (1.2 - 1.2^2 \times 0.8^2)$ [1 mark]

giving $W_{eddy} = k_e \times 50^2 \times 0.5^2 = 59.986$ W = 60 W

and $W_{hyst} = 100 - 60 = 40$ W [1 mark]

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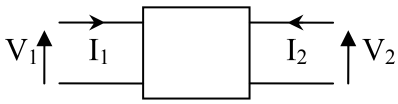
3 (a) $[Y] = \begin{bmatrix} 0.0385 - j0.182 & -0.0385 + j0.192 \\ -0.0385 + j0.192 & 0.0385 - j0.182 \end{bmatrix} S$



for the ABCD matrix parameters, for the circuit shown

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix},$$ giving $V_1 = A\,V_2 - B\,I_2$ and $I_1 = C\,V_2 - D\,I_2$

and for the admittance matrix parameters, for the circuit shown

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix},$$

from which $I_2 = y_{21}V_1 + y_{22}V_2$

which may be re-arranged to give $V_1 = (I_2 - y_{22}V_2)/y_{21}$

by comparison of coefficients,

$$A = -\frac{y_{22}}{y_{21}} = \frac{0.0385 - j0.182}{0.0385 - j0.192} = \frac{0.1860\angle -78.06^o}{0.1958\angle -78.66^o} = 0.950\angle 0.60^o$$

and $$B = -\frac{1}{y_{21}} = \frac{1}{0.0385 - j0.192} = \frac{1}{0.1958\angle -78.66^o} = 5.107\angle 78.66^o\ \Omega$$

Since the system is perfectly symmetric, A = D and A.D – B.C = 1

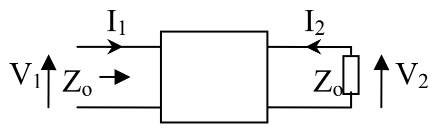
$\therefore D = 0.950\angle 0.60^o$

and $$C = \frac{A.D - 1}{B} = \frac{0.950\angle 0.60^o \times 0.950\angle 0.60^o - 1}{5.107\angle 78.66^o} = \frac{0.9025 + j0.0189 - 1}{5.107\angle 78.66^o} = \frac{-0.0975 + j0.0189}{5.107\angle 78.66^o}$$

i.e. $$C = \frac{-0.0975 + j0.0189}{5.107\angle 78.66^o} = 0.0194\angle 90.37^o\ S$$

Thus the ABCD matrix is given as $\begin{bmatrix} 0.950\angle 0.60^o & 5.107\angle 78.66^o \\ 0.0194\angle 90.37^o & 0.950\angle 0.60^o \end{bmatrix}$ [3 marks]

(b) When loaded at port 2 with Z_o, $\frac{V_2}{-I_2} = Z_o$



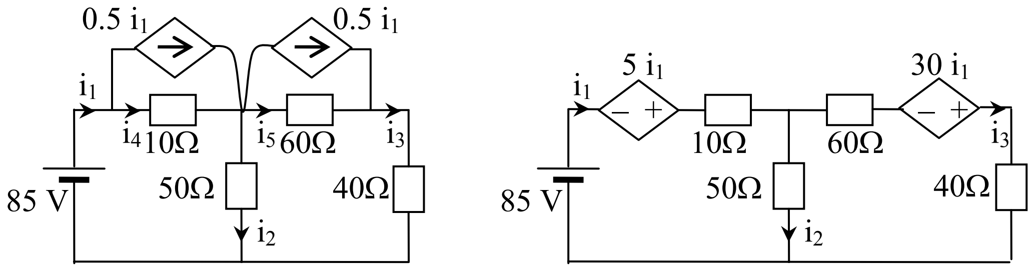
Also at port 1 input impedance, $\frac{V_1}{I_1} = Z_o$

Thus, from the ABCD parameter matrix $\frac{V_1}{I_1} = Z_o = \frac{AV_2 - BI_2}{CV_2 - DI_2} = \frac{AZ_o + B}{CZ_o + D}$

i.e. $CZ_o^2 + DZ_o = AZ_o + B$

Since A=D, $CZ_o^2 = B$, giving $Z_o = \sqrt{\frac{B}{C}} = \sqrt{\frac{5.107\angle 78.66^o}{0.0194\angle 90.37}} = 16.22\angle -5.7^o$ [2 marks]

(c) The current source can be distributed as shown and converted to voltage sources.



[2 marks]

(d) $85 + 5\, i_1 = 10\, i_1 + 50\, i_2$

$30\, i_1 = (60+40)\,(i_1 - i_2) - 50\, i_2$ [1 mark]

(e) which may be reduced to

$5\, i_1 + 50\, i_2 = 85$ or $i_1 + 10\, i_2 = 17$ and $15\, i_2 = 7\, i_1$

substitution gives $i_1 = 3$ A, $i_2 = 1.4$ A

and $i_3 = i_1 - i_2 = 1.6$ A, $i_4 = i_1 - 0.5 i_1 = 1.5$ A, $i_5 = i_3 - 0.5 i_1 = 0.1$ A [2 mark]

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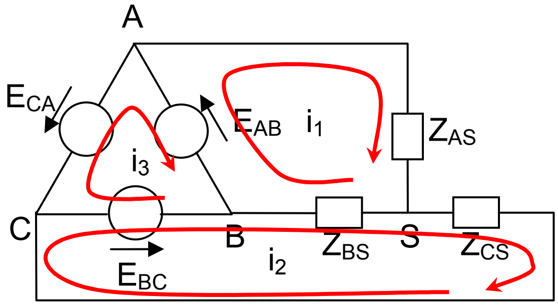
4 (a) $E_{AB} + E_{BC} + E_{CA} = 0$



$E_{BC} = -E_{AB} - E_{CA} = -100\angle 0^o - 150\angle 90^o$

$= -100 - j150 = 180.28\angle 236.31^o$ [1 mark]

(b) $[Z_m] = \begin{bmatrix} Z_{AS} + Z_{BS} & -Z_{BS} & 0 \\ -Z_{BS} & Z_{BS} + Z_{CS} & 0 \\ 0 & 0 & 0 \end{bmatrix}$, $[E_{gm}] = \begin{bmatrix} E_{AB} \\ E_{BC} \\ 0 \end{bmatrix}$

reduced equations eliminating zero column and zero row gives

$$[Z_m] = \begin{bmatrix} Z_{AS} + Z_{BS} & -Z_{BS} \\ -Z_{BS} & Z_{BS} + Z_{CS} \end{bmatrix} = \begin{bmatrix} 50\angle 0^O + 50\angle 30^O & -50\angle 30^O \\ -50\angle 30^O & 50\angle 30^O + 50\angle 60^O \end{bmatrix}$$

$$= \begin{bmatrix} 93.30 + j25 & -43.30 - j25 \\ -43.30 - j25 & 68.30 + j68.30 \end{bmatrix}$$ [1 mark]

$$[E_{gm}] = \begin{bmatrix} E_{AB} \\ E_{BC} \end{bmatrix} = \begin{bmatrix} 100\angle 0^o \\ 180.28\angle 236.31^o \end{bmatrix}$$ [1 mark]

(c) $$\begin{bmatrix} 100\angle 0^o \\ 180.28\angle 236.31^o \end{bmatrix} = \begin{bmatrix} 93.30 + j25 & -43.30 - j25 \\ -43.30 - j25 & 68.30 + j68.30 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 96.59\angle 15^o & -50\angle 30^o \\ -50\angle 30^o & 96.59\angle 45^o \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 96.59\angle 15^o & -50\angle 30^o \\ -50\angle 30^o & 96.59\angle 45^o \end{bmatrix}^{-1} \begin{bmatrix} 100\angle 0^o \\ 180.28\angle 236.31^o \end{bmatrix}$$

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \frac{1}{96.59^2\angle 60^o - 50^2\angle 60^o} \begin{bmatrix} 96.59\angle 45^o & 50\angle 30^o \\ 50\angle 30^o & 96.59\angle 15^o \end{bmatrix} \begin{bmatrix} 100\angle 0^o \\ 180.28\angle 236.31^o \end{bmatrix}$$

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \frac{1}{6829.6\angle 60^o} \begin{bmatrix} 96.59\angle 45^o & 50\angle 30^o \\ 50\angle 30^o & 96.59\angle 15^o \end{bmatrix} \begin{bmatrix} 100\angle 0^o \\ 180.28\angle 236.31^o \end{bmatrix}$$

$i_1 = 1.414\angle -15^o + 1.3198\angle 206.31^o = 1.366 - j\,0.366 - 1.1831 - j\,0.5850$

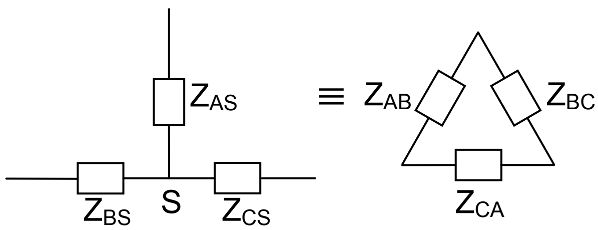
$I_{AS} = i_1 = 0.183 - j\,0.951 = 0.968\angle -79.1^o$ A

$i_2 = 0.7321\angle -30^o + 2.5497\angle 191.31^o = 0.6340 - j\,0.3660 - 2.5 - j\,0.5$

$I_{CS} = -i_2 = 1.866 - j\,0.866 = 2.057\angle -24.90^o$ [3 marks]

(d) $$Y_{AB} = \frac{Y_{AS}.Y_{BS}}{Y_{AS} + Y_{BS} + Y_{CS}}$$

$$Y_{AB} = \frac{0.02\angle 0^o \times 0.02\angle -30^o}{0.02\angle 0^o + 0.02\angle -30^o + 0.02\angle -60^o}$$



$$Y_{AB} = \frac{0.02\angle -30^o}{1 + 0.866 - j0.5 + 0.5 - j0.866} = \frac{0.02\angle -30^o}{2.366 - j1.366} = \frac{0.02\angle -30^o}{2.732\angle -30^o} = 0.00732\angle 0^o$$

$$Y_{CA} = \frac{0.02\angle -60^o \times 0.02\angle 0^o}{0.02\angle 0^o + 0.02\angle -30^o + 0.02\angle -60^o} = 0.00732\angle -30^o$$ [2 marks]

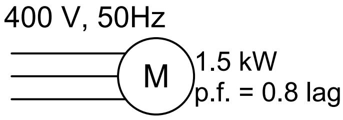
(e) $I_{AB} = 100\angle 0^o \times 0.00732\angle 0^o = 0.732\angle 0^o$

$I_{CA} = 150\angle 90^o \times 0.00732\angle -30^o = 1.098\angle 60^o$ [1 mark]

(f) $I_{AS} = I_{AB} - I_{AC} = 0.732\angle 0^o - 1.098\angle 60^o = 0.732 - 0.549 - j\,0.951$

$= 0.183 - j\,0.951 = 0.968\angle -79.1^o$ [1 mark]

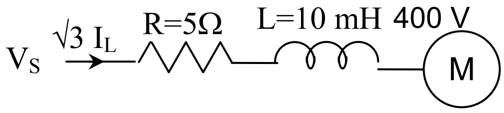
5 (a) $P = \sqrt{3}\, V_L I_L \cos\phi$



$$I_L = \frac{1500}{\sqrt{3}\times 400\times 0.6} = 3.6084A$$ [1 mark]

(b) $Q = \sqrt{3}\, V_L I_L \sin\phi = P\tan\phi = 1500\times\frac{0.8}{0.6} = 2000$ var [1 mark]

(c) Using equivalent single phase circuit



$\sqrt{3}\, I_L = \sqrt{3}\times 3.6084 = 6.25\angle{-53.13^o}$

$V_S = 400 + 6.25\angle{-53.13^o}\times(5+j\,10\times10^{-3}\times100\pi) = 400 + 6.25\angle{-53.13^o}\times5.905\angle 32.14^o$

$= 400 + 36.906\angle{-21.0^o} = 400 + 34.46 - j\,13.21 = 434.66\angle{-1.74^o}$ V [2 marks]

(d) overall power factor at source = $\cos(-1.74^o + 53.13^o) = 0.624$ lag [1 mark]

(e) new power factor = 0.95 lag, $\phi_{new} = \cos^{-1}(0.95) = 18.19^o$

new Q = 1500 tan 18.19^o = 493.0 var

Q required = 2000 – 493 = 1507 var

For 3 capacitors connected in delta, voltage = 400 V

$3\times400^2\times C\times100\pi = 1507$

$C = 9.994\times10^{-6} = 10\ \mu F$ [2 marks]

(f) $I_A = 10\angle 0^o A$, $I_B = 10\angle{-120^o}A$ and $I_C = 50\angle 90^o A$

$$\begin{bmatrix} I_{A0} \\ I_{A1} \\ I_{A2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}\begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}\begin{bmatrix} 10\angle 0^o \\ 10\angle -120^o \\ 50\angle 90^o \end{bmatrix}$$

$I_{A0} = \frac{1}{3}(10\angle 0^o + 10\angle{-120^o} + 50\angle 90^o) = \frac{1}{3}(10 - 5 - j8.66 + j50) = \frac{1}{3}(5 + j41.34)$

i.e. $I_{A0} = I_{B0} = I_{C0} = 13.88\angle 83.10^o$ A [1 mark]

$I_{A1} = \frac{1}{3}(10\angle 0^o + 10\angle{-120^o}\times1\angle 120^o + 50\angle 90^o\times1\angle{-120^o}) = \frac{1}{3}(10+10+43.3 - j25) = \frac{1}{3}(63.3 - j25)$

$I_{A1} = 22.69\angle{-21.55^o}$ A, $I_{B1} = 22.69\angle{-141.55^o}$ A, $I_{C1} = 22.69\angle 98.45^o$ A [1 mark]

$I_{A2} = \frac{1}{3}(10\angle 0^o + 10\angle{-120^o}\times1\angle{-120^o} + 50\angle 90^o\times1\angle 120^o) = \frac{1}{3}(10 - 5 + j8.66 - 43.3 - j25)$

$= \frac{1}{3}(-38.3 - j16.34)$

$I_{A2} = 18.85\angle 203.10^o$ A, $I_{B2} = 18.85\angle{-36.90^o}$ A, $I_{C2} = 18.85\angle 83.10^o$ A [1 mark]

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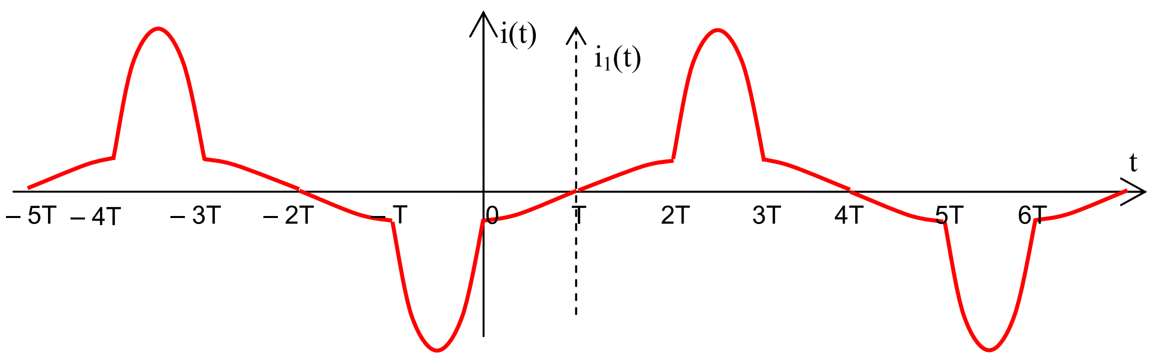
6 (a) Period of waveform = 6T, $\omega_o.6T = 2\pi$ so that $\omega_o = \frac{\pi}{3T}$ [1 mark]

(b) The y-axis may be shifted by T so that the new waveform $i_1(t)$ becomes an odd waveform.



$i_1(t) = i(t - T)$

The waveform $i_1(t)$ has both odd symmetry as well as half wave symmetry. Also it is made out of the waveforms – cos $\omega_o t$ throughout and an additional periodic waveform superposed $i_2(t)$.

Thus $i_2(t) = 0$ $\quad 0 \le t \le T$

$i_2(t) = -4\sin 3\omega_o t$ $\quad T \le t \le \frac{3}{2}T$

Thus for $i_2(t)$, the Fourier coefficients are given as

$A_o = 0$ (since mean value = 0), $A_n = 0$ (since odd function), [1½ marks]

$B_n = 0$ for even n (since half wave symmetry)

Thus for odd n,

$$B_n = 4\times\frac{2}{T}\int_T^{\frac{3T}{2}} -4\sin 3\omega_o t.\sin n\omega_o t dt = -\frac{16}{T}\int_T^{\frac{3T}{2}} [\cos(n-3)\omega_o t - \cos(n+3)\omega_o t.dt$$

$$B_n = -\frac{16}{T}\left[\frac{\sin(n-3)\omega_o t}{(n-3)\omega_o} - \frac{\sin(n+3)\omega_o t}{(n+3)\omega_o}\right]_T^{\frac{3T}{2}}$$

$$= -\frac{16}{\omega_o T}\left[\frac{\sin\frac{(n-3)\omega_o 3T}{2} - \sin(n-3)\omega_o T}{n-3} - \frac{\sin\frac{(n+3)\omega_o 3T}{2} - \sin(n+3)\omega_o T}{n+3}\right]$$

$$= -\frac{48}{\pi}\left[\frac{\sin(n-3)\frac{\pi}{2} - \sin(n-3)\frac{\pi}{3}}{n-3} - \frac{\sin(n+3)\frac{\pi}{2} - \sin(n+3)\frac{\pi}{3}}{n+3}\right]$$

$$= \frac{48}{\pi}\left[\frac{\sin(n-3)\frac{\pi}{3}}{n-3} - \frac{\sin(n+3)\frac{\pi}{3}}{n+3}\right]$$, since n is odd [2 marks]

Thus $B_1 = \frac{48}{\pi}\left[\frac{\sin\frac{2\pi}{3}}{2} - \frac{\sin\frac{4\pi}{3}}{4}\right] = \frac{18\sqrt{3}}{\pi} = 9.924$

B_3 cannot be calculated from the general expression since $(n - 3) = 0$.

Thus $B_3 = 4\times\frac{2}{T}\int_T^{\frac{3T}{2}} -4\sin 3\omega_o t.\sin 3\omega_o t dt = \frac{16}{T}\int_T^{\frac{3T}{2}}[\cos 6\omega_o t - 1].dt = \frac{16}{T}\left[\frac{\sin 6\omega_o t}{6\omega_o} - t\right]_T^{\frac{3T}{2}}$

i.e. $B_3 = \frac{16}{2\pi}[\sin 9\omega_o T - \sin 6\omega_o T - 9\omega_o T + 6\omega_o T] = -8$

and $B_5 = \frac{48}{\pi}\left[\frac{\sin\frac{2\pi}{3}}{2} - \frac{\sin\frac{8\pi}{3}}{8}\right] = \frac{18\sqrt{3}}{\pi}$ [1½ marks]

Thus the Fourier series of the original waveform is

$i(t) = -\cos\omega_o t + 9.924\sin\omega_o(t+T) - 8\sin 3\omega_o(t+T) + 9.924\sin 5\omega_o(t+T) + \ldots$ [1 mark]

$i(t) = -\cos\omega_o t + 9.924\times\cos(60^o)\sin\omega_o t + 9.924\times\sin(60^o)\cos\omega_o t - 8\sin(3\omega_o t+180^o) + 9.924\sin(5\omega_o t+300^o) + \ldots$

$i(t) = 8.808\sin(\omega_o t + 33.16^o) + 8\sin 3\omega_o t + 9.924\sin(5\omega_o t+300^o) + \ldots$ [1 mark]

(c) If the current is supplied from a source 100 sin ω_ot with internal resistance 1 Ω,

Terminal voltage v(t) = e(t) – R.i(t)

$v(t) = 100\sin\omega_o t - 8.808\sin(\omega_o t + 33.16^o) - 8\sin 3\omega_o t - 9.924\sin(5\omega_o t+300^o) + \ldots$

$= 92.63\sin\omega_o t - 4.82\cos\omega_o t - 8\sin 3\omega_o t - 9.924\sin(5\omega_o t+300^o) + \ldots$

$= 92.76\sin(\omega_o t - 0.12^o) - 8\sin 3\omega_o t - 9.924\sin(5\omega_o t+300^o) + \ldots$ [2 marks]

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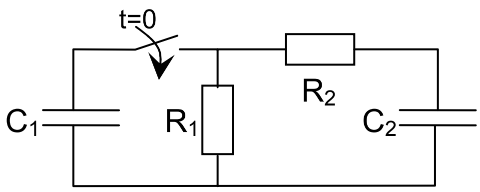
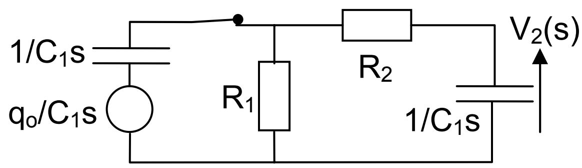
7 (a)





The given circuit has the Laplace Transformed circuit shown. [1 mark]

(b) $$V_2(s) = \frac{\frac{1}{C_2 s}}{R_2 + \frac{1}{C_2 s}} \cdot \frac{q_o}{C_1 s} \cdot \frac{R_1 // \left(R_2 + \frac{1}{C_2 s}\right)}{\frac{1}{C_1 s} + R_1 // \left(R_2 + \frac{1}{C_2 s}\right)} = \frac{1}{R_2 C_2 s + 1} \cdot \frac{q_o}{C_1 s} \cdot \frac{\dfrac{R_1 \cdot \left(R_2 + \frac{1}{C_2 s}\right)}{R_1 + R_2 + \frac{1}{C_2 s}}}{\dfrac{1}{C_1 s} + \dfrac{R_1 \cdot \left(R_2 + \frac{1}{C_2 s}\right)}{R_1 + R_2 + \frac{1}{C_2 s}}}$$

$$= \frac{1}{R_2 C_2 s + 1} \cdot \frac{q_o}{C_1 s} \cdot \frac{\dfrac{R_1 \cdot (R_2 C_2 s + 1)}{(R_1 + R_2) C_2 s + 1}}{\dfrac{1}{C_1 s} + \dfrac{R_1 \cdot (R_2 C_2 s + 1)}{(R_1 + R_2) C_2 s + 1}} = q_o \cdot \frac{R_1}{(R_1 + R_2) C_2 s + 1 + R_1 C_1 s \cdot (R_2 C_2 s + 1)}$$

$$V_2(s) = \frac{q_o \cdot R_1}{R_1 C_1 R_2 C_2 s^2 + (R_1 C_2 + R_2 C_2 + R_1 C_1) s + 1}$$ [3 marks]

(c) C_1 = 90 nF, C_2 = 10 nF, q_o = 0.0225 C, R_1 = 1 kΩ and R_2 = 100 Ω

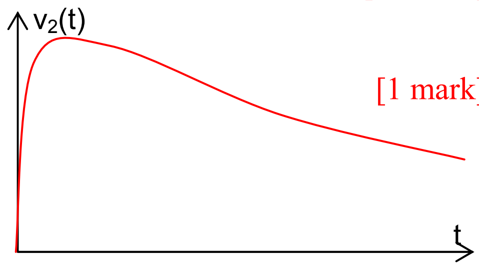
$$V_2(s) = \frac{0.0225 \times 1000}{1000 \times 90 \times 10^{-9} \times 100 \times 10 \times 10^{-9} s^2 + (10 \times 10^{-6} + 1 \times 10^{-6} + 90 \times 10^{-6}) s + 1}$$

i.e. $$V_2(s) = \frac{22.5}{90 \times 10^{-12} s^2 + 101 \times 10^{-6} s + 1} = \frac{0.25 \times 10^{12}}{s^2 + 1.1222 \times 10^6 s + 0.011111 \times 10^{12}}$$

$$V_2(s) = \frac{0.25\times 10^{12}}{(s+1.11223\times 10^6)(s+0.009989\times 10^6)}$$

$$V_2(s) = \frac{0.25\times 10^{12}}{1.1022\times 10^6}\left[\frac{1}{(s+0.009989\times 10^6)} - \frac{1}{(s+1.11223\times 10^6)}\right]$$

$v_2(t) = 227\,(e^{-0.01t} - e^{-1.112t})$ kV with t in μs [3 marks]



(d) Sketch of waveform is shown [1 mark]

(e) for maximum value, dv/dt = 0

i.e. $0.01e^{-0.01t} = 1.112e^{-1.112t}$

$e^{-1.102t} = 0.0899$

t = 2.186 μs

peak value = $227\,(e^{-0.01\times 2.186} - e^{-1.112\times 2.186})$

= 227 (0.978 – 0.088) = 202 kV [2 marks]