

**EE 201 - THEORY OF ELECTRICITY - Answers****Level 2 Examination - December 2001**

1. Impedance $Z = R + j \omega L + 1/j \omega C$
 current $I = E/Z$
 $= E/\sqrt{(R^2 + (\omega L - 1/\omega C)^2)}$

For maximum current (resonance),

$$\omega L = 1/\omega C$$

Value of maximum current $I_{\max} = 5\text{mA} = E/R$

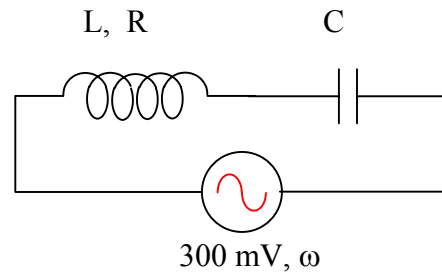
i.e. $0.005 = 0.3/R$, giving $R = 60 \Omega$

Q-factor for a simple series circuit $= L\omega/R$

i.e. $105 = L \times 2\pi \times 6000/60$, giving $L = 0.167 \text{ H}$ or **167 mH**

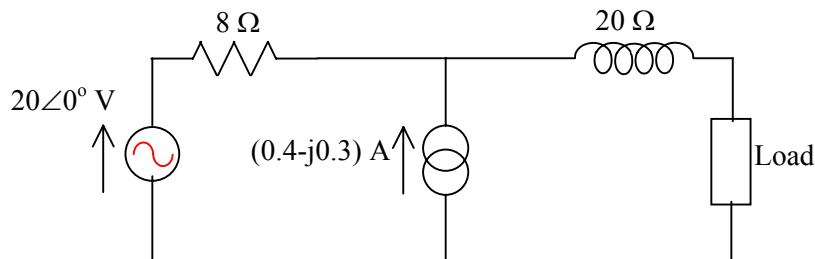
Resonance frequency $f_0 = \omega/2\pi = 1/2\pi\sqrt{LC}$

i.e. $6000 = 1/2\pi\sqrt{(0.167 \times C)}$, giving $C = 4.21 \times 10^{-9}$ or **4.21 μF**



2. a) The Maximum Power Transfer theorem states that for maximum active power to be delivered to the load, the load impedance must correspond to the conjugate of the source impedance (or in the case of direct quantities, be equal to the source impedance. With restrictive loads, other solutions would also exist).

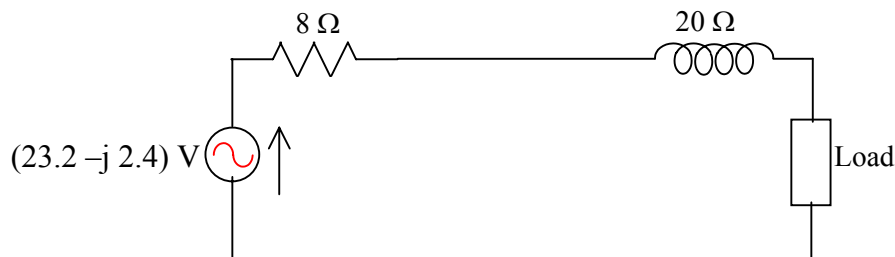
b)



Convert voltage source to equivalent current source $\Rightarrow I_{\text{source}} = 20/8 = 2.5\angle 0^\circ \text{ A}$ in parallel with a resistance of 8Ω .

This will give a total source current of $2.5\angle 0^\circ + 0.4 - j0.3 = 2.9 - j0.3 \text{ A}$ in parallel with the 8Ω resistance.

This can be converted to an equivalent voltage source of $(2.9 - j0.3) \times 8 = (23.2 - j2.4) \text{ V}$ or $23.32\angle -5.91^\circ$ in series with a resistor of 8Ω .



Thus the total effective source impedance before the Load is $(8 + j20) \Omega$.

Thus for maximum power transfer, **Load impedance = $(8 - j20) \Omega$** .

Value of **maximum power** $= 23.32^2/4 \times 8 = 17 \text{ W}$.

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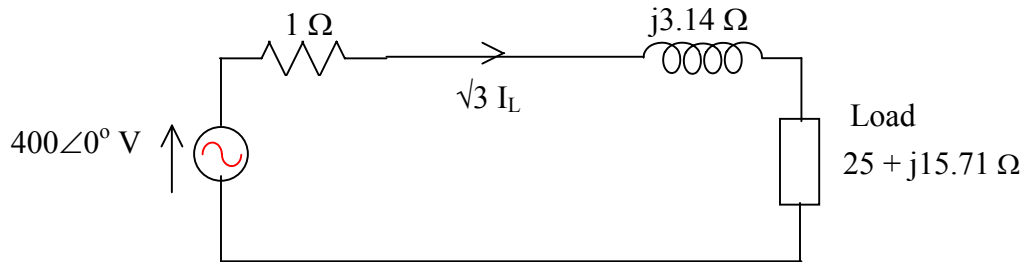
3. Impedance of load inductance at 50 Hz = $0.15 \times 2\pi \times 50 = 47.12 \Omega$

Impedance of each arm of balanced delta connected load = $75 + j 47.12 \Omega$

Impedance of line inductance at 50 Hz = $0.01 \times 2\pi \times 50 = 3.14 \Omega$

Impedance of each arm of balanced equivalent star = $(75 + j 47.12)/3 \Omega = 25 + j15.71 \Omega$

Thus the equivalent single phase diagram may be drawn as



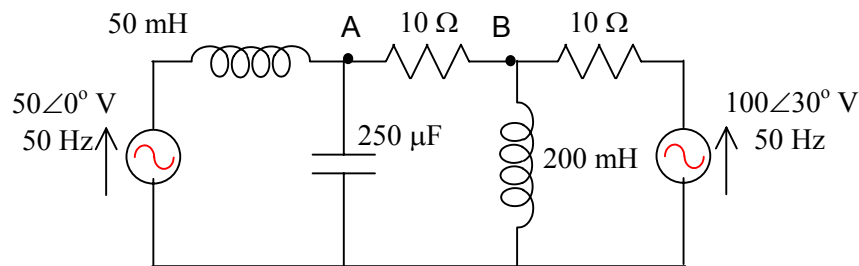
From the diagram, $\sqrt{3} I_L = 400\angle 0^\circ / (1 + j3.14 + 25 + j15.71) = 400\angle 0^\circ / 32.11\angle 35.94^\circ$

Thus **Line current $I_L = 7.19\angle -35.9^\circ$ A**

line voltage across load = $\sqrt{3} I_L Z_{\text{load}} = \sqrt{3} \times 7.19\angle -35.9^\circ \times (25 + j15.71)$
 $= \sqrt{3} \times 7.19\angle -35.9^\circ \times 29.52\angle 32.15^\circ$

Thus **Line voltage across load $V_L = 367.6\angle -3.7^\circ$ V**

4.

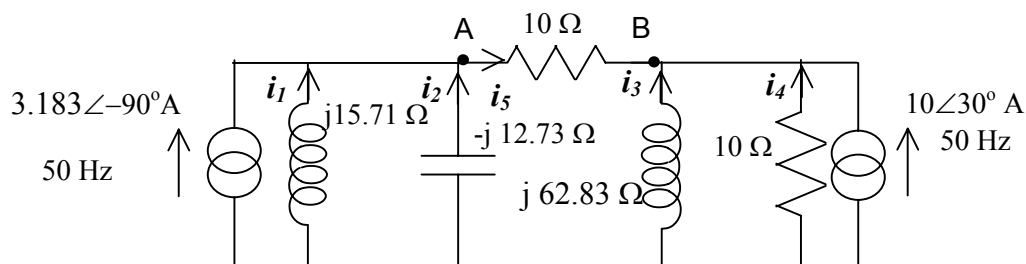


Impedance of elements at 50 Hz is as follows

$50 \text{ mH} \Rightarrow j 2\pi \times 50 \times 50 \times 10^{-3} = j 15.71 \Omega$, $200 \text{ mH} \Rightarrow j 62.83 \Omega$,

$250 \mu\text{F} \Rightarrow 1 / (j 2\pi \times 50 \times 250 \times 10^{-6}) = -j 12.73 \Omega$

The 50V source gets converted to an equivalent current source of $50\angle 0^\circ / j 15.71 = 3.183\angle -90^\circ \text{ A}$, and the 100 V source gets converted to a $100\angle 30^\circ / 10 = 10\angle 30^\circ \text{ A}$ current source.



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There are 3 nodes, of which the common node at the bottom may be taken as reference.

Thus the branch-node incidence matrix $[A]$ and the branch admittance matrix are given as

$$[A] = \begin{pmatrix} -1 & 0 \\ -1 & 0 \\ 0 & -1 \\ 0 & -1 \\ 1 & -1 \end{pmatrix}, \quad [Y_b] = \begin{pmatrix} 1/j15.71 & 0 & 0 & 0 & 0 \\ 0 & -1/j12.73 & 0 & 0 & 0 \\ 0 & 0 & 1/j62.83 & 0 & 0 \\ 0 & 0 & 0 & 1/10 & 0 \\ 0 & 0 & 0 & 0 & 1/10 \end{pmatrix}$$

The nodal admittance matrix can be obtained as $[Z_N] = [A]^t [Y_b] [A]$

$$= \begin{pmatrix} -1 & -1 & 0 & 0 & 1 \\ 0 & 0 & -1 & -1 & -1 \end{pmatrix} \begin{pmatrix} 1/j15.71 & 0 & 0 & 0 & 0 \\ 0 & -1/j12.73 & 0 & 0 & 0 \\ 0 & 0 & 1/j62.83 & 0 & 0 \\ 0 & 0 & 0 & 1/10 & 0 \\ 0 & 0 & 0 & 0 & 1/10 \end{pmatrix} \begin{pmatrix} -1 & 0 \\ -1 & 0 \\ 0 & -1 \\ 0 & -1 \\ 1 & -1 \end{pmatrix}$$

This can be shown to reduce to a (2×2) matrix $[Y_N]$

The nodal injected currents I_N at the nodes A and B are $3.183 \angle -90^\circ$ A and $10 \angle 30^\circ$ A.

Thus the nodal voltage vector V_N consisting of V_A and V_B can be determined by solving the equation $I_N = [Y_N] V_N$

5 a) $V_{s \text{ rms}} = 100/\sqrt{2} \angle 30^\circ = 70.7 \angle 30^\circ$ V

$\omega = 100$ rad/s

40 mH $\Rightarrow j 4 \Omega$

200 μ F $\Rightarrow -j 50 \Omega$.

Mesh impedance matrix $[Z_m]$ can be written as

$$[Z_m] = \begin{pmatrix} 5 - j50 & j50 \\ j50 & 15 + j4 - j50 \end{pmatrix}$$

The matrix mesh analysis equation may be written as

$$\begin{pmatrix} 70.7 \angle 30^\circ \\ 0 \end{pmatrix} = \begin{pmatrix} 5 - j50 & j50 \\ j50 & 15 + j4 - j50 \end{pmatrix} \begin{pmatrix} i_{m1} \\ i_{m2} \end{pmatrix}$$

Therefore $j 50 i_{m1} = (15 - j46) i_{m2}$ or $i_{m1} = (-0.92 - j 0.3) i_{m2}$

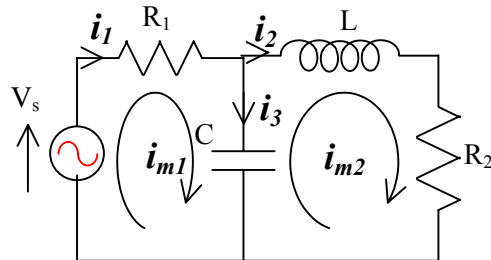
and $70.7 \angle 30^\circ = (5 - j50) i_{m1} + j50 i_{m2} = (5 - j50) (-0.92 - j 0.3) i_{m2} + j50 i_{m2}$

i.e. $70.7 \angle 30^\circ = (-19.6 + j44.5) i_{m2} = 48.63 \angle 113.8^\circ i_{m2}$ giving $i_{m2} = 1.454 \angle -83.8^\circ$ A

Thus $i_{m1} = (-0.92 - j 0.3) i_{m2} = 0.968 \angle 198.1^\circ \times 1.454 \angle -83.8^\circ = 1.407 \angle 114.3^\circ$ A

Thus the branch currents are $i_1 = 1.407 \angle 114.3^\circ$ A, $i_2 = 1.454 \angle -83.8^\circ$ A,

and $i_3 = i_{m1} - i_{m2} = (-0.579 + j1.282) - (0.157 - j1.445) = -0.736 + j2.727 = 2.825 \angle 105.1^\circ$ A





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b) The sequence components of the voltages are determined as follows.

$$\begin{pmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{pmatrix} = \frac{1}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{pmatrix} \begin{pmatrix} V_a = 200 \angle 0^\circ \\ V_b = 100 \angle -90^\circ \\ V_c = 150 \angle 150^\circ \end{pmatrix}$$

Thus

$$\begin{aligned} V_{a0} &= \frac{1}{3} (200 \angle 0^\circ + 100 \angle -90^\circ + 150 \angle 150^\circ) = \frac{1}{3} (200 - j100 - 129.9 + j75) \\ &= \frac{1}{3} (70.1 - j25) = \frac{1}{3} (102.24 \angle -19.6^\circ) \text{ V} \end{aligned}$$

Zero sequence voltage = **34.08 \angle -19.6° V**

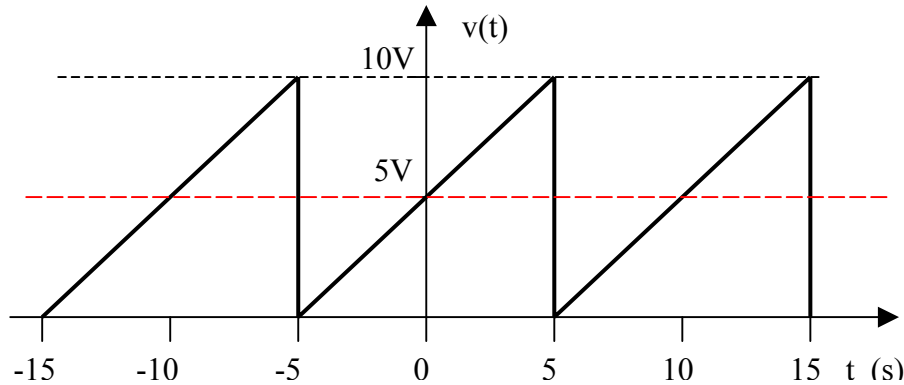
$$\begin{aligned} V_{a1} &= \frac{1}{3} (200 \angle 0^\circ + 1 \angle 120^\circ \times 100 \angle -90^\circ + 1 \angle 240^\circ \times 150 \angle 150^\circ) \\ &= \frac{1}{3} (200 + 86.6 + j50 + 129.9 + j75) = 138.83 + j41.67 = 144.95 \angle 16.71^\circ \text{ V} \end{aligned}$$

Positive sequence voltage = **144.95 \angle 16.7° V**

$$\begin{aligned} V_{a2} &= \frac{1}{3} (200 \angle 0^\circ + 1 \angle 240^\circ \times 100 \angle -90^\circ + 1 \angle 120^\circ \times 150 \angle 150^\circ) \\ &= \frac{1}{3} (200 - 86.6 + j50 - j150) = 37.80 + j33.33 = 50.40 \angle 41.40^\circ \text{ V} \end{aligned}$$

Negative sequence voltage = **50.40 \angle 41.4° V**

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Waveform $v(t)$ has a mean value of 5V. Therefore $A_0/2$ in Fourier Series = 5 V.

Consider new function $v(t) - 5$. This is an odd waveform. Therefore $A_n = 0$ for all n .

Waveform does not have half wave symmetry.

Period = 10 s, $\omega_0 = 2\pi/10 = \pi/5$

$$\begin{aligned} \text{There } B_n &= 2 \times 2/T \int [v(t) - 5] \sin n\omega_0 t \, dt \text{ from } t = 0 \text{ to } 5 \\ &= 4/10 \int t \cdot \sin n\pi t/5 \, dt \\ &= 0.4 [t \cdot \{ \cos n\pi t/5 \} / \{ n\pi/5 \} \Big|_{\text{limits}} - \int 1 \cdot \{ \cos n\pi t/5 \} / \{ n\pi/5 \} \, dt] \\ &= 0.4 [0 - 5 \times (5/n\pi) \cdot \cos n\pi - 0 + (5/n\pi)^2 \cdot \sin n\pi] \\ &= (10/n\pi) \cdot \cos n\pi \end{aligned}$$



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$$B_1 = -10/\pi, B_2 = 5/\pi$$

Thus the Fourier Series is

$$v(t) = 5 - (10/\pi) \sin \pi t/5 + (5/\pi) \sin 2\pi t/5 + \dots$$

$$\text{r m s value of waveform} = \sqrt{[5^2 + \frac{1}{2} (10/\pi)^2 + \frac{1}{2} (5/\pi)^2 + \dots]} \cong 5.60 \text{ V}$$

$$[\text{or more accurately } \sqrt{[(1/10) \int (5 + t)^2 dt]} = 5.774\text{V}]$$

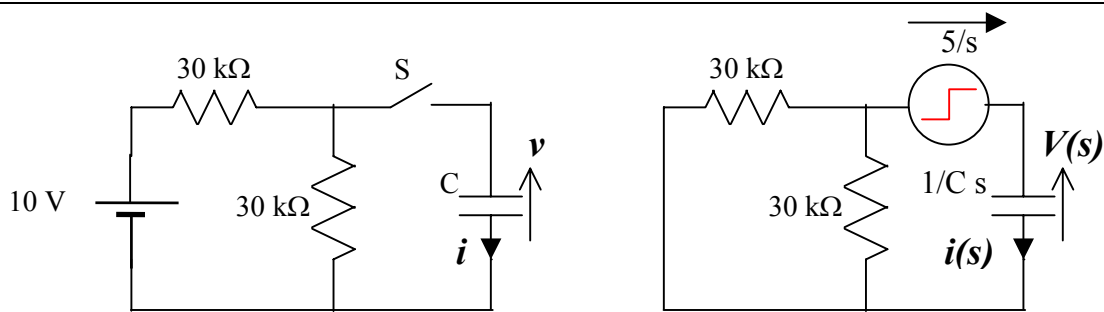
For this waveform, average value is identical to the mean value. i.e. average value = 5 V

Therefore **form factor** = $5.774/5 = 1.154$ (or $5.60/5 = 1.12$)

Peak value = 10 V

Therefore **peak factor** = $10/5.774 = 1.732$

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$$C = 3 \mu\text{F}.$$

Before closure of the switch, a steady current of $1/6$ mA would flow in the resistors giving a voltage of 5V across the switch and no voltage across the capacitor.

The closure of the switch would correspond to a cancellation of the voltage across it.

Thus the transformed circuit would be as shown.

$$\begin{aligned} i(s) &= (5/s) / (15,000 + 1/Cs) = (5 \times 3 \times 10^{-6}) / (15000 \times 3 \times 10^{-6} s + 1) \\ &= (15 \times 10^{-6}) / (0.045 s + 1) = 3.333 \times 10^{-4} / (s + 22.222) \end{aligned}$$

$$\text{therefore } i(t) = 3.333 \times 10^{-4} e^{-22.222t} \text{ A}$$

$$V(s) = (1/Cs) \cdot i(s) = 111.1 / [s(s + 22.222)] = 5 \{ 1/s - 1/(s + 22.222) \}$$

$$v(t) = 5 [1 - e^{-22.222t}] \text{ V}$$