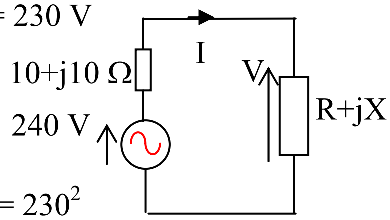
EE 201 - THEORY OF ELECTRICITY – Short Answers

Level 2 Semester 1 Examination - August 2006

1 (a) Current $I = \dfrac{240}{(10+R)+j(10+X)}$, voltage V = (R+jX).I , | V | = 230 V



active power $P = |I|^2 R = \dfrac{240^2}{[(R+10)^2+(X+10)^2]}.R$,

i.e. $|V|^2 = (R^2+X^2).|I|^2 = (R^2+X^2)\cdot\dfrac{240^2}{[(R+10)^2+(X+10)^2]} = 230^2$

2 marks

(b) $\therefore$ $(R+10)^2 + (X+10)^2 = 1.0888\,(R^2 + X^2)$

i.e. $20R + 20X + 200 = 0.0888\,(R^2 + X^2)$ or $R^2 + X^2 = 225.1\,(10 + R + X)$

i.e. $P = \dfrac{240^2}{1.0888[R^2+X^2]}\cdot R = \dfrac{240^2}{245.1[10+R+X]}\cdot R$

We can differentiate for maximum power, keeping the condition in mind.

$\dfrac{dP}{dR} = 0$, gives the condition $(10+R+X).1 - R.(1+\dfrac{dX}{dR}) = 0$

or $10+R+X = R+R\dfrac{dX}{dR}$, giving $R\dfrac{dX}{dR} = 10+X$

also $2R + 2X\dfrac{dX}{dR} = 225.1\times(0+1+\dfrac{dX}{dR})$

so that $2R + 2X(10+X)/R = 225.1\times(1 + (10+X)/R)$

i.e. $2R^2 + 20X + 2X^2 = 225.1R + 2251 + 225.1X$

i.e. $2\times225.1\times(10+R+X) + 20X = 225.1R + 2251 + 225.1X$

i.e. $2251 + 225.1R + 245.1X = 0$

i.e. $X = -0.9184\,(10+R)$

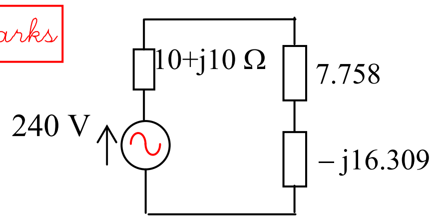
$\therefore$ $R^2 + 0.9184^2\,(10+R)^2 = 225.1\,(10 + R - 9.184 - 0.9184R)$

i.e. $1.8435R^2 + 84.35 + 16.87R = 183.68 + 18.368R$

giving $1.8435R^2 - 1.50R - 99.33 = 0$

i.e. $R = \underline{7.758\ \Omega}$, giving $X = -\underline{16.309\ \Omega}$

5 marks



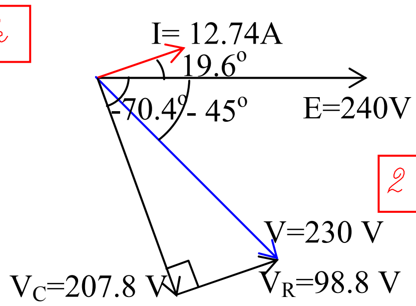
(c) substituting gives Maximum Power = 1258 W,

under the given conditions,

$I = \dfrac{240}{10+10+7.758-16.309} = \dfrac{240}{17.758-6.309}$

i.e. $I = \dfrac{240}{18.85\angle -19.56^o} = \underline{12.74\angle 19.56^o\ A}$

1 mark



2 marks

(d) terminal voltage at source (load voltage) = $12.74\angle 19.56^o \times 18.06\angle -64.56^o = 230.0\angle -45^o$ V

voltage drop across the resistive part of load = $7.758\times12.74\angle 19.56^o = 98.8\angle 19.6^o$ V

voltage drop across the capacitive part of load = $16.309\angle -90^o \times 12.74\angle 19.56^o = 207.8\angle -70.4^o$ V

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2. (a) The three methods of defining the resonance frequency of an R-L-C circuit are

i) current through a circuit is a maximum for a given voltage (or Z maximum, or voltage minimum, or Y minimum)

ii) voltage across a circuit is a maximum for a given current (or Y maximum, or current minimum, or Z minimum)

iii)power factor of the circuit is unity (or impedance or admittance is real)

2 marks

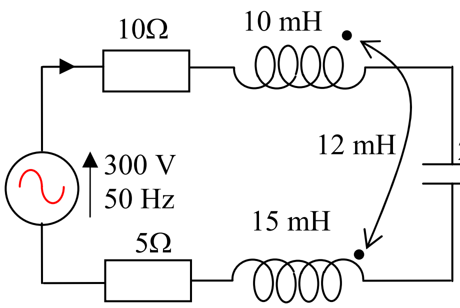
(b) 10 mH $\rightarrow$ j10×10^{-3} ×2π×100 = j6.283 Ω

15 mH $\rightarrow$ j9.425 Ω, 12 mH $\rightarrow$ j7.540 Ω

20 μF $\rightarrow$ 1/(j2π×100×20×10^{-6}) = – j159.2 Ω

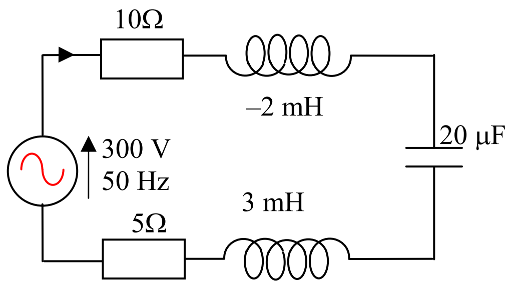
Using Kirchoff's voltage law

300 = 10 i + (j6.283 i – j7.540 i) – j 159.2i + (j9.425 i – j7.540 i) + 5 i



3 marks

This would correspond to the uncoupled circuit shown.



(c) Series resonance occurs between (3 – 2) mH and 20 μF.

i.e. resonance frequency $\omega = \dfrac{1}{\sqrt{1\times10^{-3}\times20\times10^{-6}}}$

$= 7071$ rad/s = 1125 Hz

1 mark

Current in the circuit $I = \dfrac{300}{15 + j\omega\, 0.001 + \dfrac{5\times10^4}{j\omega}}$

At resonance, current $I = \dfrac{300}{15} = 20A$

1 mark

(d) At half power points, the current magnitude would be 70.7% of the current at resonance.

i.e. $0.707\times20 = \dfrac{300}{\sqrt{15^2 + (\omega\, 0.001 - \dfrac{5\times10^4}{\omega})^2}}$

i.e. $200\times[225+(0.001\omega - 5\times10^4/\omega)^2] = 300^2$

$(0.001\omega - 5\times10^4/\omega)^2 = 225$

$0.001\omega - 5\times10^4/\omega = \pm15$

$\omega^2 \pm 15\times10^3\omega - 5\times10^7 = 0$

i.e. $\omega = \pm\, 7.5\times10^3 \pm \sqrt{(7.5\times10^3)^2 + 5\times10^7}$ = 17807 rad/s or 2807 rad/s

frequency at half power points = 2834 Hz or 452 Hz

3 marks

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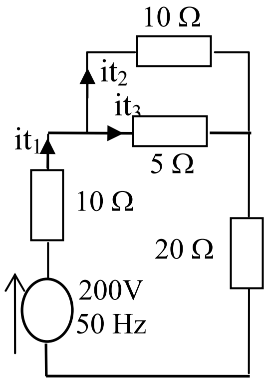
3. (a) With the 100 Ω load disconnected, the circuit can be simplified as follows.



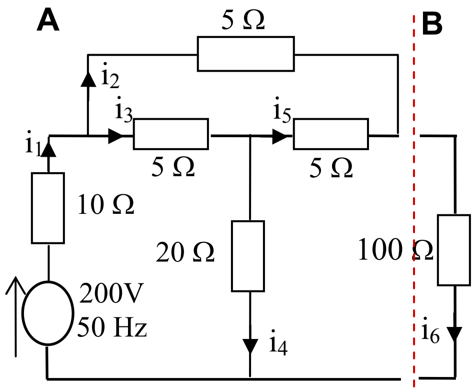
$$i_{t1} = \frac{200}{10 + 10//5 + 20}$$

$$= 6\ \text{A}$$

$i_{t2} = i_{t5} = 6\times5/15 = 2$ A

$i_{t3} = 6 - 2 = 4$ A

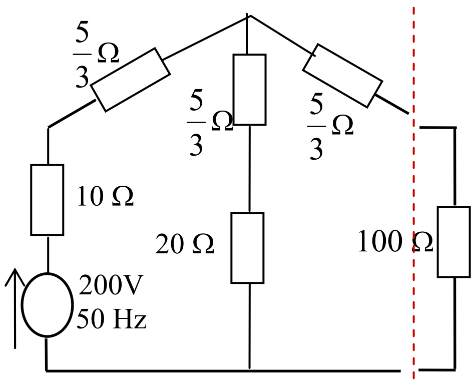
∴ Thevenin's voltage across 100 Ω at B = 200 – 10×6 – 5×2 = 130 V



1 mark

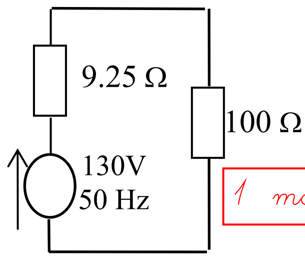
To determine Thevenin's impedance convert the delta into a star.

Thus Thevenin's impedance is given by



$$\frac{5}{3} + \frac{65}{3} // \frac{35}{3} = \frac{5}{3} + \frac{65\times35}{3\times100} = 9.25\Omega$$

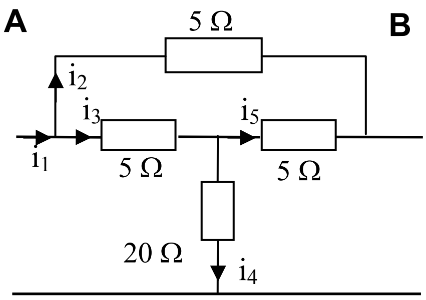
2 marks



1 mark

Thus the Thevenin's equivalent circuit is

(b) with port B on short circuit,



$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = 5//(5+5)+20 = 23.33\ \Omega,$$

1 mark

$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = 23.33\times\frac{20+5/3}{20+5/3+5/3}$$

1 mark

$$= 23.33\times\frac{65}{70} = 21.667\ \Omega$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

by symmetry $z_{22} = 23.33\ \Omega$, $z_{12} = 21.67\ \Omega$

1 mark

the z-parameter matrix is thus given by $\begin{bmatrix} 23.33 & 21.67 \\ 21.67 & 23.33 \end{bmatrix}$

1 mark

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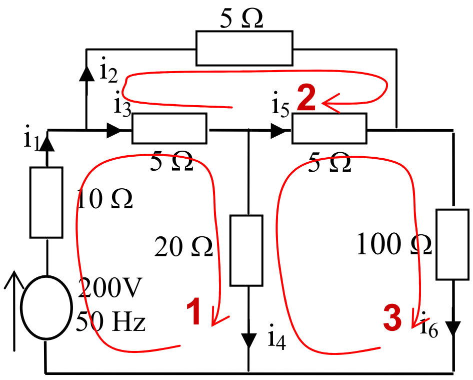
4. branch-mesh incidence matrix and branch impedance matrix



	1	2	3
1	1	0	0
2	0	1	0
3	1	−1	0
4	1	0	−1
5	0	−1	1
6	0	0	1

1 mark

$$\begin{bmatrix} 10 & 0 & 0 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 20 & 0 & 0 \\ 0 & 0 & 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 100 \end{bmatrix}$$

1 mark

mesh impedance matrix is thus given by

$$\begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 10 & 0 & 0 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 20 & 0 & 0 \\ 0 & 0 & 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 100 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & -1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 35 & -5 & -20 \\ -5 & 15 & -5 \\ -20 & -5 & 125 \end{bmatrix}$$

2 marks

The matrix mesh analysis equation is thus given by

$$\begin{bmatrix} 200 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 35 & -5 & -20 \\ -5 & 15 & -5 \\ -20 & -5 & 125 \end{bmatrix} \begin{bmatrix} I_{m1} \\ I_{m2} \\ I_{m3} \end{bmatrix}, \text{ giving } \begin{bmatrix} I_{m1} \\ I_{m2} \\ I_{m3} \end{bmatrix} = \begin{bmatrix} 35 & -5 & -20 \\ -5 & 15 & -5 \\ -20 & -5 & 125 \end{bmatrix}^{-1} \begin{bmatrix} 200 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} I_{m1} \\ I_{m2} \\ I_{m3} \end{bmatrix} = \frac{1}{54625} \begin{bmatrix} 1850 & 725 & 325 \\ 725 & 3975 & 275 \\ 325 & 275 & 500 \end{bmatrix} \begin{bmatrix} 200 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.03387 & 0.0133 & 0.0060 \\ 0.01327 & 0.0728 & 0.0050 \\ 0.00595 & 0.0050 & 0.0092 \end{bmatrix} \begin{bmatrix} 200 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 6.774 \\ 2.654 \\ 1.190 \end{bmatrix}$$

3 marks

Thus the branch currents are given by

$i_1 = I_{m1} = 6.774$ A

$i_2 = I_{m2} = 2.654$ A

$i_3 = I_{m1} - I_{m2} = 4.120$ A

$i_4 = I_{m1} - I_{m3} = 5.584$ A

$i_5 = - I_{m2} + I_{m3} = - 1.464$ A

$i_6 = I_{m3} = 1.190$ A

3 marks

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5.

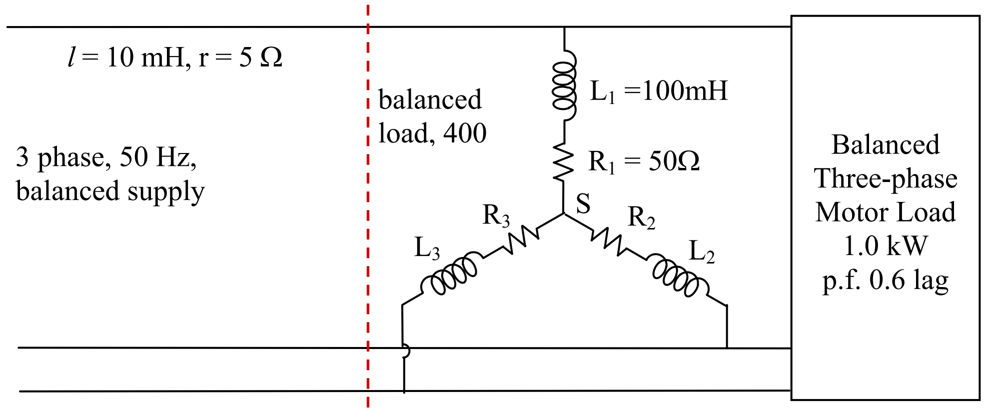


Figure Q5

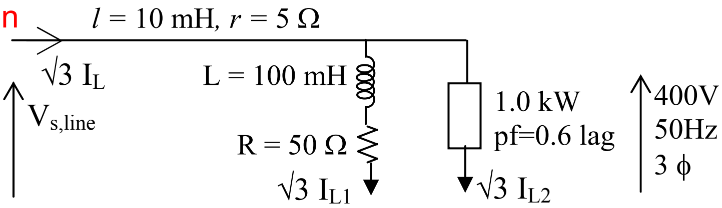
(a) (i) $i_L = (400/\sqrt{3})/(50 + j\ 100\times10^{-3}\times2\pi\times50) = 230.9/(50+j\ 31.416) = 3.911\angle-32.14^o$

(ii) $i_L = 1.0\times10^3\ /(\sqrt{3}\times400\times0.6)$ lagging the voltage at $\cos^{-1}(0.6) = 2.406\angle-53.13^o$

total line current supplied $= 3.911\angle-32.14^o + 2.406\angle-53.13^o$

$= 3.312 - j\ 2.081 + 1.443 - j\ 1.924 = 4.756 - j\ 4.005 = \underline{\underline{6.218\angle-40.10^o}}$ 3 marks

Alternate Solution



(a) (i) $\sqrt{3}\ I_{L1} = 400/(50 + j\ 100\times10^{-3}\times2\pi\times50)$

$\sqrt{3}\ I_{L2} = 1.0\times10^3/(400\times0.6)$ lagging by an angle of $\cos^{-1}(0.6) = 53.13^o$

$\sqrt{3}\ I_L = \sqrt{3}\ I_{L1} + \sqrt{3}\ I_{L2}$

$= 6.774\angle-32.14^o + 4.1667\angle-53.13^o$

$= 10.77\angle-40.10^o$ A

$I_L = 6.218\angle-40.10^o$ A

(b) total reactive power supplied $= \sqrt{3}\times400\times6.218\times\sin(40.10^o) = \underline{\underline{2775\ \text{var}}}$ 1 mark

(c) voltage at the supply end of the line $= 400 + \sqrt{3}\times6.218\angle-40.10^o\times(5+ j\ 2\pi\times50\times10\times10^{-3})$

$= 400 + \sqrt{3}\times6.218\angle-40.10^o\times5.905\angle32.14^o = 400 + 63.60\angle-7.96^o$

$= 400 + 62.98 - j\ 8.81 = 462.98 - j\ 8.81 = \underline{\underline{463.1\angle-1.09^o}}$ 2 marks

(d) overall power factor at the source $= \cos(40.10^o - 1.09^o) = \underline{\underline{0.777\ \text{lag}}}$ 1 mark

(e) the capacitances that must be connected across the combined load to improve the power factor to 0.9 lag when the load voltage is 400 V

If capacitances are connected in shunt, voltage is kept constant, active power at load is constant.

Thus $P = \sqrt{3}\times400\times6.218\times\cos 40.10^o = 3295$ W

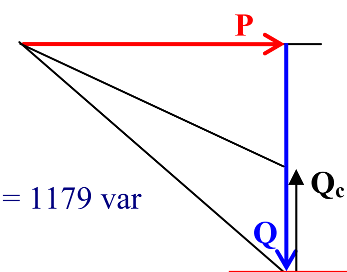
if new power factor is 0.9 lag, power factor angle $= 25.84^o$

new reactive power $= 3295\times\tan 25.84^o = 1596$ var

Reactive power to be supplied from capacitors $= 2775 - 1596 = 1179$ var

therefore $400^2\ C \times 100\ \pi = 1179/3$

$C = \underline{\underline{7.82\ \mu F}}$



3 marks

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6. (a) $$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix} \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix}$$

1 mark

(b) $$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix} \begin{bmatrix} 50\angle 90^0 \\ 200\angle 0^0 \\ 100\angle -30^0 \end{bmatrix}$$

$V_a = 50\angle 90^o + 200\angle 0^o + 100\angle -30^o = j\,50 + 200 + 86.6 - j\,50 = \underline{\underline{286.6\angle 0^o \text{ V}}}$

$V_b = 50\angle 90^o + 200\angle 0^o \times 1\angle 240^o + 100\angle -30^o \times 1\angle 120^o = j50 - 100 - j173.2 + j100$

$= -100 - j\,23.2 = \underline{\underline{102.66\angle -166.94^o \text{ V}}}$

$V_c = 50\angle 90^o + 200\angle 0^o \times 1\angle 120^o + 100\angle -30^o \times 1\angle 240^o = j50 - 100 + j173.2 - 86.6 - j50$

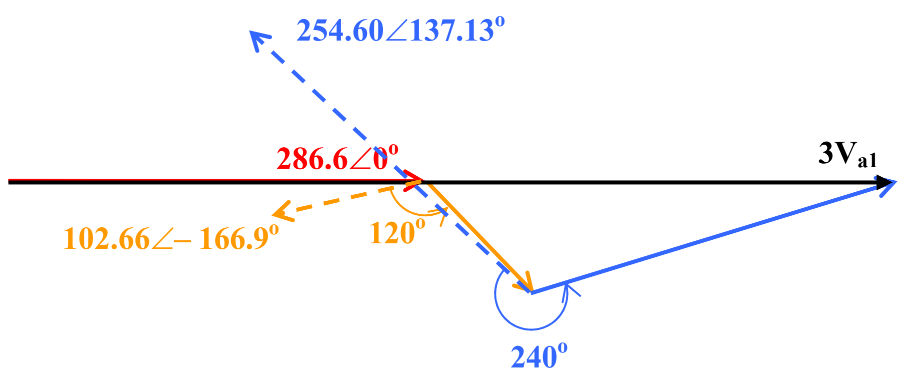
$= -186.6 + j\,173.2 = \underline{\underline{254.60\angle 137.13^o \text{ V}}}$

3 marks

(c) $$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix} \begin{bmatrix} 286.6\angle 0^0 \\ 102.7\angle -166.94^0 \\ 254.6\angle 137.13^0 \end{bmatrix}$$

3 times the value of the positive sequence component may be determined by phasor addition as follows.

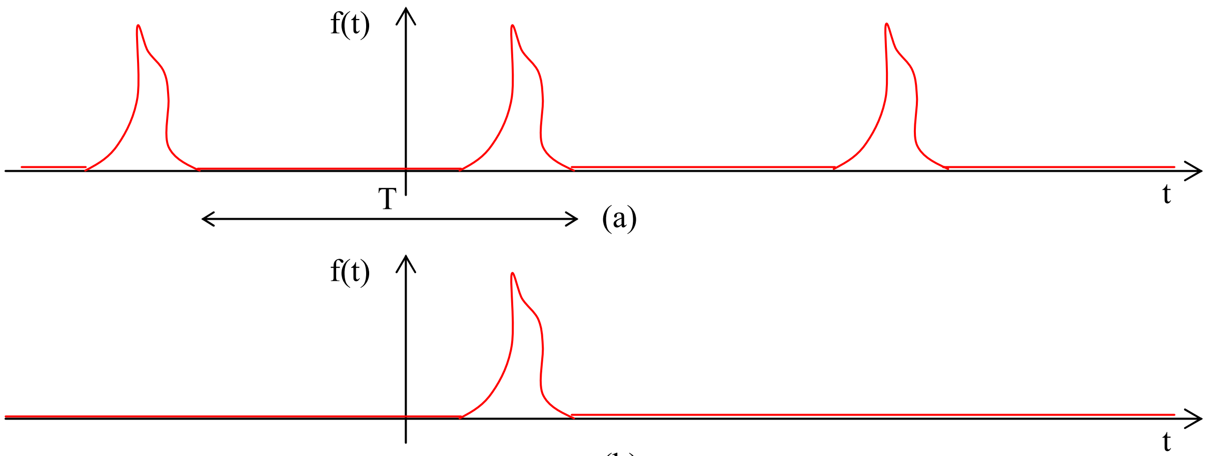
$$V_{a1} = \frac{1}{3}[286.6\angle 0^o + 102.66\angle -166.94^o \times 1\angle 120^o + 254.60\angle 137.13^o \times 1\angle 240^o]$$



2 marks

7. (a) The Fourier series states that any practical periodic function with period T, or frequency $\omega_o = 2\pi/T$, can be represented as an infinite sum of sinusoidal waveforms that have frequencies which are an integral multiple of ω_o.

$$f(t) = \frac{A_o}{2} + \sum_{n=1}^{\infty}(A_n \cos n\omega_o t + B_n \sin n\omega_o t) \text{ or in complex form } f(t) = \sum_{n=-\infty}^{\infty} C_n \cdot e^{jn\omega_o t},$$



(b)

A non-repetitive waveform, for which Fourier Transform is defined, can be considered as one with period T→ ∞, with corresponding fundamental frequency $\omega_o = \frac{2\pi}{T} = \Delta\omega \to 0$.

The frequencies involved are no longer discrete but continuous, so that the general frequency $n\omega_o$ corresponds to $\Sigma\, \Delta\omega \to \int d\omega = \omega$. Similarly the remaining variables are

$$\omega_o \to d\omega\ ,\ C_n \to dC,\ n\omega_o \to \omega\ ,\ \frac{1}{T} = f = \frac{\omega_o}{2\pi} \to \frac{d\omega}{2\pi}$$

giving the Fourier Transform

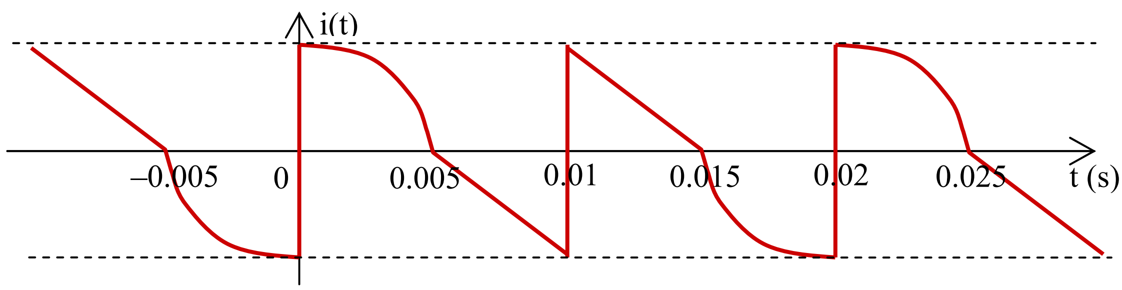
$$F(\omega) = \frac{dC}{d\omega} = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(t).e^{-j\omega t}.dt$$

With functions which do not decay (such as the causal sine waveform), the Fourier Transform cannot be evaluated. To avoid this problem, waveforms which do not decay may be artificially decayed by an exponential factor to allow the integration. The integrated result is then exponentially magnified to correct for the initial decay introduced. The Laplace Transform is defined from the Fourier Transform based on this artificial decay for ***causal functions.***

2 marks

$$\mathcal{L}\,[f(t)] = F(s) = \int_0^{\infty} f(t)\cdot e^{-st}\cdot dt, \text{ where } s = \sigma + j\,\omega \text{ is the Laplace operator}$$

(b)



i(t) = 100 cos 314 t for $0 < t \le 0.005$
i(t) = 100 – 20000.t for $0.005 < t \le 0.01$
i(t) = 300 – 20000.t for $0.01 < t \le 0.015$
i(t) = – 100 cos 314 t for $0.015 < t \le 0.02$

Period of waveform T = 0.02, ω_0 = 314 rad/s

Waveform has mean value zero. $\therefore A_o = 0$. 1 mark

Waveform has odd symmetry. $\therefore A_n = 0$ for all n.

Thus i(t) $= \sum_{n=1}^{\infty} B_n \sin n\omega_o t$,

where $B_n = \frac{2\times 2}{T}\int_{t=0}^{\frac{1}{2}T} i(t).\sin n\omega_o t.dt$

$$= 200\int_{t=0}^{0.005} 100\cos 314t.\sin n314t.dt + 200\int_{t=0.005}^{0.01}(100-20000t).\sin n314t.dt$$

$$= 10000\int_{0}^{0.005}[\sin 314(n+1)t + \sin 314(n-1)t].dt + 20000\int_{0.005}^{0.01}\sin 314nt.dt - 4000000\int_{0.005}^{0.01} t.\sin 314nt.dt$$

$$= 10000\left[\frac{\cos 314(n+1)t}{-314(n+1)} + \frac{\cos 314(n-1)t}{-314(n-1)}\right]_0^{0.005} + \left[20000\frac{\cos 314nt}{-314n}\right]_{0.005}^{0.01}$$

$$-4000000\left[\left[t.\frac{\cos 314nt}{-314n}\right]_{0.005}^{0.01} - \int_{0.005}^{0.01} 1.\frac{\cos 314nt}{-314n}.dt\right]$$ 4 marks

$$= \frac{31.83}{n+1}\left(1-\cos(n+1)\frac{\pi}{2}\right) + \frac{31.83}{n-1}\left(1-\cos(n-1)\frac{\pi}{2}\right) + \frac{63.66}{n}(\cos n\pi - \cos\frac{n\pi}{2})$$

$$+\frac{12732}{n}(0.01\cos n\pi - 0.005\cos\frac{n\pi}{2}) - \frac{40.528}{n^2}(\sin n\pi - \sin\frac{n\pi}{2})$$

B_1=8.70, B_2=74.27, B_3=6.11, B_4=32.89, B_5= –0.50, B_6=32.89, ...

$\therefore$ i(t) = 8.70 sin 314 t + 74.27 sin 628 t + 6.11 sin 942 t +.... 1 mark

(c) $\therefore$ v(t) $= 10\, i(t) + 0.01\frac{d\, i(t)}{dt}$ = 100 sin 314 t + 750 sin 628 t – 150 sin 942 t +....

31.4 cos 314 t + 471 cos 628 t – 141 cos 942 t +....

i.e. v(t) = 104.8 sin(314t +17.4°) + 885.6 sin(628t+32.1°) – 205.9 sin(942t+43.2°) 1 mark

(d) Laplace Transform of

a. causal ramp waveform ***t*** = $\int_0^{\infty} t.e^{-st}dt = t.\frac{e^{-st}}{-s}\Big|_0^{\infty} - \int_0^{\infty} 1.\frac{e^{-st}}{-s}dt = \frac{1}{s^2}$ 1 mark

b. causal step waveform ***h(t)*** = $\int_0^{\infty} 1.e^{-st}dt = \frac{e^{-st}}{-s}\Big|_0^{\infty} = \frac{1}{s}$ 1 mark

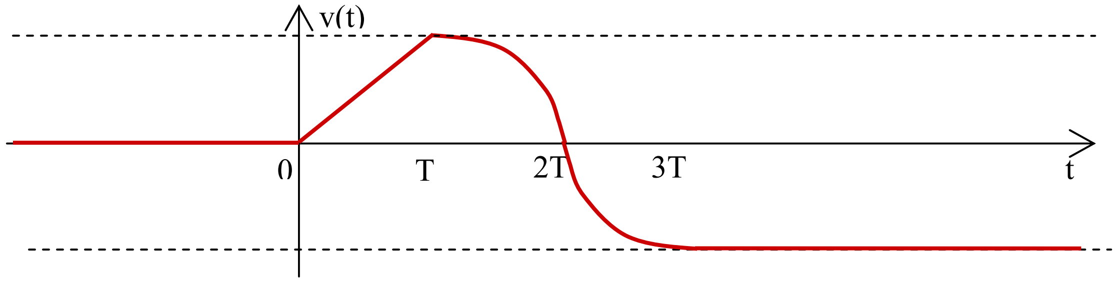
c. causal sine waveform ***sin*** ω ***t*** =

$$\int_0^\infty \sin\omega\, t.e^{-st}dt = \sin\omega\, t.\frac{e^{-st}}{-s}\Bigg|_0^\infty - \int_0^\infty \omega.\cos\omega\, t.\frac{e^{-st}}{-s}dt = \frac{\omega}{s}.\cos\omega\, t\frac{e^{-st}}{-s} + \int_0^\infty \omega^2.\sin\omega\, t.\frac{e^{-st}}{-s}dt$$

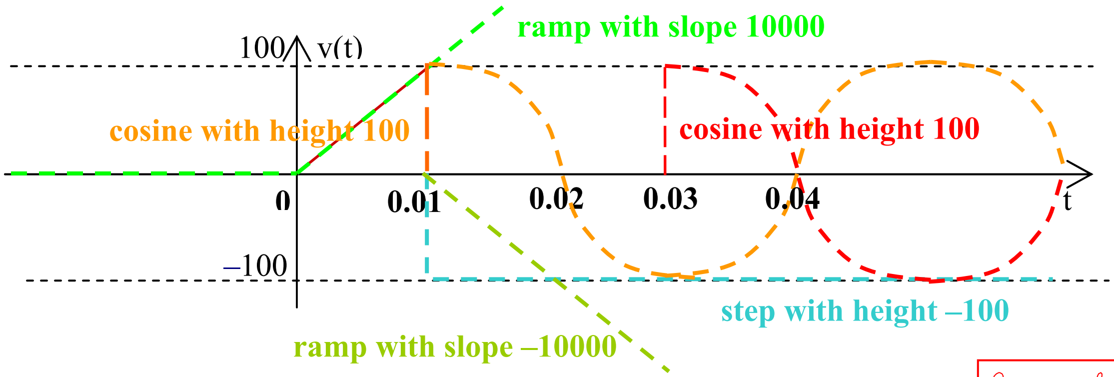
$$\rightarrow \frac{\omega}{s^2+\omega^2}$$

1 mark

(d)



The waveform shown can be made up as shown below, with shifted components.



hence the Laplace transform can be obtained as follows.

3 marks

$$V(s)=10000\times\frac{1}{s^2} - 100\times\frac{1}{s} - 10000\times\frac{1}{s^2}\times e^{0.01S} + 100\times\frac{s}{s^2+(50\pi)^2}\times e^{0.01S} + 100\times\frac{s}{s^2+(50\pi)^2}\times e^{0.03S}$$

(e) for series circuit with R=10 Ω, C = 10 μF,

I(s) = V(s)/(R + 1/Cs)

$$I(s) = \frac{1}{10+10^5/s}\times V(s)$$

1 mark