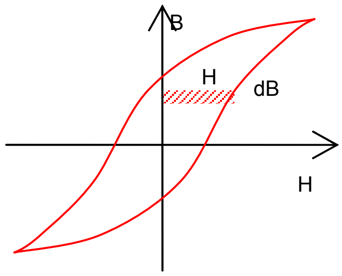
EE 201 - THEORY OF ELECTRICITY – Short Answers

Level 2 Semester 1 Examination - January 2004



1. (a) $P_h = \frac{1}{T}\oint v.i.dt = f.\oint N\frac{d\phi}{dt}i.dt = f.\oint A.dB.H.l$

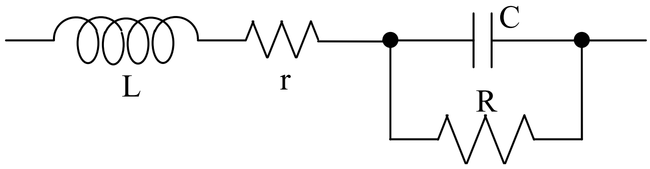
$= f.\oint H.dB.\text{volume}$

Hysteresis loss per unit volume $P_h \propto \oint H.dB.$ = area of loop

(b) Eddy current loss per unit volume $P_e \propto \text{thickness}^2$

$\therefore$ reducing thickness, hence using small thickness sheets (laminations) drastically reduces the eddy current losses in the transformer core.

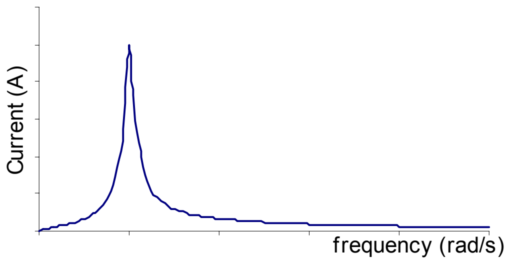
(c) Using the definition that resonance occurs when the impedance of the circuit is purely real (or power factor is unity),

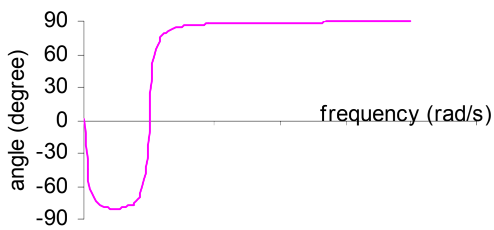


$$Z = j\omega L + r + \frac{R.\,{}^{1}\!/\!_{j\omega C}}{R + {}^{1}\!/\!_{j\omega C}}$$

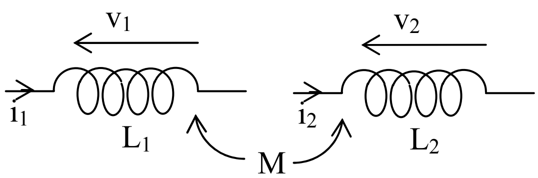
$$= j\omega L + r + \frac{R}{1 + j\omega CR} = \frac{R}{1+\omega^2 C^2 R^2} + r + j\omega\left(L - \frac{CR^2}{1+\omega^2 C^2 R^2}\right)$$

$\therefore$ at resonance $L = \dfrac{CR^2}{1+\omega^2 C^2 R^2}$





(d) energy stored = $\int v_1.i_1\, dt + \int v_2.i_2\, dt$



$$= \int\left(L_1\frac{di_1}{dt} \pm M\frac{di_2}{dt}\right)i_1.dt + \int\left(L_2\frac{di_2}{dt} \pm M\frac{di_1}{dt}\right)i_2.dt$$

$= \int L_1.i_1\, d\, i_1 \pm (M.i_1\, di_2 + M.i_2\, di_1) + L_2.i_2\, d\, i_2$

$= ½\, L_1\, i_1^2 \pm M.i_1\, i_2 + ½\, L_1\, i_1^2$

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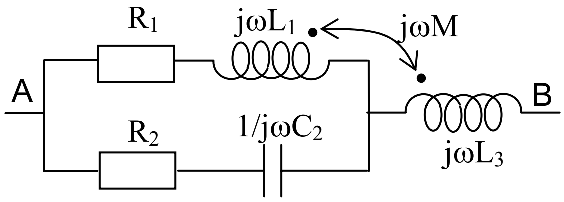
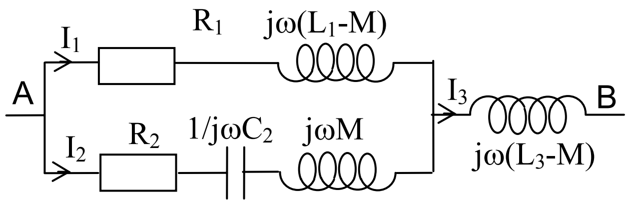


(i)



(ii)

Figure (ii) is the non-coupled equivalent circuit of figure (i).

(a) $$Z_{AB} = j\omega(L_3 - M) + \left(R_1 + j\omega(L_1 - M)\right) // \left(R_2 + \frac{1}{j\omega\, C_2} + j\omega M\right)$$

i.e. $$Z_{AB} = j\omega(L_3 - M) + \frac{\left(R_1 + j\omega(L_1 - M)\right).\left(R_2 + \frac{1}{j\omega\, C_2} + j\omega\, M\right)}{\left(R_1 + j\omega(L_1 - M) + R_2 + \frac{1}{j\omega\, C_2} + j\omega\, M\right)}$$

(b) V_{AB} = 100V, ω = 250 rad/s, L_1 = 40 mH, R_1 = 20 Ω, R_2 = 0 Ω, C_2 = 80 μF, M = 20 mH, L_3 = 40 mH

$$Z_{AB} = j250 \times 20 \times 10^{-3} + \frac{\left(20 + j250 \times 20 \times 10^{-3}\right).\left(0 + \frac{1}{j\,250 \times 80 \times 10^{-6}} + j250 \times 20 \times 10^{-3}\right)}{\left(20 + j5 - j50 + j5\right)}$$

i.e. $$Z_{AB} = j5 + \frac{(20 + j5)(-j45)}{20 - j40} = 20.25 + j0.5 \quad \Omega \text{ or } 20.26\angle 1.41^{\circ}\ \Omega$$

(c) $$I_3 = \frac{100\angle 0^{o}}{20.26\angle 1.41^{o}} = 4.936\angle -1.41^{o}\text{ A}$$

$$I_1 = \frac{-j45}{20 - j40} \times 4.936\angle -1.41^{o} = 4.967\angle\text{-}29.97^{\circ}\text{ A}$$

$$I_2 = \frac{20 + j5}{20 - j40} \times 4.936\angle -1.41^{o} = 2.275\angle 76.06^{\circ}\text{ A}$$

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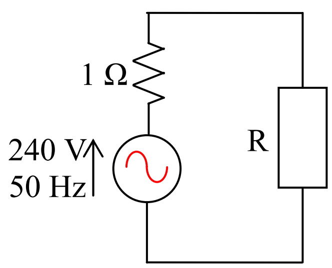
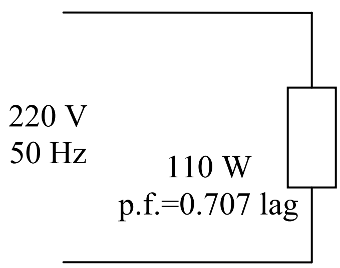
3 (a) $$P = \left(\frac{240}{R+1}\right)^2 .R$$





for max^{m} P, $\frac{dP}{dR} = 0$

$$= \frac{240^2}{(R+1)^4}\left[(R+1)^2.1 - R.2(R+1)\right]$$

i.e. R + 1 – 2 R = 0, R = 1 Ω

∴maximum theoretical power = $(240/2)^2 \times 1$ = 14400 = 14.4 kW

(b) Load voltage = 240×R/(R+1) = 120 V

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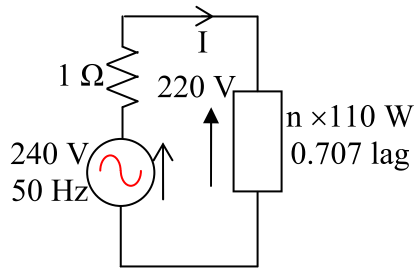
(c) $220\,I \times 0.707 = n \times 110$



considering load voltage as reference

$240 = |\,220\angle 0^o + I\angle{-45^o}\times 1\,|$

$= |\,220 + I \times 0.707 - j\,I \times 0.707\,|$

i.e. $240^2 = 220^2 + (I \times 0.707)^2 + 2\times 220\times I \times 0.707 + (I \times 0.707)^2$

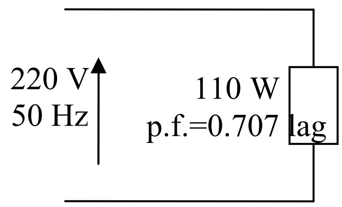
substituting for I in terms of n

$240^2 = 220^2 + (n/2)^2 + 2\times 220\times n/2 + (n/2)^2$

i.e. $n^2 + 440\,n - 18400 = 0$ giving $n = -220 \pm \sqrt{(220^2+18400)} = -220 \pm 258.46 \rightarrow 38.46$

$\therefore$ maximum number of such loads = 38.

<u>Alternate Method for part (c)</u>

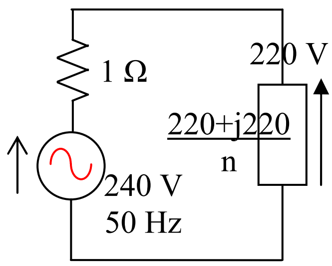


for a single load supplied at 220 V,

$VI\cos\phi = 110 \rightarrow 220\,I \times 0.707 = 110 \rightarrow I = 0.707$ A

$Z = 220/0.707\ \angle 45^o = 220 + j\,220$

voltage magnitudes are proportional to impedance magnitudes



$$\frac{|220|}{|240|} = \frac{\left|\frac{220}{n} + j\frac{220}{n}\right|}{\left|\frac{220}{n} + 1 + j\frac{220}{n}\right|} = \frac{220}{n}\frac{\sqrt{2}}{\sqrt{\left(\frac{220}{n}+1\right)^2 + j\left(\frac{220}{n}\right)^2}}$$

$$\frac{|220|}{|240|} = \frac{220\sqrt{2}}{\sqrt{(220+n)^2 + j(220)^2}} \rightarrow (220+n)^2 + 220^2 = 2\times 240^2$$

i.e. $n^2 + 440\,n - 18400 = 0$ giving $n = 38.46$ or 38 loads

(d) Active power delivered to loads = 38 × 110 = 4180 = 4.18 kW

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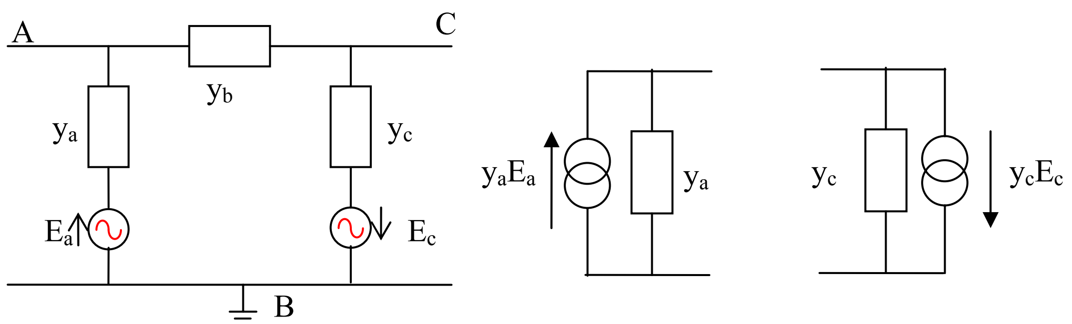
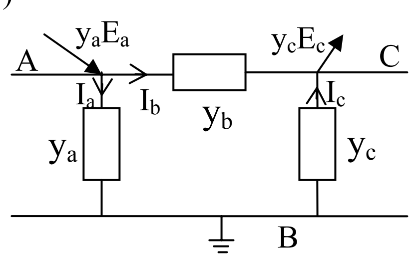
4 (a)



(b)



$$[Y_b] = \begin{bmatrix} y_a & 0 & 0 \\ 0 & y_b & 0 \\ 0 & 0 & y_c \end{bmatrix}, \quad [A] = \begin{matrix} \\ a \\ b \\ c \end{matrix}\begin{matrix} A\ \ C \\ \begin{bmatrix} 1 & 0 \\ 1 & -1 \\ 0 & -1 \end{bmatrix} \end{matrix} \quad I_{gN} = \begin{bmatrix} y_a E_a \\ -y_c Ec \end{bmatrix},$$

(c) nodal admittance matrix $[Y_N] = [A]^t [Y_b] [A]$

$$[Y_N] = \begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & -1 \end{bmatrix} \begin{bmatrix} y_a & 0 & 0 \\ 0 & y_b & 0 \\ 0 & 0 & y_c \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & -1 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} y_a + y_b & -y_b \\ -y_b & y_b + y_c \end{bmatrix}$$

(d) $$\begin{bmatrix} y_a E_a \\ -y_c E_c \end{bmatrix} = \begin{bmatrix} y_a + y_b & -y_b \\ -y_b & y_b + y_c \end{bmatrix} \begin{bmatrix} V_A \\ V_C \end{bmatrix}$$

$$V_A = \frac{1}{(y_a + y_b)(y_b + y_c) - y_b y_b} \left[(y_b + y_c)(y_a E_a + y_b(-y_c E_c) \right]$$

(e) $$V_A = \frac{1}{4y^2 - y^2} \left[2y^2 E - y^2 E \right] = \frac{E}{3}$$

(f) Using Ohm's Law, $I = \dfrac{E_a + E_c}{z_a + z_b + z_c} = \dfrac{2E}{3} y_a$

i.e. $V_A = E_a - z_a I = E - \dfrac{2E}{3} y_a . \dfrac{1}{y_a} = \dfrac{E}{3}$

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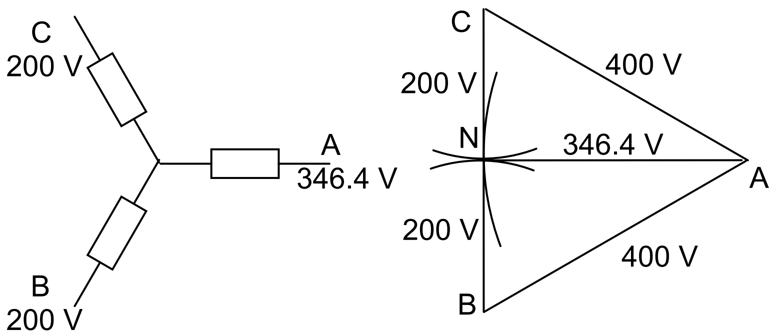
5 (a) A single phasor diagram can be drawn using the phase voltages and the line voltages to give the directions of the voltages as follows.



$V_{AN} = 346.4\angle 0^o$ V

$V_{BN} = 200.0\angle -90^o$ V

$V_{CN} = 200.0\angle 90^o$ V

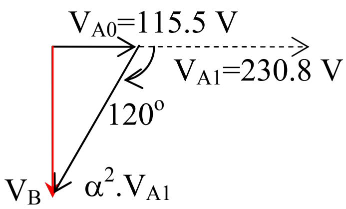
(b) $$\begin{bmatrix} V_{A0} \\ V_{A1} \\ V_{A2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix} \begin{bmatrix} 346.4\angle 0^o \\ 200\angle -90^o \\ 200\angle 90^o \end{bmatrix}$$

V_{A0} $= ⅓ [346.4\angle 0^o + 200.0\angle -90^o + 200.0\angle 90^o] = ⅓ [346.4\angle 0^o]$
$= 115.5\angle 0^o$ V

V_{A1} $= ⅓ [346.4\angle 0^o + 1\angle 120^o \times 200.0\angle -90^o + 1\angle 240^o \times 200.0\angle 90^o]$
$= ⅓ [346.4\angle 0^o + 200\cos 30^o + j200\sin 30^o + 200\cos 330^o + j200\sin 330^o]$
$= 230.8 \angle 0^o$ V

V_{A2} $= ⅓ [346.4\angle 0^o + 1\angle 240^o \times 200.0\angle -90^o + 1\angle 120^o \times 200.0\angle 90^o]$
$= ⅓ [346.4\angle 0^o + 200\cos 150^o + j200\sin 150^o + 200\cos 210^o + j200\sin 210^o]$
$= 0$ V

(c) V_B $= V_{A0} + \alpha^2 V_{A1} + \alpha V_{A2}$
$= 115.5\angle 0^o + 1\angle 240^o \times 230.8 \angle 0^o + 1\angle 120^o \times 0$



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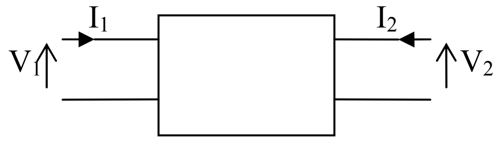
6 (a) $\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_a + y_b & -y_b \\ -y_b & y_a + y_b \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$



$I_2 = -\, y_b V_1 + (y_a + y_b) V_2$

$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$

i.e. $V_1 = \dfrac{y_a + y_b}{y_b} V_2 - \dfrac{1}{y_b} I_2, \quad \therefore A = 1 + \dfrac{y_a}{y_b},\ B = \dfrac{1}{y_b}$

$I_1 = (y_a + y_b) V_1 - y_b V_2$ $V_1 = A\, V_2 - B\, I_2$

i.e. $I_1 = \dfrac{(y_a + y_b)^2}{y_b} V_2 - \dfrac{(y_a + y_b)}{y_b} I_2 - y_b V_2 = \dfrac{(y_a^2 + 2 y_a y_b)}{y_b} V_2 - \dfrac{(y_a + y_b)}{y_b} I_2$

$\therefore C = \dfrac{y_a^2 + 2 y_a y_b}{y_b}, \quad D = 1 + \dfrac{y_a}{y_b}, \qquad \begin{bmatrix} A & C \\ B & D \end{bmatrix} = \begin{bmatrix} 1 + \dfrac{y_a}{y_b} & \dfrac{1}{y_b} \\ \dfrac{y_a^2 + 2 y_a y_b}{y_b} & 1 + \dfrac{y_a}{y_b} \end{bmatrix}$

(b) A . D – B . C = $\left(1 + \dfrac{y_a}{y_b}\right)^2 - \dfrac{1}{y_b} \cdot \dfrac{y_a^2 + 2 y_a y_b}{y_b} = \dfrac{y_b^2}{y_b^2} = 1$

(c) if $y_b = y_a$, $\begin{bmatrix} A & C \\ B & D \end{bmatrix} = \begin{bmatrix} 2 & \dfrac{1}{y_b} \\ 3 y_a & 2 \end{bmatrix}$

(d) for cascade connection, multiply [ABCD] matrices together.

$$\begin{bmatrix} A & C \\ B & D \end{bmatrix} = \begin{bmatrix} 2 & \dfrac{1}{y_b} \\ 3 y_a & 2 \end{bmatrix} \begin{bmatrix} 2 & \dfrac{1}{y_b} \\ 3 y_a & 2 \end{bmatrix} = \begin{bmatrix} 7 & \dfrac{4}{y_b} \\ 12 y_a & 7 \end{bmatrix}$$

(e) A . D – B . C = 7 × 7 - $\dfrac{4}{y_a} \times 12 y_a$ = 49 – 48 = 1

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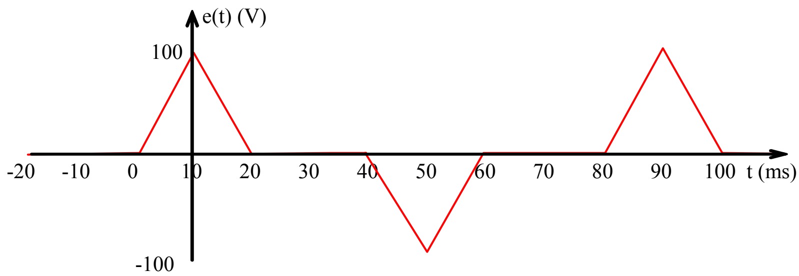
7 (a) mean value = 0

peak value = 100 V



average = $\dfrac{\frac{1}{2} \times 100 \times 20}{40}$ = 25 V

rms value = $\sqrt{\dfrac{4}{80} \int_0^{10} (10t)^2\, dt}$

$= \sqrt{\dfrac{1}{20} \times 100 \times \dfrac{10^3}{3}} = \dfrac{100}{\sqrt{6}}$ = 40.82 V, period of waveform T = 80 ms

peak factor = 100/40.82 = 2.45, $\omega \times 80 \times 10^{-3} = 2\pi$, gives $\omega = 25\pi$ rad/s

form factor = 40.82/25 = 1.63

(b) if the waveform is shifted by 10 ms, it has both even symmetry and half-wave symmetry.

mean value = 0 giving $a_o = 0$.

for modified waveform, even symmetry gives $b_n = 0$ for all n, and half-wave symmetry gives even harmonics = 0.

$e(t) = \sum a_n \cos n\omega(t-0.01)$ for odd n

$$a_n = 4\times\frac{2}{T}\int(100-10000t)\cos n\omega\, t.dt$$

$$= \frac{8}{0.08}\left[(100-10000t).\frac{\sin n\omega\, t}{n\omega}\Bigg|_0^{0.01}\right] - \int_0^{0.01}(-10000).\frac{\sin n\omega\, t}{n\omega}.dt$$

$$= 100\left[\frac{0.\sin 0.25n\pi}{25n\pi} + 10000\frac{(-\cos n\omega\, t}{(25n\pi)^2}\Bigg|_0^{0.01}\right] = \frac{1600}{(n\pi)^2}(1-\cos\frac{n\pi}{4})$$

$$a_1 = \frac{1600}{\pi^2}(1-\cos\frac{\pi}{4}) = \frac{1600}{\pi^2}\times 0.293 = 47.48$$

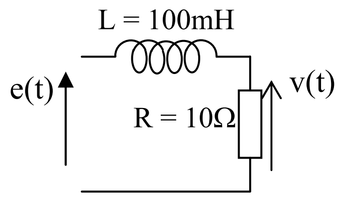
$$a_3 = \frac{1600}{9\pi^2}(1-\cos\frac{3\pi}{4}) = \frac{1600}{\pi^2}\times 0.190 = 30.75$$

$$a_5 = \frac{1600}{25\pi^2}(1-\cos\frac{5\pi}{4}) = \frac{1600}{\pi^2}\times 0.068 = 11.07$$

$\therefore e(t) = 47.48\cos(78.54(t-0.01)) + 30.75\cos(3\times78.54(t-0.01)) + 11.07\cos(5\times78.54(t-0.01)) + \ldots$

or $e(t) = 47.48\cos(78.54t-\pi/4)) + 30.75\cos(3\times78.54t-3\pi/4)) + 11.07\cos(5\times78.54t-5\pi/4)) + \ldots$

(c) for each component frequency, the output voltage can be obtained by potential divider action [peak values can be used instead of rms values to give peak values].



$$V_n = \frac{10}{10+jn\omega L}.E_n = \frac{10}{10+jn25\pi\times0.1}.E_n = \frac{10}{10+j2.5\pi\, n}.E_n$$

$$V_1 = \left|\frac{10}{10+j2.5\pi}\right|\times 47.48 = \frac{10}{\sqrt{10^2+(2.5\pi)^2}}\times 47.48 = 37.34\angle-38.1^0$$

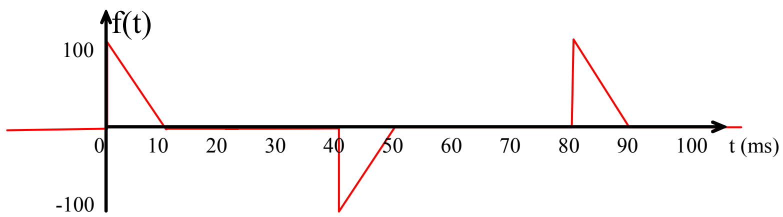
similarly, $V_3 = 12.01\angle\text{-}67.0^0$, $V_5 = 2.73\angle\text{-}75.7^0$

$\therefore v(t) = 37.34\cos(78.54t-83.1^o) + 12.01\cos(3\times78.54t-202.0^o) + 2.73\cos(5\times78.54t-300.7^o) + \ldots$

(d) $$\mathcal{L}[\,f(t-\tau)\,] = \int_0^\infty f(t-\tau).e^{-st}dt = \int_{-\tau}^\infty f(t).e^{-s(t+\tau)}dt = \int_0^\infty f(t).e^{-st}.e^{-s\tau}dt$$

$$= e^{-s\tau}.\int_0^\infty f(t).e^{-st}.dt = e^{-s\tau}.F(s)$$

(e)



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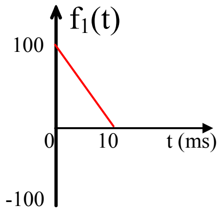


Consider the function $f_1(t)$ which has the same properties as f(t) in the region 0 to 10 ms.

$$\mathcal{L}[f_1(t)] = \int_0^{0.01}(100-10000t)e^{-st}.dt$$

$$= (100-10000t)\frac{e^{-st}}{-s}\Bigg|_0^{0.01} - \int_0^{0.01}(-10000).\frac{e^{-st}}{-s}.dt$$

$$= 0 + 10000\frac{e^{-st}}{(-s)^2}\Bigg|_0^{0.01} = \frac{10000}{s^2}(e^{-0.01s}-1)$$

function f(t) can be obtained by shifting and algebraically adding $f_1(t)$

i.e. $\mathcal{L}[f(t)] = \mathcal{L}[f_1(t)].[1 - e^{-0.04s} + e^{-0.08s} - e^{-0.12s} + e^{-0.16s} ....]$

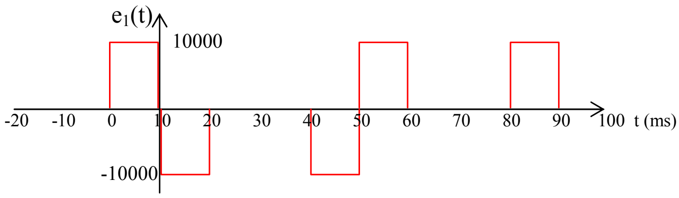
$$= \frac{10000}{s^2}(e^{-0.01s}-1).\frac{1}{(1+e^{-0.04s})}$$

<u>Alternate solution for (b)</u>

consider the waveform $e_1(t)$ which has e(t) as its derivative

$\omega T = 2\pi$, T = 80 ms

$\omega = 25\pi$ rad/s



mean value = 0

giving $a_o = 0$.

the waveform is shifted by 10 ms, it has both odd symmetry and half-wave symmetry.

for modified waveform, even symmetry gives $a_n = 0$ for all n,

and half-wave symmetry gives even harmonics = 0.

$e(t) = \sum b_n \sin n\omega(t-0.01)$ for odd n

$$b_n = 4\times\frac{2}{T}\int_0^{T/4} e_1(t).dt = \frac{0}{0.08}\int_0^{0.01}(-)10000\sin n\omega\, t.dt + \int_{0.01}^{0.02} 0\sin n\omega\, t.dt$$

$$= (-)100\times 10000\frac{\cos n\omega\, t}{(-)n\omega}\Bigg|_0^{0.01} = 10^6\frac{\cos n25\pi\, t}{n25\pi}\Bigg|_0^{0.01} = \frac{40000}{n\pi}(\cos 0.25 n\pi - 1)$$

$$e_1(t) = \frac{40000}{\pi}[(\cos 0.25\pi - 1)\sin 25\pi t + \tfrac{1}{3}(\cos 0.75\pi - 1)\sin 75\pi t + \tfrac{1}{5}(\cos 1.25\pi - 1)\sin 125\pi t + ...$$

$\therefore e(t) = \int e_1(t)\, dt$

$$= \frac{40000}{\pi}\left[\left(\frac{1}{\sqrt{2}}-1\right)\frac{\cos 25\pi t}{-25\pi} + \frac{1}{3}\left(-\frac{1}{\sqrt{2}}-1\right)\frac{\cos 75\pi t}{-75\pi} + \frac{1}{5}\left(-\frac{1}{\sqrt{2}}-1\right)\frac{\cos 125\pi t}{-125\pi} + ...\right]$$

$= 47.48\cos(78.54t-\pi/4)) + 30.75\cos(3\times 78.54t-3\pi/4)) + 11.07\cos(5\times 78.54t-5\pi/4)) + ...$