University of Moratuwa, Sri Lanka

Faculty of Engineering

Department of Electrical Engineering

B. Sc. Engineering Honours Degree Course

Level 2 – Semester 1 Examination

EE201 – THEORY OF ELECTRICITY

Time Allowed: 3 Hours | October 2007

Additional Material

- Graph Paper will be provided if required.
- Laplace transform pairs are provided on the other side of this page.

Instructions to Candidates

- This paper contains 7 questions in 5 pages, including the cover page.
- Answer **All** Questions.
- This examination accounts for 70% of the module assessment.
- Each question carries a total of 10 marks. Marks allocated to each part of a question are indicated in square brackets at the end of the part.
- Total allocation for the paper is 70 marks.
- This is a closed book examination and only authorised calculators will be permitted.

Technical Data:

- Permeability of free space $\mu_0 = 4\pi \times 10^{-7}$ H/m
- Permittivity of free space $\varepsilon_0 = 8.854 \times 10^{-12}$ F/m
- Velocity of light in free space = 2.998×10^{8} m/s

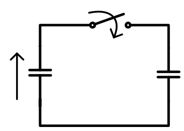
LAPLACE TRANSFORMS

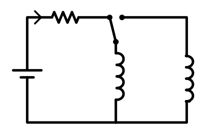
$f(t)$	$F(s) = \mathrm{L}[f(t)]$
Unit impulse $-\delta t$	1
Unit step $-\ U(t)$	$\frac{1}{s}$
t	$\frac{1}{s^2}$
t^n	$\frac{n!}{s^{n+1}}$
e^{-at}	$\frac{1}{(s+a)}$
$1-e^{-at}$	$\frac{a}{s(s+a)}$
$t\,e^{-at}$	$\frac{1}{(s+a)^2}$
$t^n\,e^{-at}$	$\frac{n!}{(s+a)^{n+1}}$
$e^{-at} - e^{-bt}$	$\frac{b-a}{(s+a)(s+b)}$
$\sin(\omega t)$	$\frac{\omega}{(s^2+\omega^2)}$
$\sin(\omega t + \phi)$	$\frac{\omega\cos(\phi) + s\sin(\phi)}{(s^2+\omega^2)}$
$t\sin(\omega t)$	$\frac{2\omega s}{(s^2+\omega^2)^2}$
$\cos(\omega t)$	$\frac{s}{(s^2+\omega^2)}$
$\cos(\omega t + \phi)$	$\frac{s\cos(\phi) - \omega\sin(\phi)}{(s^2+\omega^2)}$
$t\cos(\omega t)$	$\frac{s^2-\omega^2}{(s^2+\omega^2)^2}$
$e^{-at}\sin(\omega t)$	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{-at}\cos(\omega t)$	$\frac{s+a}{(s+a)^2+\omega^2}$
$\sinh(\omega t)$	$\frac{\omega}{(s^2-\omega^2)}$
$\cosh(\omega t)$	$\frac{s}{(s^2-\omega^2)}$

Q1. **(a)** Figure Q1-(a) shows an initially charged capacitor C_1 with a voltage V_1 across it and an uncharged capacitor C_2. Show that after closure of the switch S_1 the voltage across the capacitors is

$$V = C_1V_1/(C_1 + C_2).$$

Find an expression for the total energy stored then by the capacitors. [3 marks]





(b) Figure Q1-(b) shows an inductor L_1 with a direct current I_1 flowing through it. Show that when the *make-before-break* switch S_2 is changed from contact *a* to contact *b*, the subsequent steady state current through the inductors is

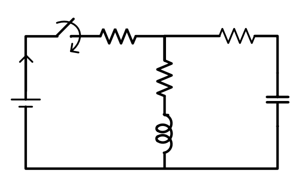
$$I = L_1I_1/(L_1 + L_2).$$

Find an expression for the total energy stored then by the inductors. [3 marks]

(c) Compare the energy stored before and after switch operation of each of the circuits in parts (a) and (b) above. Comment on your results. [4 marks]

Q2. **(a)** In the circuit of Figure Q2, the capacitor is initially uncharged and the switch is closed at time $t = 0$. Find the values of the following quantities, [4 marks]

i. initial current *i(t)* just after closure of switch,
ii. final steady state current *i(t)*, and
iii. final value of voltage across the capacitor.



(b) The input voltage *v(t)* of a particular circuit is related to its output current *i(t)* by the following differential equation

$$\frac{d^2 i(t)}{dt^2} + 2\frac{d\,i(t)}{dt} + 2\,i(t) = \frac{d\,v(t)}{dt} + v(t)$$

Find an expression for the output current *i(t)* at steady state when the input voltage *v(t)* is

i. a direct source of 10 V, [2 marks]
ii. an alternating source of 10 cos (2t) V. [4 marks]

Q3. **(a)** Determine one possible circuit that can be represented by the impedance function

$$Z(s) = \frac{s(s^2+4)(s^2+16)}{5(s^2+1)(s^2+9)} \quad k\Omega$$

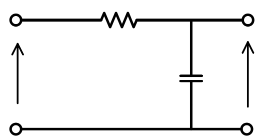
where s is the complex frequency in Mrad/s. [10 marks]

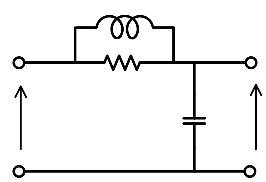
Q4. **(a)** Determine the transfer function $V_{out}(s)/V_{in}(s)$ for the circuit in Figure Q4-(a), where the circuit parameters are such that $RC = 1$. Show that the response to a unit impulse is, [2 marks]

$$V_{out}(t) = e^{-t}$$

(b) Determine the transfer function $V_{out}(s)/V_{in}(s)$ for the circuit in Figure Q4-(b), where the circuit parameters are such that $RC = L/R = 1$. Show that the response to a unit impulse is, [4 marks]

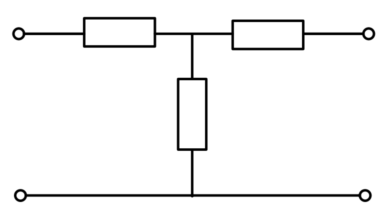
$$V_{out}(t) = \frac{2\sqrt{3}}{3} e^{-t/2} \cos\left(\frac{\sqrt{3}}{2}t - \frac{\pi}{6}\right)$$





(c) Sketch approximately to scale, the impulse responses of the two circuits above for $0 \le t \le 1$ and compare these with the response of an ideal integrating circuit. [4 marks]

Q5. The two port T-network shown in Figure Q5 is an equivalent circuit for a 100 meter length of a particular type of a coaxial cable at a given frequency. All the values indicated are equivalent impedances in ohms.



i. Derive an equivalent T-network for a 400 meter length of the same cable. [6 marks]

ii. What is the characteristic impedance of the 100 meter length of the cable? Would it be same or different for the 400 meter length of the cable? Explain your answer. [4 marks]

Q6. A 3-phase 100V, 50 Hz, 3-wire balanced supply with the phase sequence *ABC* is feeding a star connected balanced 3-phase load of (4+j3) Ω per phase. A sudden short circuit fault occurs on the load and as a result, load impedance on phase *A* is replaced by a fault resistance of 1.0 Ω. The other two impedances of the 3-phase load remains intact.

- **i.** Determine the magnitude of fault current in phase *A*. [4 marks]
- **ii.** Calculate the symmetrical components of the line currents after the fault. [4 marks]
- **iii.** Taking the phase voltage *A* of the 3-phase supply as your reference, draw the 3 phasor diagrams corresponding to positive, negative and zero symmetrical components. [2 marks]

Q7. **(a)** Explain why in most practical cases of circuit analysis, higher order harmonics of an input voltage could be ignored. [2 marks]

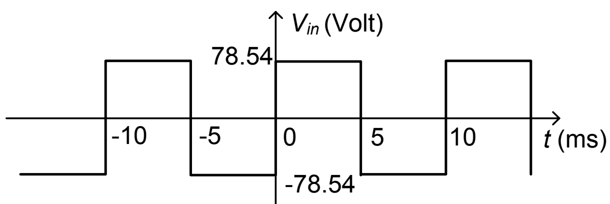


Figure Q7-(a)

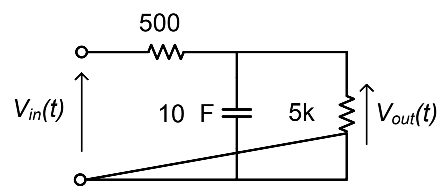


Figure Q7-(b)

(b) The square voltage waveform $V_{in}(t)$ shown in Figure Q7-(a) is applied to the circuit in Figure Q7-(b). Find the first 4 significant terms of the Fourier series of the resulting output voltage $V_{out}(t)$. [6 marks]

(c) Determine the rms value of the output voltage found in part (b) above. What would be the rms value of this voltage waveform if you approximated it with the fundamental component only? Calculate the percentage error in the rms value due to this approximation. [2 marks]