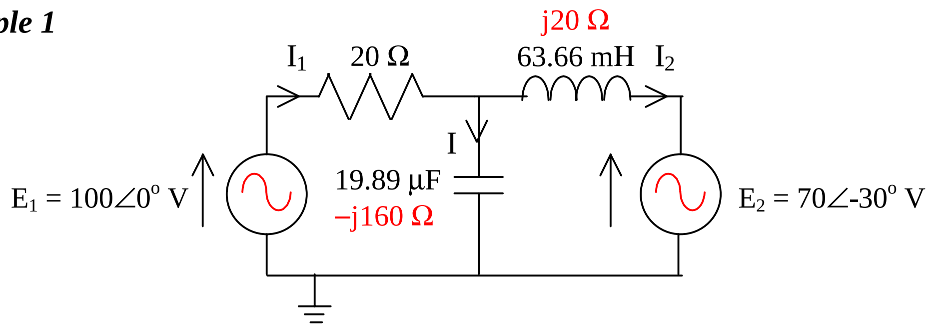
Network Theorems - Alternating Current examples - *J. R. Lucas*

In the previous chapter, we have been dealing mainly with direct current resistive circuits in order to the principles of the various theorems clear. As was mentioned, these theories are equally valid for a.c.

Example 1



For the circuit shown in the figure, if the frequency of the supply is 50 Hz, determine using Ohm's Law and Kirchoff's Laws the current I in the 160 Ω capacitor.

Impedance of capacitor and inductor at 50 Hz are

$$j\,X_L = j\,2\pi \times 50 \times 63.66 \times 10^{-3} = j\,20\ \Omega$$

and $\quad -j\,X_C = 1/(\,j\,2\pi \times 50 \times 19.89 \times 10^{-6}) = -j\,160\ \Omega$

Using Kirchoff's current law

$$I = I_1 - I_2$$

Using Kirchoff's voltage law

$$100\angle 0^o = 20\,I_1 - j160\,(I_1 - I_2) \quad \Rightarrow \quad 5 = (1\text{-}j8)\,I_1 + j8\,I_2 \qquad \text{...... (1)}$$

and $\quad 70\angle\text{-}30^o = -j\,20\,I_2 - j\,160\,(I_1 - I_2) \quad \Rightarrow \quad 7\angle\text{-}30^o = -j\,16\,I_1 + j\,14\,I_2 \qquad \text{...... (2)}$

multiplying equation (1) by 7 and equation (2) by 4 and subtracting gives

$$35 - 28\angle\text{-}30^o = (7 - j\,56 + j\,64)\,I_1 + 0 \quad \Rightarrow \quad 10.751 + j\,14 = (7 + j\,8)I_1$$

i.e. $\quad I_1 = \dfrac{17.65\angle 52.48^o}{10.63\angle 48.81^o} = 1.660\angle 3.67^o\ \text{A}$

substituting in (1),

$$j8\,I_2 = 5 - (1\text{-}j8) \times 1.660\angle 3.67^o$$

i.e. $\quad 8\angle 90^o\,I_2 = 5 - 8.062\angle\text{-}82.87^o \times 1.660\angle 3.67^o = 5 - 13.383\angle\text{-}79.20^o = 2.492 + j13.146$

i.e. $\quad I_2 = \dfrac{13.38\angle 79.27^o}{8\angle 90^o} = 1.673\angle\text{-}10.73^o\ \text{A}$

Thus the required current I is $= 1.660\angle 3.67^o - 1.673\angle\text{-}10.73^o$

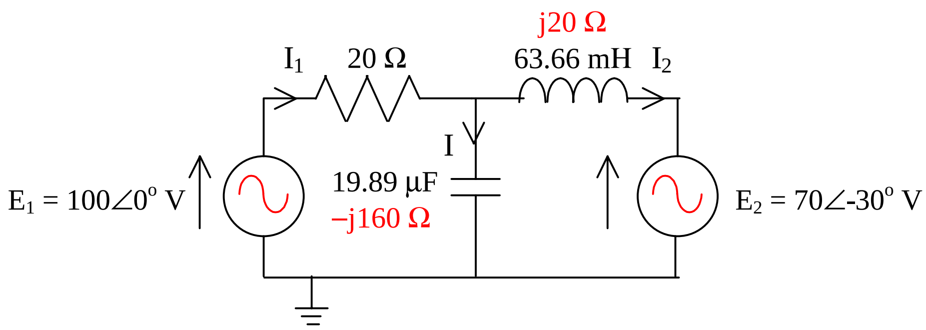
$= 1.657 + j\,0.106 - 1.644 + j\,0.311 = 0.013 + j\,0.417$

$= \underline{0.42\angle 88.2^o\ \text{A}}$

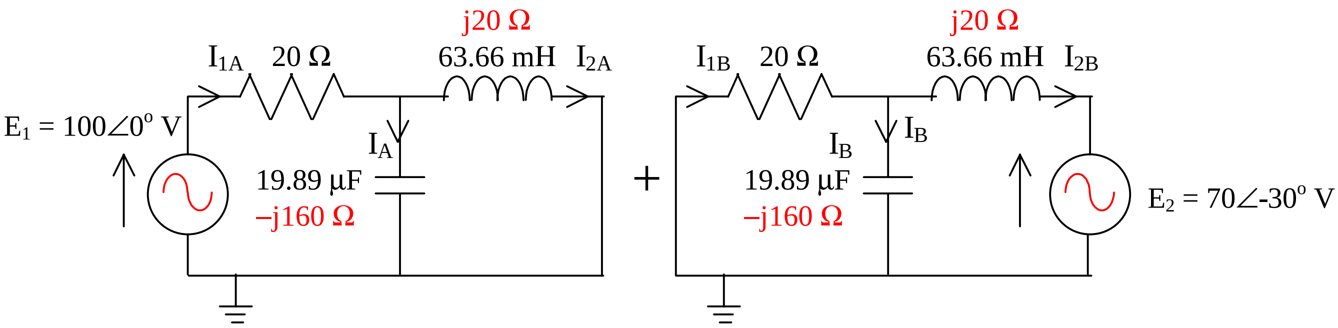
The problem could probably have been worked out with lesser steps, but I have done it in this manner so that you can get more familiarised with the solution of problems using complex numbers.

Example 2

Let us solve the same problem as earlier, but using Superposition theorem.



This circuit can be broken into its two constituent components as shown.



Using series parallel addition of impedances, we can obtain the supply currents as follows.

Equivalent $Z_{s1} = 20 + (-j160)//j20$, $\qquad Z_{s2} = j20 + 20//(-j160)$

$= 20 + \dfrac{-j160 \times j20}{-j140}$, $\qquad = j20 + \dfrac{20 \times (-j160)}{20 - j160}$

$= 20 + j\,22.857$, $\qquad = j20 + 19.692 - j\,2.462 = 19.692 + j17.538$

$= 30.372\angle 48.81^\circ\ \Omega$, $\qquad = 26.370\angle 41.69^\circ\ \Omega$

source current $I_{1A} = \dfrac{100\angle 0^o}{30.372\angle 48.81^o}$, $\qquad -I_{2B} = \dfrac{70\angle -30^o}{26.370\angle 41.69^o}$

$= 3.293\angle\text{-}48.81^\circ$, $\qquad = 2.655\angle\text{-}71.69^\circ$,

Using the current division rule (note directions of currents and signs),

$I_A = 3.293\angle -48.81^o \times \dfrac{j20}{-j140}$, $\qquad I_B = 2.655\angle -71.69^o \times \dfrac{20}{20 - j160}$

$= 0.470\angle 131.19^\circ$, $\qquad = \dfrac{53.10\angle -71.69^o}{161.25\angle -82.87^o} = 0.329\angle 11.18^\circ$

Using superposition theorem, the total current in

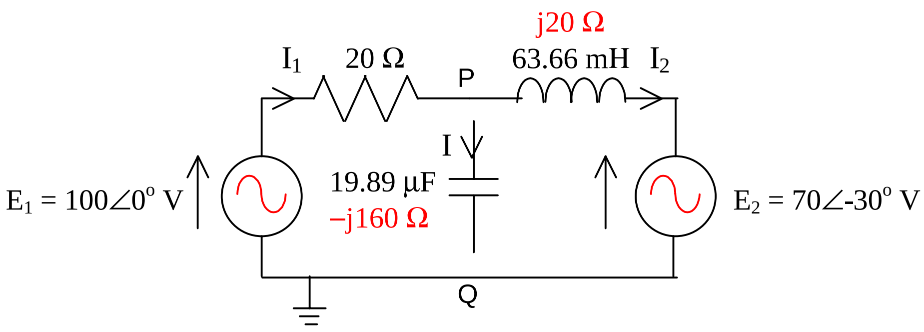
$I = 0.470\angle 131.19^\circ + 0.329\angle 11.18^\circ = -0.310 + j\,0.354 + 0.323 + j0.064$

$= 0.013 + j\,0.419 = \underline{\underline{0.42\angle 88.2^\circ\ A}}$

which is the same answer obtained in the earlier example.

Example 3

Let us again consider the same example to illustrate Thevenin's Theorem.



Consider the capacitor disconnected at P and Q.

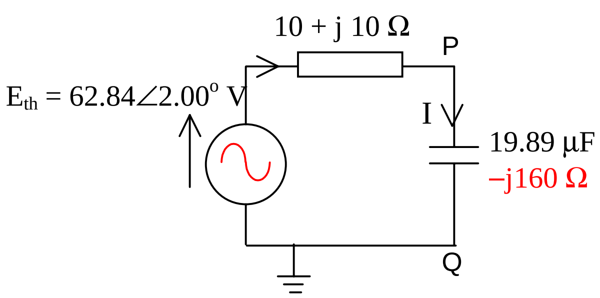
Current flowing in the circuit under this condition = $\dfrac{100\angle 0^o - 70\angle -30^o}{20 + j20}$

$$= \frac{100 - 60.62 + j35}{20 + j20} = \frac{52.69\angle 41.63^o}{28.28\angle 45^o} = 1.863\angle -3.37^o$$

$\therefore$ Thevenin's voltage source = $100\angle 0^o - 20 \times 1.863\angle\text{-}3.37^o = 62.80 + j\,2.19 = 62.84\angle 2.00^o$

Also, Thevenin's impedance across Q = 20//j20 = $\dfrac{20 \times j20}{20 + j20}$ = $14.142\angle 45^o = 10 + j\,10$

$\therefore$ Thevenin's equivalent circuit is



From this circuit, it follows that

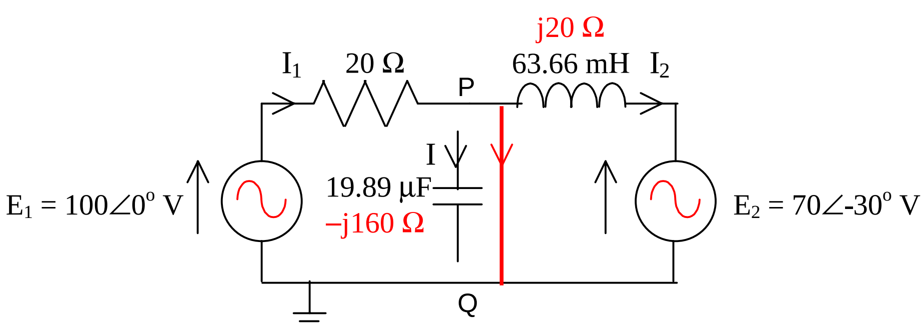
$$I = \frac{62.84\angle 2.00^o}{10 + j10 - j160} = \frac{62.84\angle 2.00^o}{150.33\angle -86.19^o}$$

$= \underline{\underline{0.418\angle\text{-}88.2^o\ \text{A}}}$

which is again the same result.

Example 4

Let us again consider the same example to illustrate Norton's Theorem.



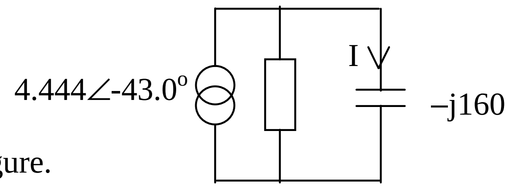
Consider the capacitor short-circuited at P and Q.

The Norton's current source is given as $\dfrac{100\angle 0^o}{20} + \dfrac{70\angle -30^o}{j20}$ = 5 –1.75 –j3.031

$= 3.25 - j\,3.031 = 4.444\angle\text{-}43.00^o$

Norton's admittance = $\frac{1}{20}+\frac{1}{j20}=0.05-j0.05$ S

or same as Thevenin's impedance 10 + j10 Ω



The Norton's equivalent circuit is as shown in the figure.

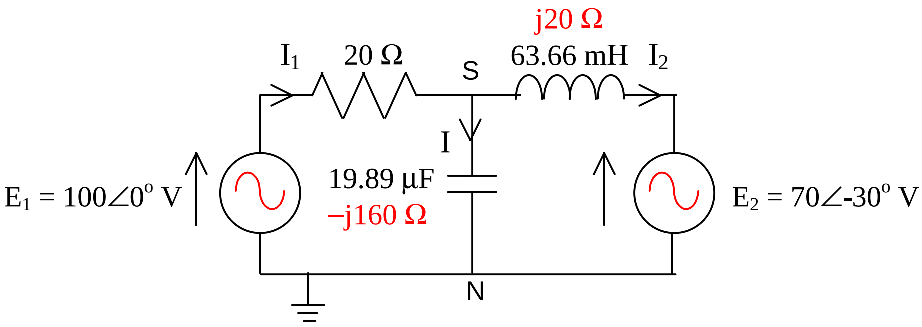
The current through the capacitor can be determined using the current division rule.

$$I = 4.444\angle{-43.0^o}\times\frac{10+j10}{10+j10-j160} = 4.444\angle{-43.0^o}\times\frac{14.142\angle 45^o}{150.33\angle-86.19^o}$$

$$= \underline{\underline{0.418\angle{-88.2^o}\ A}}$$

Example 5

Using Millmann's theorem find the current in the capacitor.



$$V_{SN} = \frac{\sum Y.V}{\sum Y} = \frac{\frac{1}{20}\cdot 100\angle 0^o + \frac{1}{-j160}\cdot 0 + \frac{1}{j20}\cdot 70\angle-30^o}{\frac{1}{20}+\frac{1}{-j160}+\frac{1}{j20}}$$

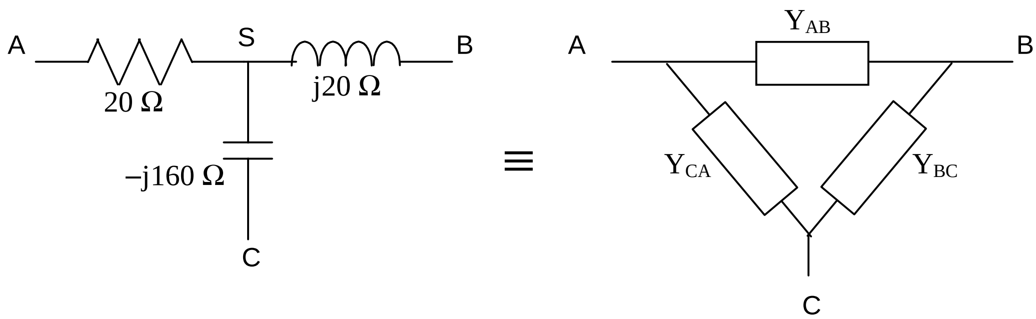
$$= \frac{5+0-1.75-j3.031}{0.05+j0.00625-j0.05} = \frac{3.25-j3.031}{0.05-j0.04375} = \frac{4.444\angle-43.00^o}{0.06643\angle-41.19}$$

$$= 66.89\angle{-1.81^o}\ V$$

$$\therefore \quad I = 66.89\angle{-1.81^o}/(-j160) = \underline{\underline{0.418\angle 88.19^o\ A}}$$

Example 6

Determine the delta equivalent of the star connected network shown.



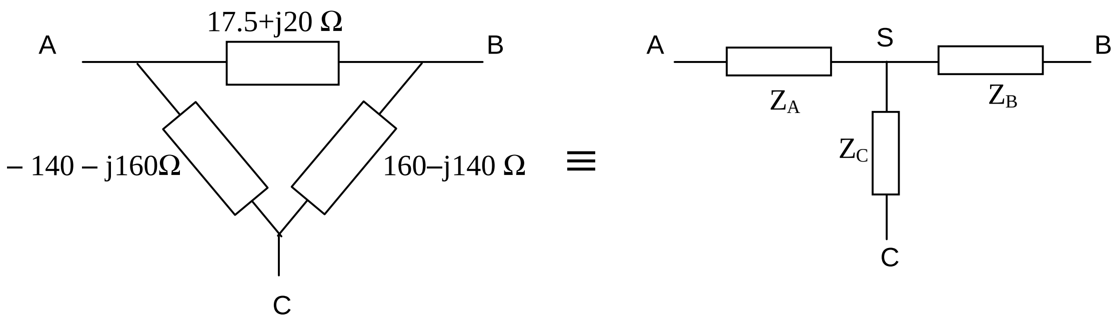
$$Y_{AB} = \frac{\frac{1}{20}\times\frac{1}{j20}}{\frac{1}{20}+\frac{1}{-j160}+\frac{1}{j20}} = \frac{1}{j20-2.5+20} = \frac{1}{17.5+j20}, \quad \therefore Z_{AB} = 17.5 + j\,20\ \Omega$$

$$Y_{BC} = \frac{\frac{1}{-j160}\times\frac{1}{j20}}{\frac{1}{20}+\frac{1}{-j160}+\frac{1}{j20}} = \frac{1}{160+j20-j160} = \frac{1}{160-j140}, \quad \therefore Z_{BC} = 160 - j\,140\ \Omega$$

$$Y_{CA} = \frac{\frac{1}{20}\times\frac{1}{-j160}}{\frac{1}{20}+\frac{1}{-j160}+\frac{1}{j20}} = \frac{1}{-j160+20-160} = \frac{1}{-140-j160}, \quad \therefore Z_{CA} = -140 - j\,160\ \Omega$$

Example 7

Determine the star equivalent of the delta connected network shown.



$$Z_A = \frac{(17.5+j20)(-140-j160)}{17.5+j20-140-j160+160-j140} = \frac{26.575\angle 48.81^o \times 212.603\angle -131.19^o}{37.5-j280}$$

$$= \frac{5650\angle -82.38^o}{282.5\angle -82.37^o} = 20.00\angle -0.01^o = 20\ \Omega$$ (same as original value in Ex 6).

$$Z_B = \frac{(17.5+j20)(160-j140)}{17.5+j20-140-j160+160-j140} = \frac{26.575\angle 48.81^o \times 212.603\angle -41.19^o}{37.5-j280}$$

$$= \frac{5650\angle 7.62^o}{282.5\angle -82.37^o} = 20.00\angle 89.99^o = j20\,\Omega$$ (same as original value in Ex 6).

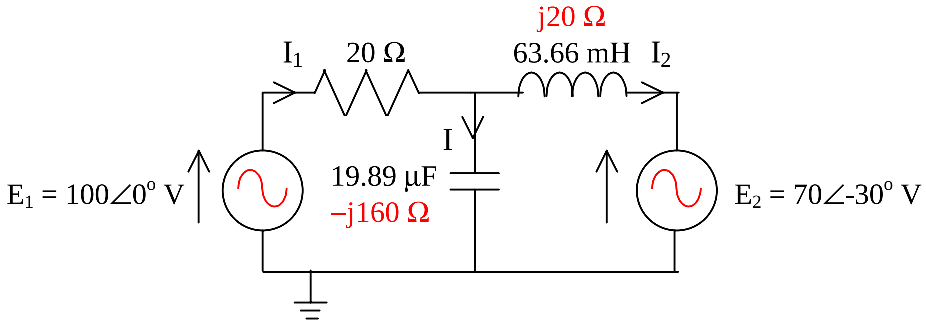
$$Z_C = \frac{(160-j140)(-140-j160)}{17.5+j20-140-j160+160-j140} = \frac{212.603\angle -41.19^o \times 212.603\angle -131.19^o}{37.5-j280}$$

$$= \frac{45200\angle -172.38^o}{282.5\angle -82.37^o} = 160.00\angle -90.01^o = -j160\,\Omega$$ (same as original value in Ex 6).

In order to show that the working is correct, I have selected the reverse problem for this example and used the results of the previous example to find the original quantities. You can see that the answers differ only due to the cumulative calculation errors.

Example 8

Determine using compensation theorem, the current I, if the available capacitor is 20 μF, instead of the 19.89 μF already assumed in the earlier problems.



Solution

20 μF corresponds to $\dfrac{1}{j20\times10^{-6}\times2\times\pi\times50}$ = -j159.15 Ω

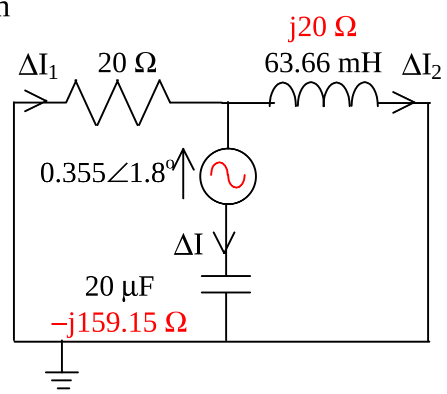
change of impedance ΔZ = –j159.15 – (–j160) = j0.85 Ω.

from earlier calculations

I = $0.418\angle\text{-}88.2^{o}$ A

∴ using compensation theorem, I . ΔZ = $0.418\angle\text{-}88.2^{o}\times j0.85 = 0.355\angle1.8^{o}$ V

∴ changes in current in the network can be obtained from



Note that the direction of ΔI is marked in the same direction as the original I, so that the source would in fact send a current in the opposite direction.

i.e. $-\Delta I = \dfrac{0.355\angle1.8^{o}}{-j159.15+20//j20}$

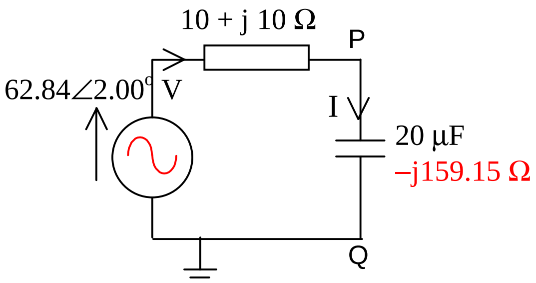
$$= \frac{0.355\angle1.8^{o}}{-j159.15+\dfrac{20\times j20}{20+j20}} = \frac{0.355\angle1.8^{o}}{-j159.15+10+j10}$$

$= \dfrac{0.355\angle1.8^{o}}{149.48\angle-86.16^{o}}$ = $0.00237\angle88.0^{o}$, giving ΔI as $-0.00237\angle88.0^{o}$, or $0.00237\angle268.0^{o}$ A

i.e. correct current I = $0.418\angle\text{-}88.2^{o} + 0.00237\angle268.0^{o}$ = 0.013 – j 0.4177 –0.00008–j0.00237

= 0.013 – j 0.420 = $\underline{0.420\angle\text{-}88.2^{o}\text{ A}}$

Comparing result using Thevenin's equivalent circuit derived in example 3



From this circuit, it follows that

$$I = \frac{62.84\angle2.00^{o}}{10+j10-j159.15} = \frac{62.84\angle2.00^{o}}{149.48\angle-86.16^{o}}$$

= $\underline{\underline{0.420\angle\text{-}88.2^{o}\text{ A}}}$ which is the same result.