University of Moratuwa, Sri Lanka

Faculty of Engineering

Department of Electrical Engineering

B. Sc. Engineering Honours Degree Course

Level 2 – Semester 2 Examination

EE201 – THEORY OF ELECTRICITY

Time Allowed: 3 Hours January 2007

Additional Material

- Graph Paper will be provided if required.
- Laplace transform pairs are provided on the other side of this page.

Instructions to Candidates

- This paper contains 7 questions in 4 pages, including the cover page.
- Answer **All** Questions.
- This examination accounts for 70% of the module assessment.
- The marks allocated to each part of a question are indicated in square brackets at the end of each part.
- Total allocation for the paper is 70 marks.
- This is a closed book examination and only authorised calculators will be permitted.

Technical Data:

- Permeability of free space $\mu_0 = 4\pi \times 10^{-7}$ H/m
- Permittivity of free space $\varepsilon_0 = 8.854 \times 10^{-12}$ F/m
- Velocity of light in free space = 2.998×10^{8} m/s

Some Laplace Transform Pairs

	$f(t)$	F(s)
1.	1	$1/s$
2.	t	$1/s^2$
3.	e^{-at}	$1/(s+a)$
4.	te^{-at}	$1/(s+a)^2$
5.	$\frac{t^{n-1}}{(n-1)!}e^{-at}$	$(s+a)^{-n}$
6.	$\sin(\omega t)$	$\omega/(s^2+\omega^2)$
7.	$\cos(\omega t)$	$s/(s^2+\omega^2)$
8.	$e^{-at}\sin(\omega t)$	$\omega/[(s+a)^2+\omega^2]$
9.	$e^{-at}\cos(\omega t)$	$(s+a)/[(s+a)^2+\omega^2]$
10.	$\sqrt{c^2+d^2}\, e^{-at}\cos\left(\omega t - \tan^{-1}\left(\frac{d}{c}\right)\right)$	$\frac{c(s+a)+d\omega}{(s+a)^2+\omega^2}$
11.	$\sinh(\omega t)$	$\omega/(s^2-\omega^2)$
12.	$\cosh(\omega t)$	$s/(s^2-\omega^2)$
Special Transforms		
13.	$\frac{df}{dt}$	$s\,F(s) - f(0^+)$
14.	$\int_0^t f(\tau)d\tau$	$F(s)/s$
15.	$c_1 f_1(t) + c_2 f_2(t)$	$c_1 F_1(s) + c_2 F_2(s)$
16.	$\int f_1(\tau) f_2(t-\tau)d\tau$	$F_1(s)\,F_2(s)$
17.	$\delta(t-T)$ (delta function, $T > 0$)	e^{-sT}
18.	$\frac{d^n}{dt^n}(\delta(t)) = \delta^{(n)}(t)$ (n-th derivative of delta function)	s^n
19.	$\delta^{(-n)}(t)$ (n-th integral of delta function)	s^{-n}

Q1. **(a)** Prove from first principles that two capacitors C_1 and C_2 connected in series can be replaced with an equivalent capacitance of $C_1C_2/(C_1+C_2)$. [2 marks]

(b) Find the voltage across each capacitor of the circuit in Figure Q1-(b). [4 marks]

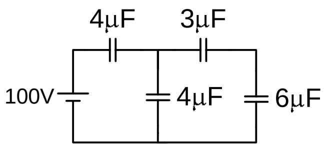

Figure Q1-(b)

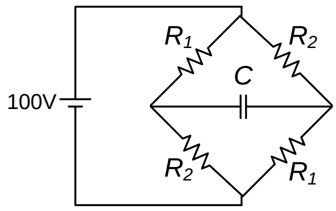

Figure Q1-(c)

(c) Find an expression for the charging time constant of the capacitor C shown in Figure Q1-(c). Also find the steady state voltage across the capacitor. [4 marks]

Q2. **(a)** Write a differential equation relating output current $i(t)$ to the input voltage $v(t)$ for the circuit in Figure Q2. [5 marks]

(b) By means of the differential equation derived above, answer the following.

i. What is the order of the differential equation? How is the order of the differential equation related to the circuit in question? [1 mark]

ii. If the input voltage $v(t)$ is a DC source of magnitude E, what is the current at steady state? [1 mark]

iii. If the input voltage $v(t)$ is an AC source of angular frequency ω, derive an expression for the input impedance of the circuit. [2 marks]

iv. Validate your answer in part (iii) above using AC theory. [1 mark]

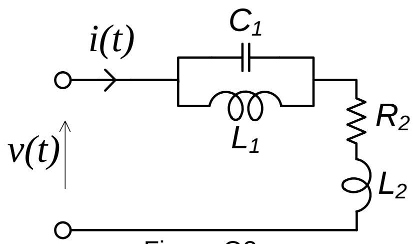

Figure Q2

Q3. **(a)** Derive an expression for the total energy stored in the coupled circuit in Figure Q3-(a). [2 marks]

(b) The circuit in Figure Q3-(b) is supplied by a sinusoidal voltage source V_s of variable frequency. The output V_{out} is taken across the inductor.

i. Write an expression for the output voltage V_{out} as a function of the input voltage V_s . [1 mark]

ii. Find an expression for the resonance frequency of the circuit. [1]

iii. Show that half-power frequencies are given by, [2 marks]

$$f_1, f_2 = \pm\frac{1}{4\pi RC} + \frac{1}{2\pi}\sqrt{\frac{1}{(2RC)^2} + \frac{1}{LC}}$$

iv. If the circuit is to be designed with a resonance frequency of 3.2 kHz and a bandwidth of 1 kHz, calculate the appropriate values for L and C, given that R = 2.0 kΩ. [2 marks]

v. With the circuit parameters as found in part (iv) above, determine the output voltage at the frequencies of f_1 and f_2, when V_s = 10V. [2 marks]

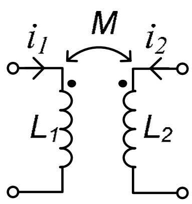

Figure Q3-(a)

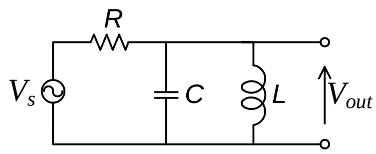

Figure Q3-(b)

Q4. **(a)** For the circuit in Figure Q4, find the input impedance across *XY*. [2 marks]

(b) Find the short circuit current through *XY*. [2 marks]

(c) Find the Norton's equivalent circuit across *XY*. [2 marks]

(d) If the voltage source is moved to the terminals *XY*, What would be the Norton's equivalent circuit across *AB*? [2 marks]

(e) Are the two Norton's equivalent circuits identical? Comment on your results. [2 marks]

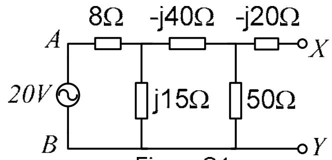

Figure Q4

Q5. **(a)** An alternating voltage source V_s has an internal impedance of $r + jx$. Find an expression for the purely resistive load R_p which when connected to the source, maximises the real output power. [4 marks]

(b) Derive an expression for the source efficiency for the above circuit when it delivers maximum power. [3 marks]

(c) You need to design a 10V, AC source such that for purely resistive loads, it delivers a maximum of 12.5W at a source efficiency of 75 percent. Determine the internal impedance of the source. [3 marks]

Q6. **(a)** Find the ABCD parameters for the transformer circuit in Figure Q6-(a). [5 marks]

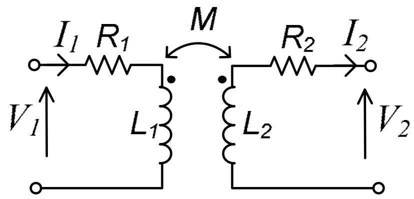

Figure Q6-(a)

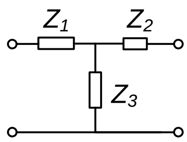

Figure Q6-(b)

(b) Find appropriate values for Z_1, Z_2 and Z_3 of the T-network in Figure Q6-(b) such that it is equivalent to the transformer circuit in Figure Q6-(a). [5 marks]

Q7. **(a)** Using the first principles and general properties of Laplace transform, find the Laplace transform of causal time function $f(t) = e^{-at}\sin\omega t$. [2 marks]

(b) In the circuit of Figure Q7, the capacitor has an initial charge of 500μC. If the switch *S* is closed at $t = 0$, using Laplace transform method, find an expression in time *t* for the subsequent variation of current *i(t)*. [5 marks]

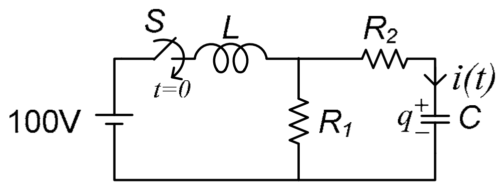

Figure Q7

(c) If *L*=1 mH, R_1=400Ω, R_2=600Ω and *C*=25μF, determine the following values related to *i(t)*, [2 marks]

- **i.** initial current,
- **ii.** time at which current becomes zero momentarily,
- **iii.** peak value of current and the time it occurs,
- **iv.** steady state value of current.

(d) Sketch, approximately to scale, the current wave form *i(t)* from $t = 0$ to 100ms. [1 mark]