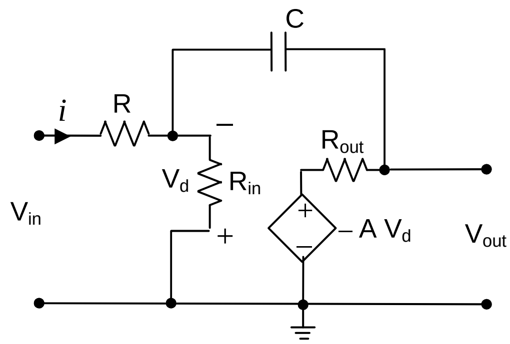
Level 2 Semester 2 Examination – June 2009



1. (a) i. Complete equivalent circuit as shown ***[1 mark]***

ii. $i = \dfrac{V_{in} - V_d}{R} = \dfrac{V_d}{R_{in}} + C\dfrac{d(V_d - V_{out})}{dt}$

$$C\frac{d(V_d - V_{out})}{dt} = \frac{V_{out} + AV_d}{R_{out}}$$ ***[2 marks]***

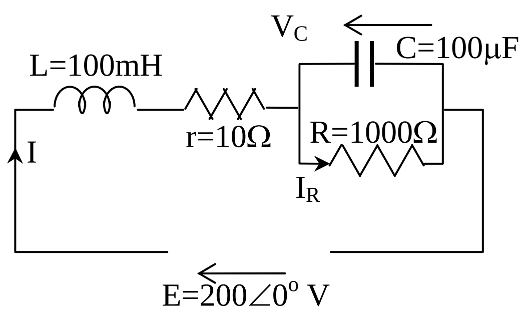
iii. With an ideal Operational amplifier,

$R_{out} = 0$, and $R_{in} = \infty$, $V_{out} = -A\,V_d$

$$\frac{V_{in} - V_d}{R} = 0 + C\frac{d(V_d - V_{out})}{dt}$$

With A→∞, V_d→0 giving $\dfrac{V_{in}}{R} = -C\dfrac{dV_{out}}{dt}$

Thus $V_{out} = -\dfrac{1}{CR}\int V_{in}\,dt$ ***[2 marks]***



(b) $Z = j\omega L + r + \dfrac{R \times \dfrac{1}{j\omega C}}{R + \dfrac{1}{j\omega C}} = j\omega L + r + \dfrac{R}{1 + j\omega CR}$

ω = 250 rad/s

Lω = 0.1×250 = 25Ω, 1/Cω = 1/0.0001×250 = 40Ω

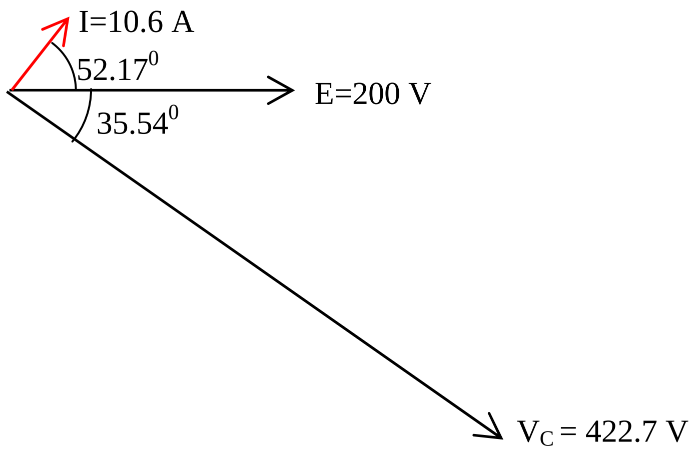
$Z = j25 + 10 + \dfrac{1000}{1 + j25} = (11.597 - j14.936) = 18.910\angle\text{-}52.17^0\ \Omega$

I = 200/18.910∠-52.17^0 = 10.576∠52.17^0 A ***[1 mark]***

V_C = 200 – (10+j25)×10.576∠52.17^0 = 200 – 26.926∠68.20^0×10.576∠52.17^0

= 200 – 284.77∠120.37^0 = 200 – (–143.98+j245.69) = 343.98 – j245.69

= 422.71∠–35.54^{0}V ***[1 mark]***



[1 mark]

(c) Resonance would occur when imaginary part of $\left[j\omega L + r + \dfrac{R}{1 + j\omega CR} \right]$ is equal to 0.

L = 100 mH, r = 10 Ω, C = 100 μF, R = 1000 Ω

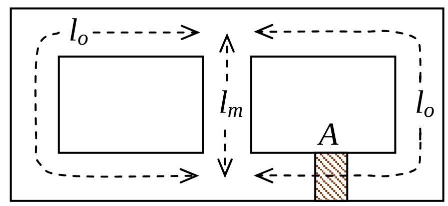
i.e. $\omega L - \dfrac{\omega CR^2}{1+\omega^2 C^2 R^2} = 0$, $0.1\omega - \dfrac{\omega 0.0001 \times 1000^2}{1+\omega^2 0.0001^2 1000^2} = 0$

i.e. $1 + 0.01\omega^2 = 1000$, since $\omega \neq 0$

i.e. $\omega^2 = 99900$, $\omega = 316.07$ rad/s or 50.30 Hz **[1 mark]**

and circuit current at resonance = $\left[\dfrac{200}{10 + \dfrac{1000}{1 + (316.07 \times 0.0001 \times 1000)^2}} \right]$ = 18.18 A **[1 mark]**

2. (a) Reluctance = $l/\mu A$



$A = 2$ cm^2, l_o=20 cm, l_m= 6 cm, $\mu_r = 2000$

outer limb reluctance $S_o = l_o/\mu_o\mu_r A$

$S_o = 0.2/(4\pi\times10^{-7}\times2000\times2\times10^{-4}) = 0.3979\times10^6$ H^{-1}

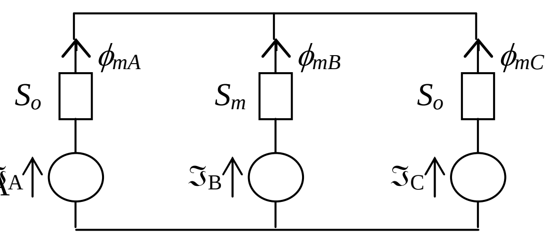
middle limb reluctance $S_m = l_m/\mu_o\mu_r A$

$S_m = 0.06/(4\pi\times10^{-7}\times2000\times2\times10^{-4}) = 0.1194\times10^6$ H^{-1}

$I_A = 20\angle0^o$ A, N = 100

$\Im_A = 2000\angle0^o$ A, $\Im_B = 2000\angle-120^o$ A, $\Im_C = 2000\angle120^o$ A

Equivalent circuit with values **[4 marks]**



(b) Since the resistors on either side are equal,

$V_{Thevenin} = (100\angle0^o + 70.7\angle45^o)/2$

$= (100 + 50 + j\,50)/2 = 75 + j25$ V

$Z_{Thevenin} = 30//30 = 15\ \Omega$ **[2 marks]**

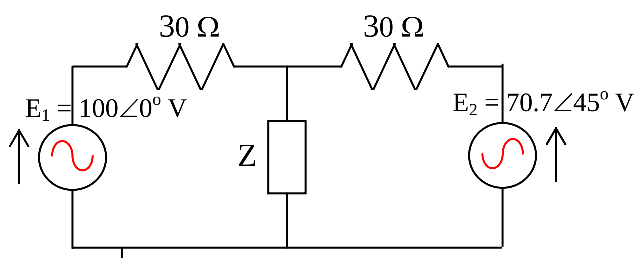


Figure Q2b

i. for inductance, Z = j 40 Ω

current = $(75 + j25)/(15+j40) = 79.057\angle18.43^o/42.720\angle69.44^o$

$= 1.851\angle{-51.01^o}$ A **[1 mark]**

ii. for resistance, Z = 40 Ω

current = $(75 + j25)/(15+40) = 79.057\angle18.43^o/55 = 1.437\angle{-18.43^o}$ A **[1 mark]**

iii. for maximum power transfer, Z = 15 Ω

current in the circuit = $(75+j25)/(15+15) = 79.057\angle18.43^o/30 = 2.635\angle18.43^o$

maximum power = $2.635^2\times15 = 104$ W **[2 marks]**

3. (a)

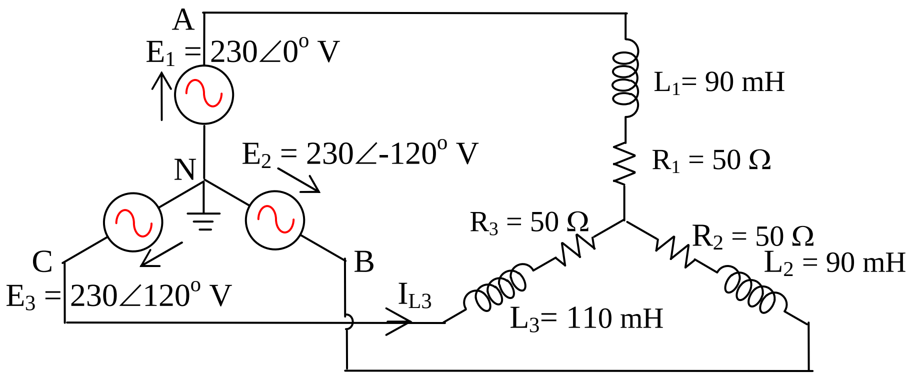
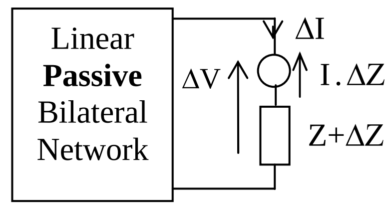


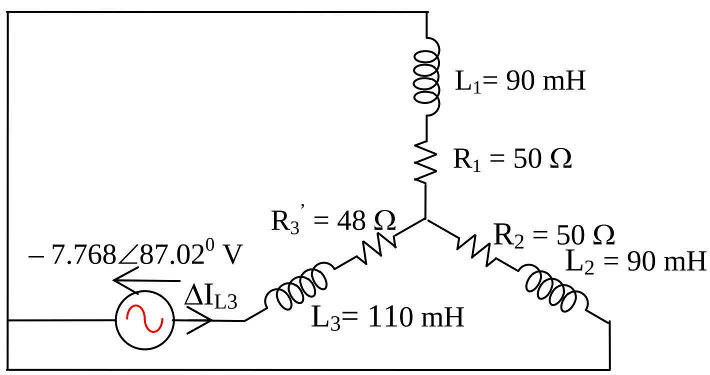
Figure Q3a

Using compensation theorem, changes can be obtained from

$I = 3.884\angle 87.02^0$ A

$\Delta Z = -2\ \Omega$, $I.\Delta Z = -7.768\angle 87.02^0$ V





At 50 Hz, 90 mH $\rightarrow$ j28.27 Ω, 110 mH $\rightarrow$ j34.56 Ω

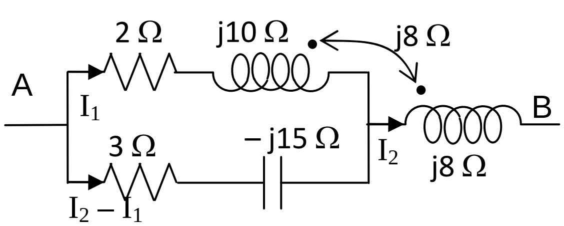
ΔI_{L3} $= -(-7.768\angle 87.02^0)/(48 + j34.56 + 25 + j17.28) = 7.768\angle 87.02^0/89.53\angle 35.38^0$

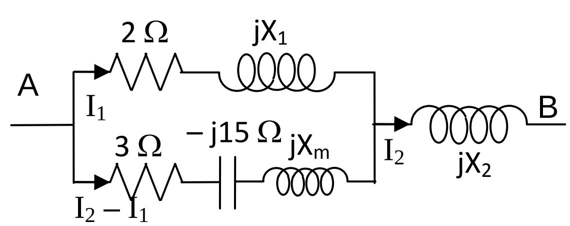
$= 0.08676\angle 51.64^0$ A

New I_{L3} $= 3.884\angle 87.02^0 + 0.08676\angle 51.64^0 = 0.202 + j3.879 + 0.054 + j\,0.068 = 0.256 + j3.947$

$= 3.955\angle 86.29^0$ A **[2 marks]**

(b)





Consider the uncoupled equivalent circuit shown.

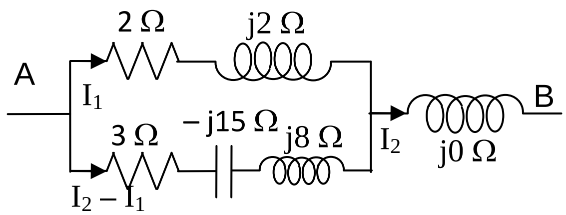
$V_{AB} = 2\,I_1 + (j10\,I_1 - j8\,I_2) + (j8\,I_2 - j8\,I_1) \equiv 2\,I_1 + jX_1\,I_1 + j\,X_2\,I_2$

Comparison of coefficients of I_1 gives $2 + j10 - j8 = 2 + j\,X_1$ or $X_1 = 2\ \Omega$

Comparison of coefficients of I_2 gives $-j8 + j8 = j\,X_2$ or $X_2 = 0\ \Omega$

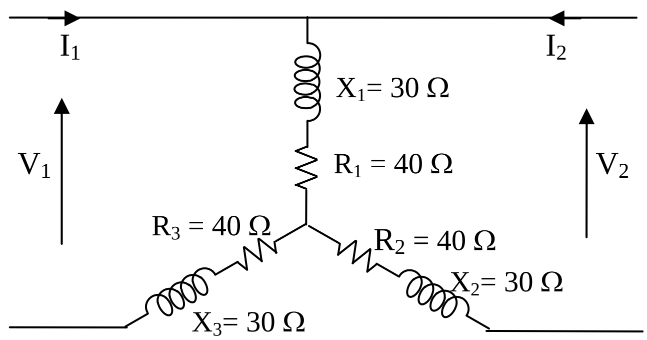
Also, $V_{AB} = (3 - j15)(I_2 - I_1) + (j8\, I_2 - j8\, I_1) \equiv (3 - j15 + jX_m)(I_2 - I_1) + j\, X_2\, I_2$

Comparison of coefficients of I_1 gives $-3 + j15 - j8 = -3 + j15 - jX_m$ or $X_m = 8\ \Omega$



[2 marks]

(c) The two port admittance matrix is given by



$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_2 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}$$

$$z_{11}\big|_{V_2=0} = 40+j30+(40+j30)//(40+j30) = 60 + j\,45 = 75\angle 36.87^0$$

$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = \frac{1}{z_{11}} = 0.01333\angle{-36.87^0}\ \text{S} \quad (\text{or } 0.01066 - j\,0.008)\ \text{S}$$

$$y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = \frac{I_1}{V_1} \times \left.\frac{I_2}{I_1}\right|_{V_2=0} = 0.01333\angle -36.87^0 \times \left(-\frac{1}{2}\right) = 0.00667\angle 143.13^0\ \text{S (or} - 0.00533+j0.004)$$

By symmetry

$y_{12} = y_{21} = 0.00667\angle 143.13^0$ S

$y_{22} = y_{11} = 0.01333\angle{-36.87^0}$ S

The two port admittance matrix is

$$[Y] = \begin{bmatrix} 0.0133\angle -36.9^0 & 0.0067\angle 143.1^0 \\ 0.0067\angle 143.1^0 & 0.0133\angle -36.9^0 \end{bmatrix}$$

[3 marks]

(b) Characteristic impedance Z_0 is given when

$V_1/I_1 = Z_o = V_2/(-I_2)$

i.e. using two port admittance parameters,

$I_1 = y_{11}Z_oI_1 - y_{12}Z_oI_2$

$I_2 = y_{21}Z_oI_1 - y_{22}Z_oI_2$

$$\therefore \quad \frac{I_2}{I_1} = \frac{y_{11}Z_0 - 1}{y_{12}Z_0} = \frac{y_{21}Z_0}{y_{22}Z_0 + 1}$$

Since $y_{22} = y_{11}$, and $y_{12} = y_{21}$

$y_{11}^2\, Z_0^2 - 1 = y_{12}^2\, Z_0^2$

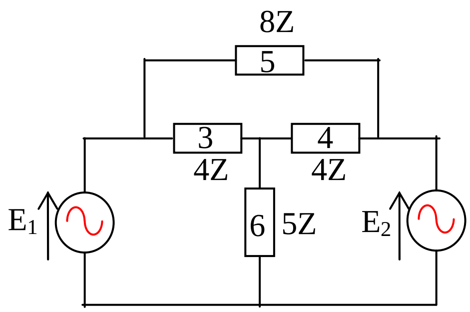
$$Z_0 = \sqrt{\frac{1}{y_{11}^2 - y_{12}^2}} = \sqrt{\frac{1}{0.01333^2\angle -36.87\times 2^0 - 0.0.00667^2\angle 143.13\times 2^0}}$$

$$= \sqrt{\frac{1}{0.0001777\angle -73.74^0 - 0.00004444\angle 286.26^0}} = \sqrt{\frac{1}{37.3\times 10^{-6} - j128\times 10^{-6}}}$$

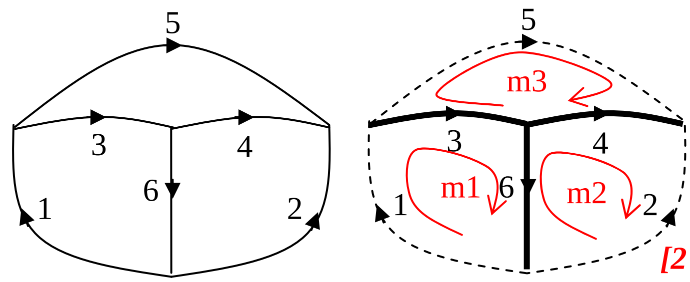
$$= \sqrt{\frac{1}{1.33\times 10^{-4}\angle -73.75^0}}$$

Z_0 = $\underline{\underline{86.7\angle 36.87^0\ \Omega}}$ ***[3 marks]***

4. (a)



The oriented graph of the network is



[2 marks]

The selected **tree** of the network is shown in a full line and those of the co-tree, forming the independent loops, are shown in a broken line.

(b) Branch impedance matrix $[Z_b]$, Branch source voltage vector $\underline{E}_{gb}$ and the mesh-branch incidence matrix [B] are as follows.

$$[Z_b] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4Z & 0 & 0 & 0 \\ 0 & 0 & 0 & 4Z & 0 & 0 \\ 0 & 0 & 0 & 0 & 8Z & 0 \\ 0 & 0 & 0 & 0 & 0 & 5Z \end{bmatrix}, \quad [E_{gb}] = \begin{bmatrix} E_1 \\ E_2 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad [B] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$$

[1+1+1 marks]

$$[Z_m] = [B]^t[Z_b][B] = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & -1 & 0 & 1 & 0 & -1 \\ 0 & 0 & -1 & -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4Z & 0 & 0 & 0 \\ 0 & 0 & 0 & 4Z & 0 & 0 \\ 0 & 0 & 0 & 0 & 8Z & 0 \\ 0 & 0 & 0 & 0 & 0 & 5Z \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$$

$$[Z_m] = \begin{bmatrix} 9Z & -5Z & -4Z \\ -5Z & 9Z & -4Z \\ -4Z & -4Z & 16Z \end{bmatrix}$$

[1 mark]

$$\underline{E}_{gm} = [B]^t \underline{E}_{gb} = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & -1 & 0 & 1 & 0 & -1 \\ 0 & 0 & -1 & -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} E_1 \\ -E_2 \\ 0 \end{bmatrix}$$ **[1 mark]**

$$\begin{bmatrix} E \\ -E \\ 0 \end{bmatrix} = \begin{bmatrix} 9Z & -5Z & -4Z \\ -5Z & 9Z & -4Z \\ -4Z & -4Z & 16Z \end{bmatrix} \begin{bmatrix} I_{m1} \\ I_{m2} \\ I_{m3} \end{bmatrix}, \quad \begin{bmatrix} I_{m1} \\ I_{m2} \\ I_{m3} \end{bmatrix} = \begin{bmatrix} 9Z & -5Z & -4Z \\ -5Z & 9Z & -4Z \\ -4Z & -4Z & 16Z \end{bmatrix}^{-1} \begin{bmatrix} E \\ -E \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} I_{m1} \\ I_{m2} \\ I_{m3} \end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} 128Z^2 & 96Z^2 & 56Z^2 \\ 96Z^2 & 128Z^2 & 56Z^2 \\ 56Z^2 & 56Z^2 & 56Z^2 \end{bmatrix}^{-1} \begin{bmatrix} E \\ -E \\ 0 \end{bmatrix}, \quad \Delta = (128\times9 - 96\times5 - 56\times4)Z^3 = 448\ Z^3$$

$$\begin{bmatrix} I_{m1} \\ I_{m2} \\ I_{m3} \end{bmatrix} = \frac{1}{Z} \begin{bmatrix} 0.2857 & 0.2143 & 0.125 \\ 0.2143 & 0.2857 & 0.125 \\ 0.125 & 0.125 & 0.2857 \end{bmatrix} \begin{bmatrix} E \\ -E \\ 0 \end{bmatrix}, \quad I_{m1} = 0.0714E/Z,\ I_{m2} = -0.0714E/Z,\ I_{m3} = 0$$

i.e. $I_1 = 0.0714E/Z$, $I_2 = 0.0714E/Z$, $I_3 = 0.0714E/Z$, $I_4 = 0.0714E/Z$, $I_5 = 0$, $I_6 = 0.1428E/Z$ **[3 marks]**

5. (a) 3-phase, 400 V, 50 Hz star-connected load, (80 + j60) Ω each arm.

current drawn = $(400/\sqrt{3})/(80 + j60) = 2.309\angle{-36.87^0}$ A

active power consumed = $\sqrt{3}\times400\times2.309\times\cos(36.87^0) = 1.28$ kW (or $3\times2.309^2\times80$)

power factor of the load = $\cos(36.87^0) = 0.8$ lag **[2 marks]**

(b) 3-phase motor, 1 kW, p.f. = 0.72 lag added.

line current supplied from motor = $1000/(\sqrt{3}\times400\times0.72) = 2.005\angle{-43.95^0}$ A

line current supplied from the source = $2.309\angle{-36.87^0} + 2.005\angle{-43.95^0}$

$= 1.847 - j\,1.385 + 1.443 - j\,1.392 = 3.290 - j\,2.777 = 4.305\angle{-40.2^0}$ A

supply power factor = $\cos(40.2^0) = 0.764$ lag **[1 mark]**

(c) Reactive power = $\sqrt{3}\times400\times4.305\times\sin(40.2^0) = 1925$ var, Active Power = 1+1.28 = 2.28 kW

improve load power factor to 0.95 lag, power factor angle = 18.19^0

New reactive power = $2280 \tan 18.19^0 = 749.4$

Reactive power required to be supplied = 1925 – 749 = 1176 var

Rating of each capacitor = 1176/3 = 392 var

For delta connected capacitors, $C = 392/(400^2\times100\pi) = 7.80\ \mu F$ each. **[2 marks]**

(d) $I_A = 10.0\angle{-30^o}$A, $I_B = 4.0\angle{-120^o}$A and $I_C = 6.0\angle 115^o$A.

The symmetrical components of the currents in phase A are given by

$$\begin{bmatrix} I_{A0} \\ I_{A1} \\ I_{A2} \end{bmatrix} = \tfrac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}\begin{bmatrix} 10.0\angle -30^0 \\ 4.0\angle -120^0 \\ 6.0\angle 115^0 \end{bmatrix}$$

$I_{A0} = (10.0\angle{-30^o} + 4.0\angle{-120^o} + 6.0\angle 115^o)/3 = (2.887 - j1.667 - 0.667 - j1.155 - 0.845 + j1.813)$

$= 1.375 - j1.009 = 1.705\angle{-36.3^o}$ A

$I_{A1} = (10.0\angle{-30^o} + 4.0\angle 0^o + 6.0\angle 355^o)/3 = (2.887 - j1.667 + 1.333 + 1.992 - j0.174)$

$= 6.212 - j1.841 = 6.480\angle{-16.5^o}$ A

$I_{A2} = (10.0\angle{-30^o} + 4.0\angle 120^o + 6.0\angle 235^o)/3 = (2.887 - j1.667 - 0.667 + j1.155 - 1.147 - j1.638)$

$= 1.073 - j2.150 = 2.403\angle{-63.5^o}$ A ***[3 marks]***

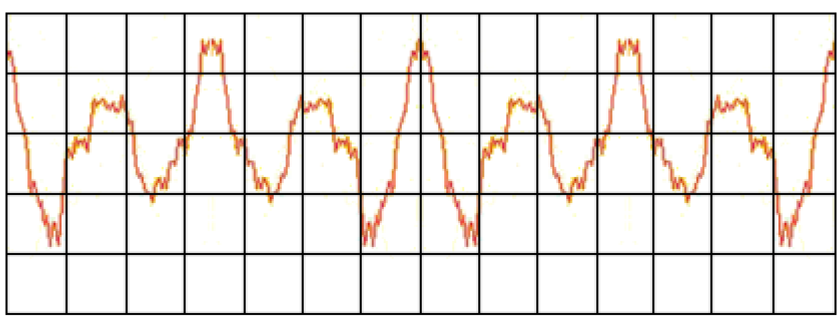
(e) Only positive sequence voltage of 230.9 V is present, and hence only positive sequence power will be present.

Power associated with zero sequence component = 0

Power associated with positive sequence component = $\sqrt{3}\times 400\times 6.480\times\cos(16.5) = 4305$ W

Power associated with negative sequence component = 0 ***[2 marks]***

6.



(a) 1 div = 2.5 ms

i. Period of the waveform → 7 div → 7×2.5 = 17.5 ms ***[1 mark]***

ii. Has even symmetry. This gives only cosine terms in the Fourier series or $B_n = 0$ ***[1 mark]***

iii. The most dominant harmonic frequency repeats itself 4 time within 1 cycle.

i.e. frequency = $4/17.5\times 10^{-3} = 228.6$ Hz ***[2 marks]***

(b) $v(t) = 100 \sin 250\, t$ volt, $i(t) = 20 + 10 \sin(250t + \pi/3) - 4 \sin(750t - \pi/6)$ ampere

i. rms value of i(t) = $\sqrt{20^2 + \frac{10^2}{2} + \frac{4^2}{2}} = 21.40$ A ***[2 marks]***

ii. Average power supplied = $\frac{100}{\sqrt{2}} \times \frac{10}{\sqrt{2}} \times \cos\frac{\pi}{3} = 250$ W ***[2 marks]***

(c) *i(t) = 20 + 10 sin (250t + π/3) – 4 sin (750t – π/6)* A, series L = 10 mH, r = 1 Ω

voltage drop = L di/dt + r i = 0.01 di/dt + 1×i

= 0.01×[10×250×cos(250t +π/3) – 4×750×cos(750t – π/6)] + 20 + 10sin(250t +π/3) – 4sin(750t – π/6)

= 20 + 10sin(250t +π/3) + 25cos(250t +π/3) – 4sin(750t – π/6) – 15×cos(750t – π/6)

Since the sine and cosine terms have the same phase angles, they can be easily combined to give

rms voltage drop = $\sqrt{20^2 + \frac{10^2 + 25^2}{2} + \frac{4^2 + 30^2}{2}}$ = 34.94 V **[2 marks]**

7. (a) Proof of the Laplace transform of

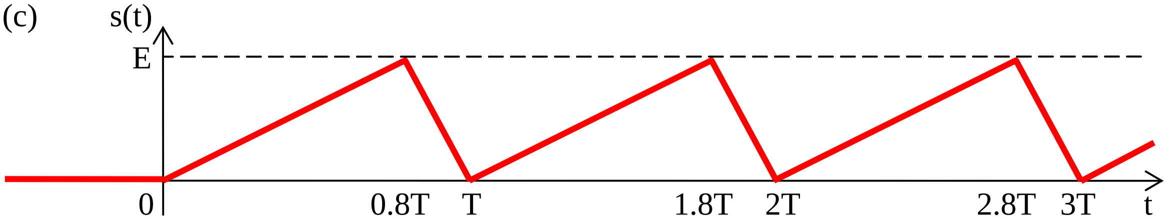
(i) unit step h(t) → $\frac{1}{s}$ **[1 mark]**

(ii) unit ramp r(t) → $\frac{1}{s^2}$ **[1 mark]**

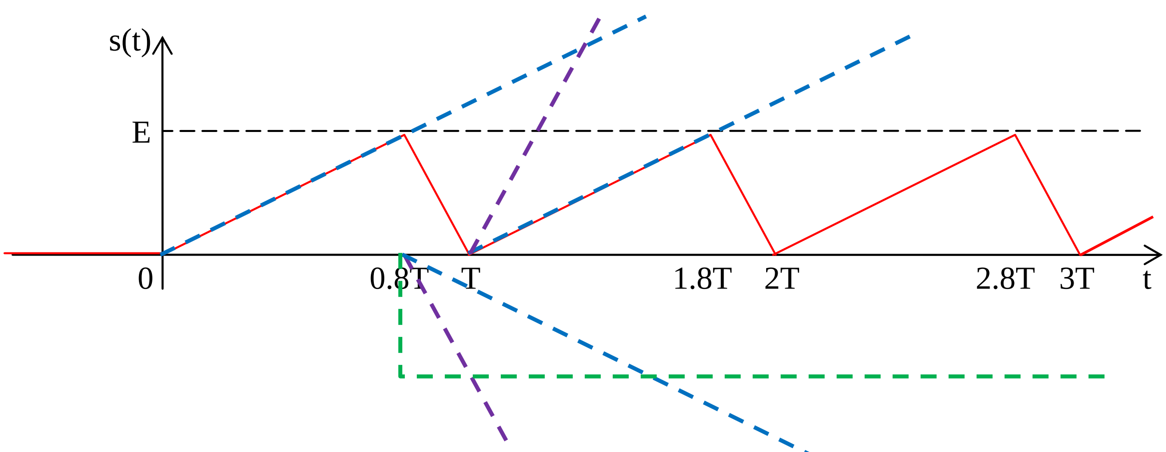
(b) $f_1(t)$ defined for 0 < t ≤ T, Laplace transform $F_1(s)$.
Laplace transform F(s) of repetitive waveform f(t) with period T, where f(t) = $f_1(t)$ for 0 < t ≤ T

Proof of $F(s) = \frac{1}{1-e^{-sT}} F_1(s)$ **[2 marks]**

(c)



Laplace transform of the causal repetitive sawtooth waveform s(t) can be obtained by the superposition of ramp and step waveforms as follows.

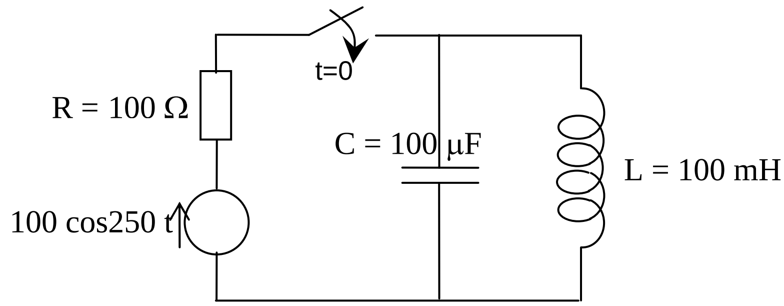


$$s(t) = \frac{1.25E}{T} \cdot r(t) - E \cdot h(t-0.8T) - \left(\frac{1.25E}{T} + \frac{5E}{T}\right) \cdot r(t-0.8T) \cdot h(t-0.8T) \quad \text{for } 0 < t < T$$

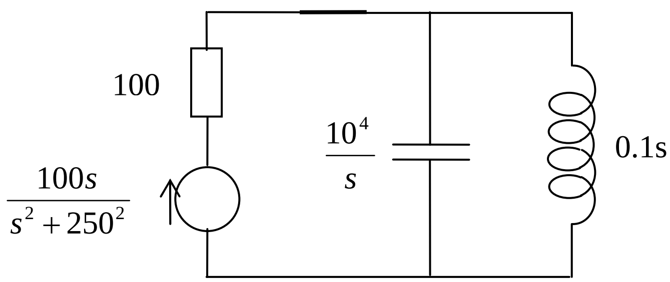
$$S(s) = \frac{1}{1-e^{-sT}} \cdot \left[\frac{1.25E}{T} \cdot \frac{1}{s^2} - E \cdot \frac{1}{s+0.8T} - \frac{6.25E}{T} \cdot \frac{1}{(s+0.8T)^2}\right]$$

[4 mark]

(d)



The Laplace transformed equivalent circuitis shown



[2 marks]