University of Moratuwa, Sri Lanka
Faculty of Engineering
Department of Electrical Engineering
B. Sc. Engineering Honours Degree Course
Level 2 – Semester … Examination

EE2010 – THEORY OF ELECTRICITY

Time Allowed: 3 Hours ……… 2009

Additional Material

Graph Paper will be provided if required.
A table of Laplace transforms is provided on the other side of this page.

Instructions to Candidates

This paper contains 7 questions in 6 pages, including the cover page.
Answer All Questions.
This examination accounts for 70% of the module assessment.
Each question carries a total 10 marks. Maximum marks allocated for each part of a question is indicated in square brackets at the end of the part.
Total allocation for the paper is 70 marks.
This is a closed book examination and only authorised calculators will be permitted.

Technical Data:

Permeability of free space μ_0 $= 4\pi \times 10^{-7}$ H/m
Permittivity of free space ε_0 $= 8.854 \times 10^{-12}$ F/m
Velocity of light in free space $= 2.998 \times 10^{8}$ m/s

Table of Laplace Transforms of common causal functions *f(t)*

$f(t)$	$F(s) = \mathrm{L}[f(t)]$
Unit impulse – δt	1
Unit step – $U(t)$	$\frac{1}{s}$
t	$\frac{1}{s^2}$
t^n	$\frac{n!}{s^{n+1}}$
e^{-at}	$\frac{1}{(s+a)}$
$1-e^{-at}$	$\frac{a}{s(s+a)}$
$t\,e^{-at}$	$\frac{1}{(s+a)^2}$
$t^n\,e^{-at}$	$\frac{n!}{(s+a)^{n+1}}$
$e^{-at}-e^{-bt}$	$\frac{b-a}{(s+a)(s+b)}$
$\sin(\omega t)$	$\frac{\omega}{(s^2+\omega^2)}$
$\sin(\omega t+\phi)$	$\frac{\omega\cos(\phi)+s\sin(\phi)}{(s^2+\omega^2)}$
$t\sin(\omega t)$	$\frac{2\omega s}{(s^2+\omega^2)^2}$
$\cos(\omega t)$	$\frac{s}{(s^2+\omega^2)}$
$\cos(\omega t+\phi)$	$\frac{s\cos(\phi)-\omega\sin(\phi)}{(s^2+\omega^2)}$
$t\cos(\omega t)$	$\frac{s^2-\omega^2}{(s^2+\omega^2)^2}$
$e^{-at}\sin(\omega t)$	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{-at}\cos(\omega t)$	$\frac{s+a}{(s+a)^2+\omega^2}$
$\sinh(\omega t)$	$\frac{\omega}{(s^2-\omega^2)}$
$\cosh(\omega t)$	$\frac{s}{(s^2-\omega^2)}$

Question 1

(a) Figure Q1a shows a practical operational amplifier with a resistor R and a capacitor C connected as shown.

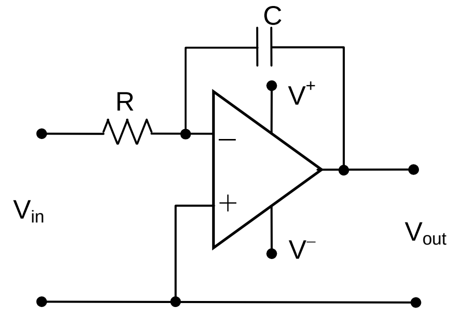

Figure Q1a

i. Draw the complete equivalent circuit showing also the input resistance R_{in}, output resistance R_{out}, and open loop gain A. *[1 mark]*

ii. Obtain expressions relating the output voltage V_{out} with the input voltage V_{in}. *[2 marks]*

iii. If the operational amplifier is ideal, derive a simplified relationship between V_{out} and V_{in}. *[2 marks]*

(b) A circuit consists of a series combination of a 100 mH inductance with a winding resistance of 10Ω, and a 100 μF capacitor with an effective parallel resistance of 1000 Ω. Calculate the current supplied and the voltage across the capacitor when supplied at 200V an angular frequency 250 rad/s. Sketch the phasor diagram showing these voltages and the current. *[3 marks]*

(c) If the supply frequency is varied from 250 rad/s, at determine the frequency at which resonance would occur, and the circuit current at resonance. *[2 marks]*

Question 2

(a) Figure Q2a shows the magnetic circuit of a three phase transformer. If I_A = 20∠0° A and the currents in the three phases are balanced, draw the magnetic equivalent circuit, with values, for the calculation of flux, if N=100 turns in each coil, the effective cross-section of each limb is A = 2 cm^2, the effective magnetic lengths of the outer limb l_o=20 cm and of the middle limb l_m=6 cm. The relative permeability of the magnetic material is μ_r = 2000, and the transformer is operating in the linear region. *[4 marks]*

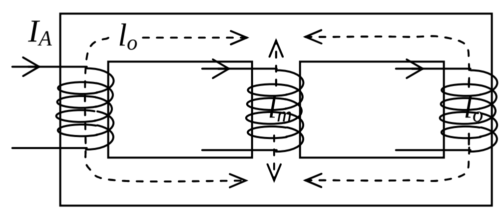

Figure Q2a

(b) Figure Q2b shows an a.c. circuit with ideal components.

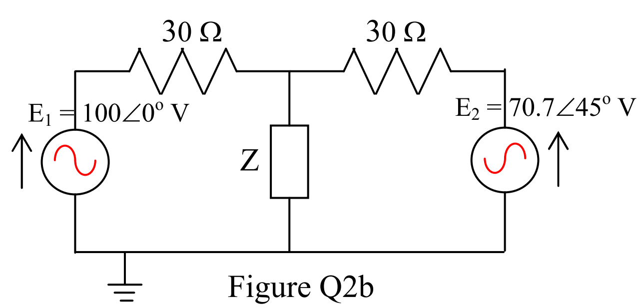

Figure Q2b

i. If Z is inductance with a reactance of 40 Ω, using Thevenin's theorem, determine the current in the inductor. *[3 marks]*

ii. What would be the corresponding current if Z is a 40 Ω resistor. *[1 mark]*

iii. What is the value of Z that will extract the maximum active power, and what is the value of this maximum power ? *[2 marks]*

Question 3

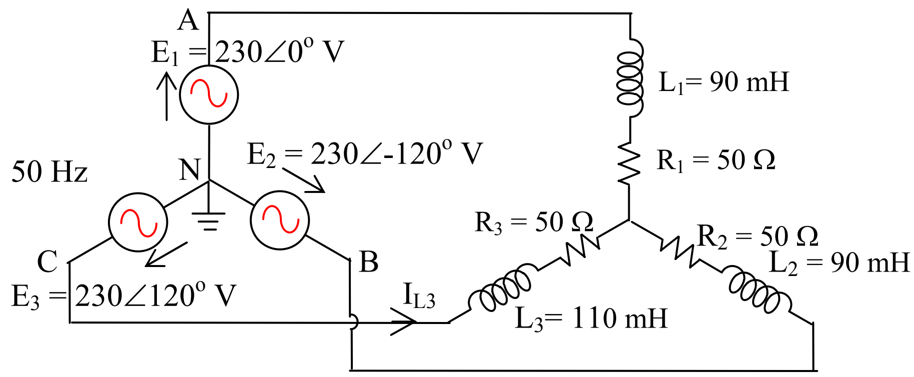

Figure Q3a

(a) For the 3-phase network shown in figure Q3a, the current I_{L3} is measured to be $3.884\angle 87.02^0$ A. Using compensation theorem, determine the new value of the current, if R_3 is decreased to 48 Ω. *[2 marks]*

(b) For the circuit shown in figure Q3b, determine from first principles, a non-coupled equivalent circuit. *[2 marks]*

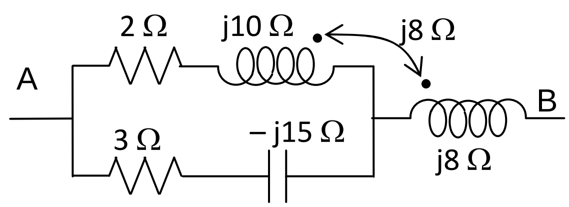

Figure Q3b

(c) For the network shown in figure Q3cd, determine the two-port admittance matrix *[3 marks]*

(d) For the network in Fig Q3c determine also the characteristic impedance Z_o, which when connected at port 2 as a load would give the input impedance at port 1 also as Z_o. *[3 marks]*

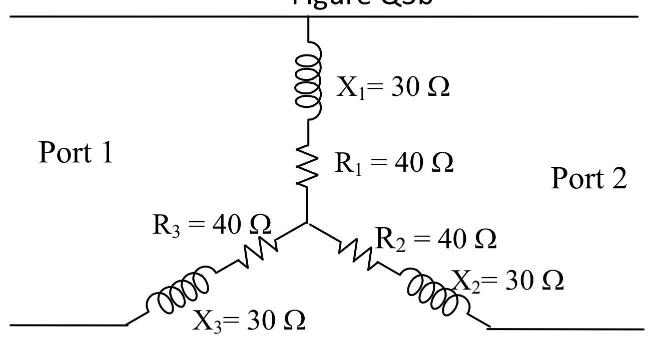

Figure Q3cd

Question 4

For the circuit shown in figure Q4,

(a) Draw the oriented graph of the network. Using a suitable tree, draw the corresponding loops.*[2 mark]*

(b) Write down the
   i. branch impedance matrix *[1 mark]*
   ii. branch source voltage vector *[1 mark]*
   iii. branch-mesh incidence matrix *[1 mark]*

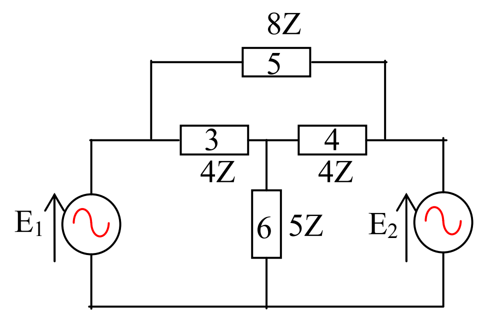

Figure Q4

(c) Hence obtain the mesh-impedance matrix and the mesh voltage vector. *[2 mark]*

(d) If $E_1 = E_1 = E$ obtain the currents in all the branches. *[3 mark]*

Question 5

(a) A balanced 3-phase, 400 V, 50 Hz supply feeds a 3-phase star-connected balanced load consisting of arms of value (80 + j60) Ω each. Determine the current drawn, the active power consumed and the power factor of the load. *[2 marks]*

(b) A 3-phase motor consuming 1 kW at a power factor of 0.72 lag is added to the system. Determine the line current supplied from the source, and the supply power factor. *[1 marks]*

(c) It is now required to improve the load power factor to 0.95 lag by connecting a bank of delta-connected capacitors. Determine the rating of each capacitor. *[2 marks]*

The above balanced supply, with phase sequence ABC and phase A voltage taken as reference, now feeds an unbalanced load of $I_A = 10.0\angle{-30^o}$A, $I_B = 4.0\angle{-120^o}$A and $I_C = 6.0\angle{115^o}$A.

(d) Determine the symmetrical components of the currents in phase A. *[3 marks]*

(e) Determine also the power associated with each sequence component *[2 marks]*

Question 6

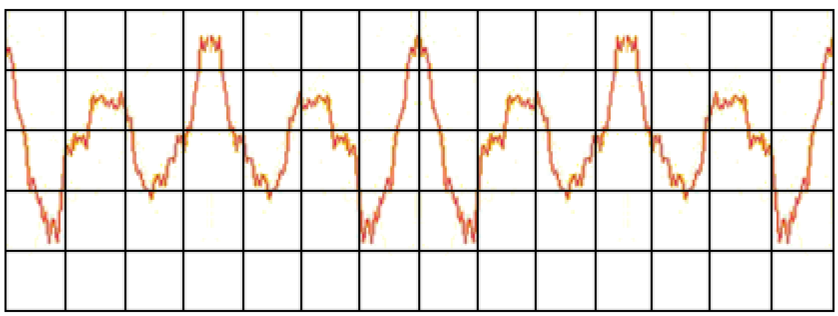

Figure Q6a

(a) Figure Q6a shows a periodic waveform observed on a digital oscilloscope screen, with 1 div on the horizontal scale corresponding to 2.5 ms.

i. Determine the period of the waveform. *[1 mark]*

ii. Identify any symmetrical properties in the waveform, and how you may make use of it in the Fourier analysis of the waveform. *[1 mark]*

iii. Identify the most dominant harmonic frequency in the waveform. *[2 marks]*

(b) A voltage $v(t) = 100 \sin 250\, t$ volt when applied across a certain non linear circuit produces a current $i(t) = 20 + 10 \sin(250t + \pi/3) - 4 \sin(750t - \pi/6)$ ampere.

i. Determine the rms value of the current i(t) *[2 marks]*

ii. Determine the average power supplied from the source. *[2 marks]*

(c) If the current i(t) passes through a series combination of L = 10 mH and a resistance of 1 Ω, determine the rms value of the voltage drop across the combination *[2 marks]*

Question 7

(a) Determine from first principles the Laplace transform of (i) the unit step H(t), where H(t) = 0 for $t<0$ and H(t) = 1 for $t \geq 0$, and (ii) a unit ramp R(t). where R(t) = 0 for $t<0$ and R(t) = t for $t \geq 0$. *[2 marks]*

(b) A function $f_1(t)$ is defined for the period $0 < t \leq T$, and its Laplace transform is $F_1(s)$. Determine from first principles, the Laplace transform F(s) of the repetitive waveform f(t)

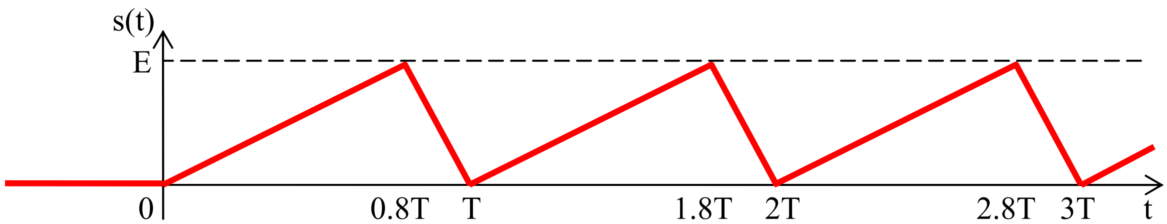

Figure Q7c

with period T, where $f(t) = f_1(t)$ for $0 < t \leq T$ *[2 marks]*

(c) Using the result of Q7(a), determine the Laplace transform of the causal repetitive sawtooth waveform s(t) shown in figure Q7c. *[4 marks]*

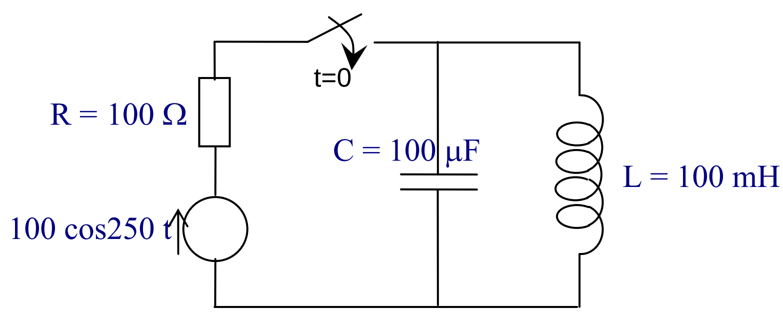

Figure Q7d

(d) Sketch the Laplace transformed equivalent circuit for the circuit shown in figure Q7d, if the switch shown is closed at time t=0. *[2 marks]*

[END OF QUESTION PAPER]