University of Moratuwa, Sri Lanka
Faculty of Engineering
Department of Electrical Engineering
B. Sc. Engineering Honours Degree Course
Level 2 – Semester 1 Examination

EE2010 – THEORY OF ELECTRICITY

Time Allowed: 3 Hours — January 2010

Additional Material

Graph Paper will be provided if required.
A table of Laplace transforms is provided on the other side of this page.

Instructions to Candidates

This paper contains 7 questions in 6 pages, including the cover page.
Answer All Questions.
This examination accounts for 70% of the module assessment.
Each question carries a total 10 marks. Maximum marks allocated for each part of a question is indicated in square brackets at the end of the part.
Total allocation for the paper is 70 marks.
This is a closed book examination and only authorised calculators will be permitted.

Technical Data:

Permeability of free space μ_0 $= 4\pi \times 10^{-7}$ H/m
Permittivity of free space ε_0 $= 8.854 \times 10^{-12}$ F/m
Velocity of light in free space $= 2.998 \times 10^{8}$ m/s

Table of Laplace Transforms of common causal functions *f(t)*

$f(t)$	$F(s) = \mathrm{L}[f(t)]$
Unit impulse – δt	1
Unit step – $U(t)$	$\frac{1}{s}$
t	$\frac{1}{s^2}$
t^n	$\frac{n!}{s^{n+1}}$
e^{-at}	$\frac{1}{(s+a)}$
$1-e^{-at}$	$\frac{a}{s(s+a)}$
$t\,e^{-at}$	$\frac{1}{(s+a)^2}$
$t^n\,e^{-at}$	$\frac{n!}{(s+a)^{n+1}}$
$e^{-at}-e^{-bt}$	$\frac{b-a}{(s+a)(s+b)}$
$\sin(\omega t)$	$\frac{\omega}{(s^2+\omega^2)}$
$\sin(\omega t+\phi)$	$\frac{\omega\cos(\phi)+s\sin(\phi)}{(s^2+\omega^2)}$
$t\sin(\omega t)$	$\frac{2\omega s}{(s^2+\omega^2)^2}$
$\cos(\omega t)$	$\frac{s}{(s^2+\omega^2)}$
$\cos(\omega t+\phi)$	$\frac{s\cos(\phi)-\omega\sin(\phi)}{(s^2+\omega^2)}$
$t\cos(\omega t)$	$\frac{s^2-\omega^2}{(s^2+\omega^2)^2}$
$e^{-at}\sin(\omega t)$	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{-at}\cos(\omega t)$	$\frac{s+a}{(s+a)^2+\omega^2}$
$\sinh(\omega t)$	$\frac{\omega}{(s^2-\omega^2)}$
$\cosh(\omega t)$	$\frac{s}{(s^2-\omega^2)}$

Question 1

(a) Figure Q1a shows a practical operational amplifier with resistors R_{1A} , R_{1B} and R_2 connected as shown.

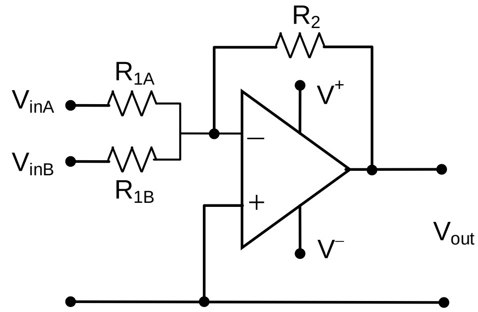

Figure Q1a

i. Draw the complete equivalent circuit showing also the input resistance R_{in}, output resistance R_{out}, and open loop gain A. *[1 mark]*

ii. Obtain expressions relating the output voltage V_{out} with the input voltages V_{in}. *[1 mark]*

iii. If the operational amplifier is ideal, derive a simplified relationship between V_{out} and V_{in}. *[1 mark]*

(b) A circuit made up of a choke (with resistance R, an inductance L), and a Capacitor C connected in series, is supplied from a 50 Hz a.c. source. Measurements with a.c. meters indicate a current of 2.0 A in the circuit and a voltage of 390 V across the choke. If the Q-factor of the choke is 2.4 and the peak energy stored in the capacitor is 1.02 J, calculate the values of R, L and C and the supply voltage. *[4 marks]*

Sketch a phasor diagram showing all the voltages and currents in the circuit. *[1 mark]*

If the frequency was allowed to vary, keeping the supply voltage constant, at what frequency would resonance occur in the circuit and what would be the current at resonance. *[1 mark]*

Determine also the current at the half-power points and an expression for determining the frequencies at which they occur? *[1 mark]*

Question 2

(a) A certain electrical equipment designed for use at 50 Hz needs the transformer to be changed for operation at 60 Hz. If the dimensions of the core is to remain unchanged, the total core losses need to be contained. If the laminations of the original core is of thickness 0.5 mm at 50 Hz, and the hysteresis and eddy current losses at 50 Hz are equal, determine the required thickness of the laminations to keep the total core loss unchanged. *[2 marks]*

(b) Figure Q2b shows an a.c. circuit with ideal components. Using Millmann's Theorem, determine the potential of S, if Z is inductance with a reactance of 40 Ω. *[3 marks]*

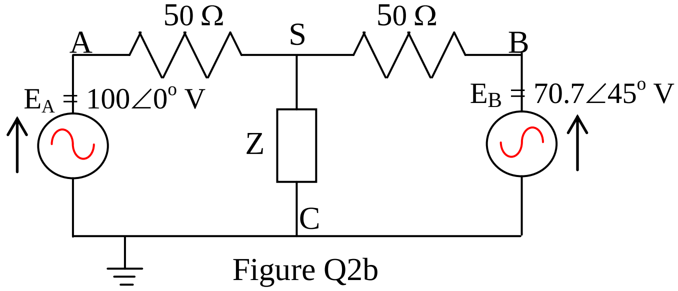

Figure Q2b

(c) An earth electrode is made from a galvanised iron pipe of outer diameter 50 mm, driven to a depth of 2m in uniform soil of resistivity 100 Ωm. Determine from first principles the effective resistance of electrode to ground. *[3 marks]*

(d) Determine the Norton's equivalent circuit for the circuit shown in figure Q2d. *[2 marks]*

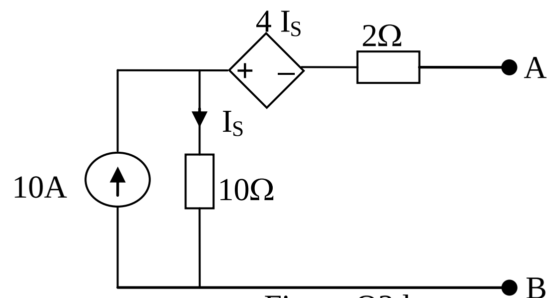

Figure Q2d

Question 3

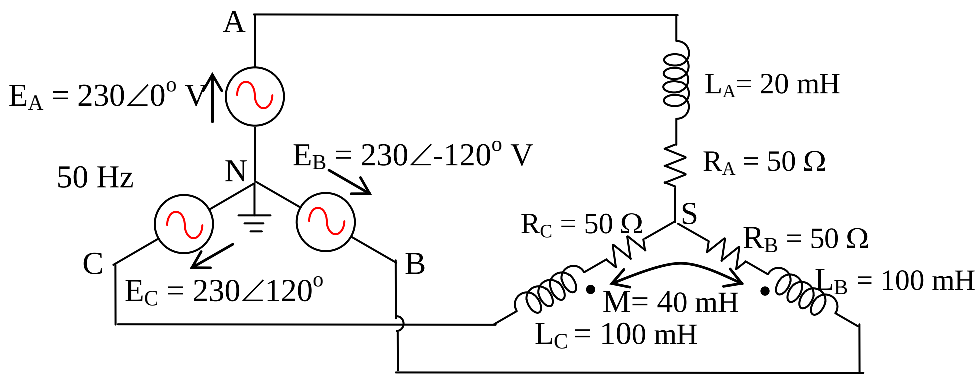

Figure Q3a

(a) For the 3-phase network shown in figure Q3a, convert the circuit to a non-coupled equivalent circuit. *[2 marks]*

(b) Determine the currents in all three phases. *[2 marks]*

(c) Determine the potential of the star-point S. *[1 mark]*

(d) For the network shown in figure Q3b, determine the ABCD parameter matrix. *[3 marks]*

(e) If a resistive load of 100 Ω is connected across Port 2, determine the impedance seen from the terminals of Port 1. *[2 marks]*

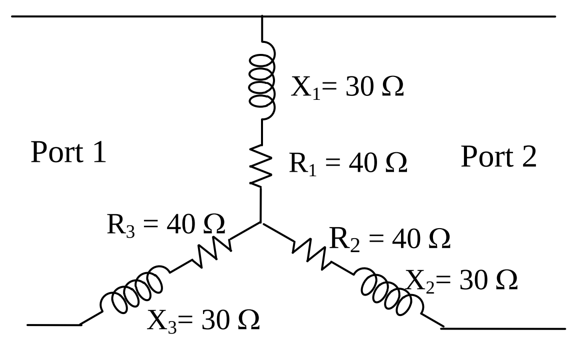

Figure Q3b

Question 4

For the circuit shown in figure Q4, $Z_3 = Z_4 = Z_5 = Z_6 = Z$

(a) Convert the voltage sources E_1 and E_2 to equivalent current sources (show steps). *[2 marks]*

(b) By selecting suitable nodes, write down the

i. branch admittance matrix *[1 mark]*

ii. branch source current vector *[1 mark]*

iii. branch-node incidence matrix *[1 mark]*

(c) Hence obtain the nodal-admittance matrix and the nodal current vector. *[3 marks]*

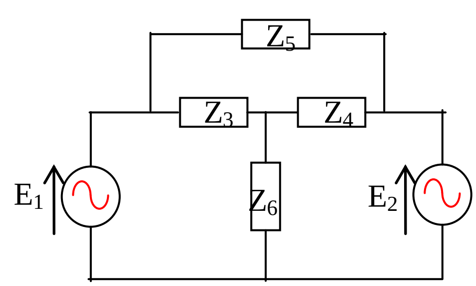

Figure Q4

(d) If $E_1 = E_2 = E$ obtain the currents in all the branches. *[2 marks]*

Question 5

A three-phase motor delivers a mechanical output of 10.5 kW, with an efficiency of 84% and a power factor of 0.70. It is supplied at its terminals at 400 V from wires which have resistances of 0.2 ohm each.

(a) Calculate the line current, the supply line voltage, the kVA input of motor, and the reactive power taken by the motor. *[4 marks]*

(b) Determine the approximate rating of capacitances, connected in delta, necessary to improve the power factor to 0.95. *[2 marks]*

State any assumptions made in your calculations.

(c) If the symmetrical components of the currents in a certain circuit are $I_{A0} = 1.0\angle{-30^o}$A, $I_{A1} = 2.0\angle 0^o$A and $I_{A2} = 1.0\angle 30^o$A, determine the phase currents. *[2 marks]*

(d) Show that for a perfectly symmetrical 3phase impedance system, with self impedances Z_s and mutual impedances Z_m, the equivalent symmetrical component matrix is diagonal.

[2 marks]

Question 6

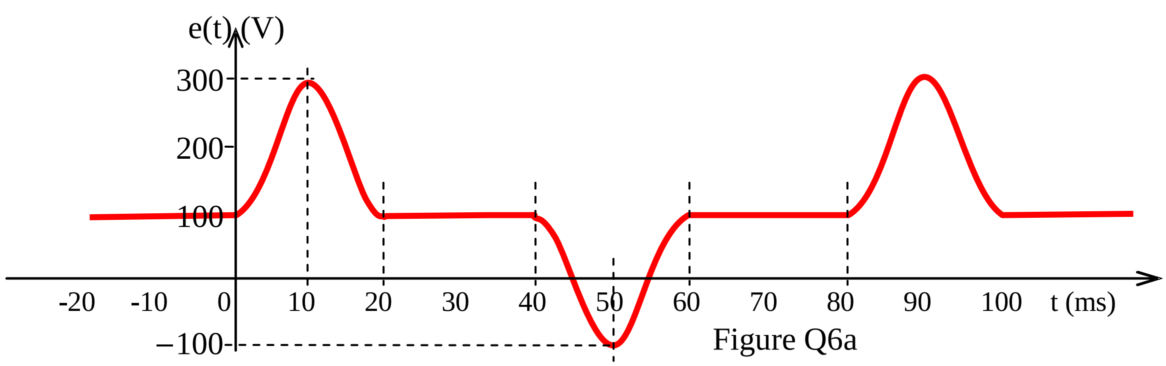

Figure Q6a

$$e(t) = 200 - 100\cos 100\pi t \qquad 0 \le t < 20 \text{ ms}$$
$$= 100 \qquad 20 \le t < 40 \text{ ms}$$
$$= 100\cos 100\pi t \qquad 40 \le t < 60 \text{ ms}$$
$$= 100 \qquad 60 \le t < 80 \text{ ms}$$

(a) Determine the first 3 significant terms of the Fourier Series of the repetitive waveform e(t) shown in figure Q6a. *[5 marks]*

(b) Determine the mean value, and approximate rms value the periodic waveform e(t) shown in figure Q6a. *[2 marks]*

(c) If the waveform e(t) is applied across the series R C circuit shown in figure Q6b, determine the Fourier Series of the resulting current i(t) in the circuit. *[3 marks]*

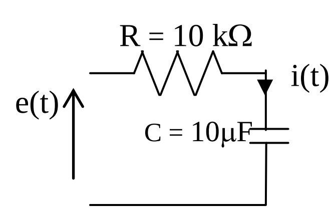

Figure Q6b

Question 7

(a) For the circuit shown in figure Q7bc, if the switch S is initially closed and steady state is reached, determine expressions for the time-varying steady-state voltage $v_c(t)$ across the capacitor and current $i_L(t)$ through the inductor. *[3 marks]*

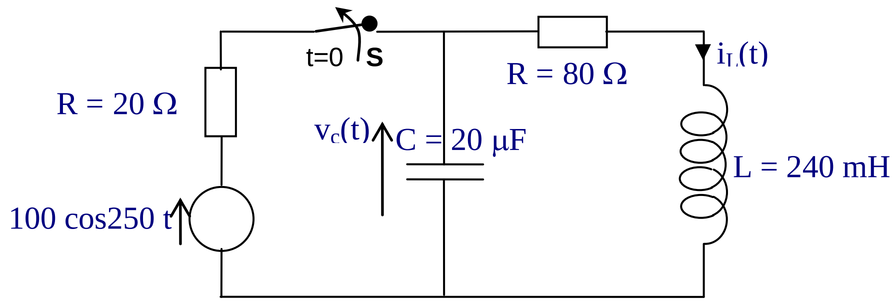

Figure Q7bc

(b) Considering the expressions derived in 7(a), sketch the Laplace transformed equivalent circuit, if the switch shown is opened at time t=0. *[2 marks]*

(c) Determine also the subsequent variation of the current $i_L(t)$ through the inductor. *[3 marks]*

[END OF QUESTION PAPER]