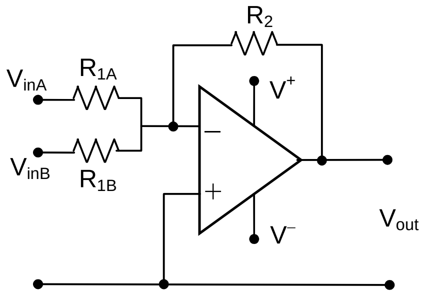
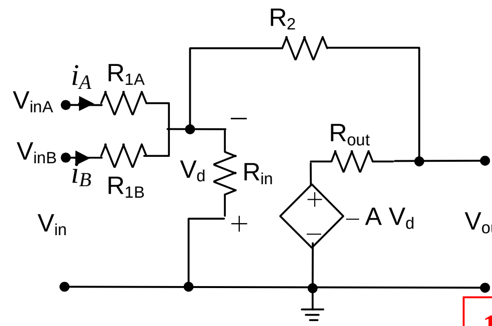
EE 2010 - THEORY OF ELECTRICITY – Short Answers

Level 2 Semester 1 Examination - January 2010

1. (a)





1 mark

$$i = i_A + i_B = \frac{V_{inA} - V_d}{R_{1A}} + \frac{V_{inB} - V_d}{R_{1B}} = \frac{V_d}{R_{in}} + \frac{V_d - V_{out}}{R_2}$$

$$\frac{V_d - V_{out}}{R_2} = \frac{V_{out} + AV_d}{R_{out}}$$

1 mark

for an ideal Op Amp, $R_{out} = 0$, and $R_{in} = \infty$, $A \to \infty$,

$V_{out} = -A\, V_d$

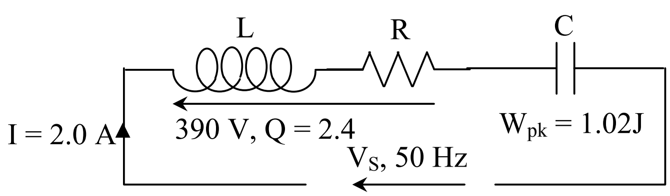
$V_d = -V_{out}/A \to 0$

$$\frac{V_{inA} - 0}{R_{1A}} + \frac{V_{inB} - 0}{R_{1B}} = 0 + \frac{0 - V_{out}}{R_2}$$

giving $V_{out} = -R_2\left(\frac{V_{inA}}{R_{1A}} + \frac{V_{inB}}{R_{1B}}\right)$

1 mark

(b)



For the choke,

$| R + j\omega L | = 390/2 = 195\ \Omega$, $\omega L/R = 2.4 \to \sqrt{(R^2 + 2.4^2 R^2)} = 195 \to R = 75\ \Omega$ **1 mark**

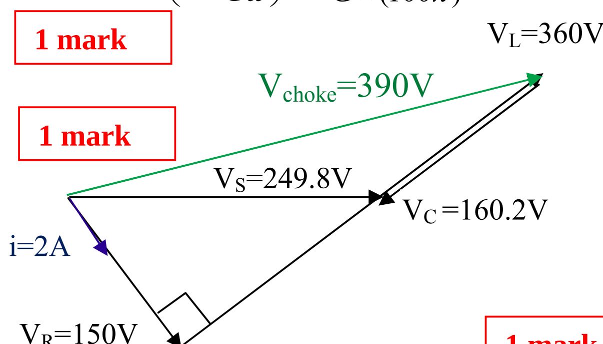
$\omega L = 2.4\, R = 180\ \Omega \to L = 180/100\pi = 0.573 = 573$ mH **1 mark**

Peak energy stored in capacitor $= \frac{1}{2} C V_{Cm}^2 = C V_{Crms}^2 = C \times \left(I \times \frac{1}{C\omega} \right)^2 = \frac{2^2}{C \times (100\pi)^2} = 1.02$

Giving C = 39.73 μF and $1/C\omega = 80.1\ \Omega$. **1 mark**

Supply voltage $V_s = 2.0 \times (75 + j\,180 - j80.1)$

$= 2.0 \times (75 + j\,99.9) = 249.84\angle 53.1^0$ V **1 mark**



1 mark

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Resonance frequency = $\omega_0 = 1/\sqrt{LC} = 1/\sqrt{(0.573\times39.73\times10^{-6})} = 209.59$ rad/s → 33.36 Hz

Current at resonance = 249.84/75 = 3.331 A

1 mark

At half-power points, current $I_{½} = 3.331/\sqrt{2} = 2.356$ A

And the half-power frequencies are obtained from

$$2.356 = \left| \frac{249.84}{75 + j\left(\omega\times0.573 - \frac{1}{\omega\times39.73\times10^{-6}}\right)} \right| \quad \rightarrow \quad \omega = 153.8,\ 284.0 \text{ rad/s}, \quad f = 24.5,\ 45.2 \text{ Hz}$$

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2. (a) For 50 Hz transformer, lamination thickness t = 0.5 mm

Hysteresis loss $\propto B_m^{1.6} f$

Eddy current loss $\propto B_m^2 f^2 t^2$

If the alternating flux density peak value = B_m and the total core loss P are to be the same for a given size of transformer,

Hysteresis loss $P_h = K_h f$

Eddy current loss $P_e = K_e f^2 t^2$

at 50 Hz, $P_h = P_e \rightarrow 50 K_h = 50^2 \times 0.5^2 K_e \rightarrow K_h = 12.5 K_e$

total core loss = $P = P_h + P_e = K_h f + K_e f^2 t^2$ at both frequencies 50 Hz and 60 Hz.

i.e. $P = 50 K_h + 50^2 \times 0.5^2 K_e = 60 K_h + 60^2 \times t^2 K_e$

or $50 \times 12.5 K_e + 50^2 \times 0.5^2 K_e = 60 \times 12.5 K_e + 60^2 \times t^2 K_e$

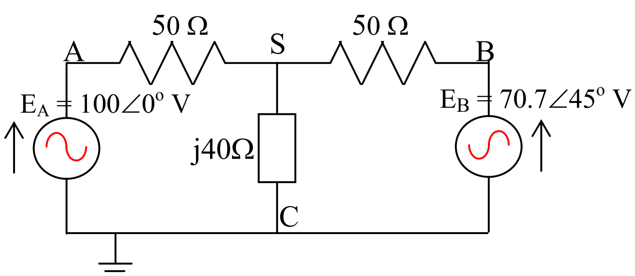
i.e. $(625 + 625) K_e = (750 + 3600 t^2) K_e$

$t^2 = 500/3600 \rightarrow t = 0.373$ mm thick laminations are required at 60 Hz.

2 marks

(b) voltages at A, B and C are $100\angle0^\circ$ V, $70.7\angle45^\circ$ V and 0 V respectively.

Thus using Millmann's theorem,



$$V_s = \frac{\sum Y \times V}{\sum Y}$$

$$V_s = \frac{\frac{1}{50}\times100\angle0^\circ + \frac{1}{50}\times70.7\angle45^\circ + 0}{\frac{1}{50}+\frac{1}{50}+\frac{1}{j40}}$$

i.e. $$V_s = \frac{2+(1+j1)+0}{0.02+0.02-j0.025} = \frac{3+j1}{0.04-j0.025} = \frac{3.162\angle18.43^0}{0.047\angle-32.01^0} = 67.04\angle50.44^0$$

3 marks

(c) diameter = 50 mm, depth = 2m, soil resistivity 100 Ωm.

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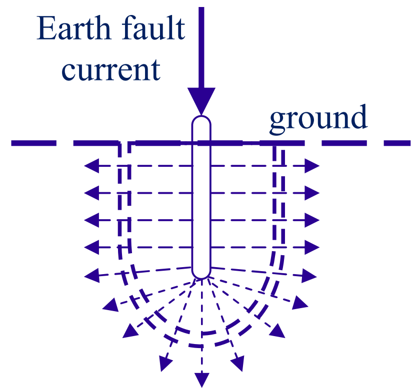
Consider an elemental cylinder of thickness dx .

The resistance dR of this can be calculated as follows.



$$\text{using } R=\frac{\rho\, l}{A}, \quad dR=\frac{\rho.dx}{2\pi\, x^2+2\pi\, x.l}$$

$$=\frac{\rho.dx}{2\pi\, x(x+l)}=\frac{\rho}{2\pi\, l}\left[\frac{1}{x}-\frac{1}{x+l}\right].dx$$

$$\int_0^R dR=\int_r^x \frac{\rho}{2\pi\, l}\left[\frac{1}{x}-\frac{1}{x+l}\right]dx \quad \Rightarrow$$

$$R=\frac{\rho}{2\pi\, l}\left[\ln\left(\frac{x}{r}\right)-\ln\left(\frac{x+l}{r+l}\right)\right]$$

as x tends to infinity, that is as a large mass of the earth is taken into consideration, the limiting value of the electrode resistance would be

$$R=\frac{\rho}{2\pi l}\ln\left[\frac{r+l}{r}\right]=\frac{100}{2\pi 2}\ln\left[\frac{0.025+2}{0.025}\right]=34.97\ \Omega$$

3 marks

(d) Norton's equivalent impedance = 10+2 = 12 Ω

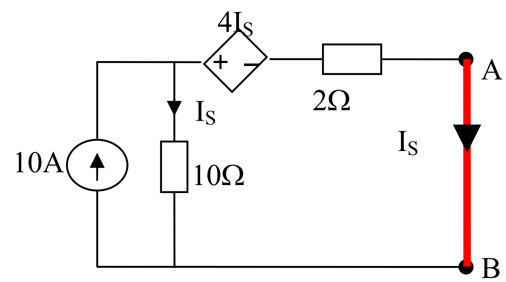
Norton's equivalent current source is determined as follows

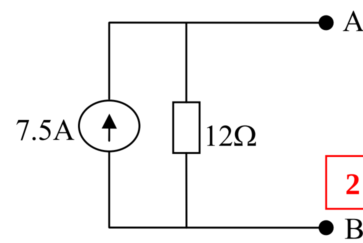
$10 = I_S + I_{SC}$

$10\, I_S = 4I_S + 2I_{SC} \quad \rightarrow I_{SC} = 3I_S$

$I_S = 2.5$ A, $I_{SC} = 3\times 2.5$ A $= 7.5$ A

The Norton's equivalent circuit is





2 marks

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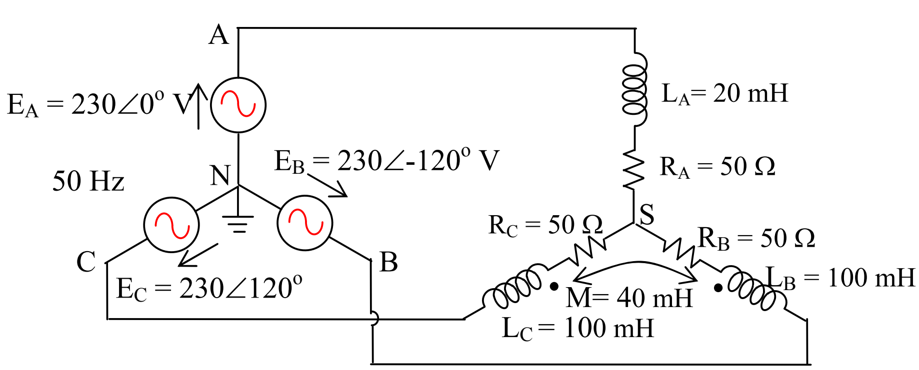
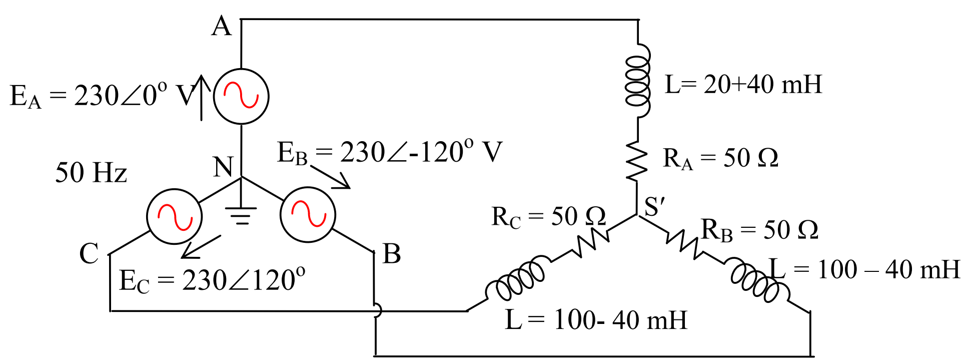
3. (a)



The uncoupled equivalent circuit may be drawn as follows



2 marks

(b) Note that S is no longer the star-point of the modified network. It is seen that the modified network is perfectly balanced, and hence the current in each phase is given by

$I_A = 230\angle 0^o/(50+j0.06\times 100\times\pi) = 230/(50+j18.85) = 4.30\angle{-20.7^o}$ A

$I_B = 4.30\angle{-140.7^o}$ A, $I_B = 4.30\angle 99.3^o$ A

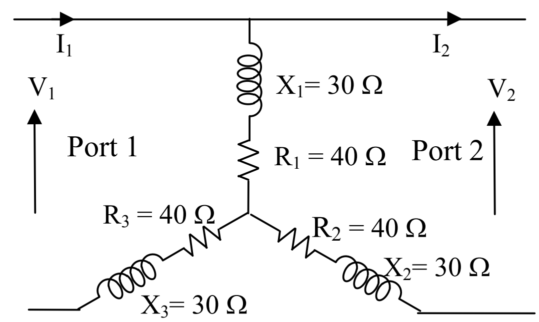
2 marks

(c) The potential of the new star point S′ is 0. The original star point S is located at a point 40 mH above S′ .

Potential of original star point S = $0 - j0.04\times 100\times\pi\times 4.30\angle{-20.7^o} = 54.0\ \angle{-110.7^o}$ V **2 marks**

(d)

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 40+j30 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \dfrac{1}{40+j30} & 1 \end{bmatrix} \begin{bmatrix} 1 & 40+j30 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 40+j30 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 40+j30 \\ \dfrac{1}{40+j30} & 2 \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 2 & 120+j90 \\ \dfrac{1}{40+j30} & 2 \end{bmatrix}$$

2 marks

> Alternate Method
>
> With port 2 on open circuit, $A = \left.\dfrac{V_1}{V_2}\right|_{I_2=0} = \dfrac{80+j60}{40+j30} = 2$,
>
> $C = \left.\dfrac{I_1}{V_2}\right|_{I_2=0} = \dfrac{1}{40+j30} = 0.02\angle\text{-}36.87^0$ S
>
> With port 2 on short circuit, $D = \left.\dfrac{I_1}{I_2}\right|_{V_2=0} = \dfrac{80+j60}{40+j30} = 2$,
>
> $B = \left.\dfrac{V_1}{I_1}\times\dfrac{I_1}{I_2}\right|_{V_2=0} = (Z_1 // Z_2 + Z_3)\times 2 = (20+j15+40+j30)\times 2 = 120+j90 = 150\angle 36.87^0\Omega$
>
> giving the same ABCD matrix.

(e) When a resistive load of 100 Ω is connected across port 2, $V_2 = 100\ I_2$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 2 & 120 + j90 \\ \dfrac{1}{40 + j30} & 2 \end{bmatrix} \begin{bmatrix} 100 I_2 \\ I_2 \end{bmatrix}$$

$$\frac{V_1}{I_1} = \frac{(200 + 120 + j90) I_2}{\left(\dfrac{100}{40 + j30} + 2\right) I_2} = \frac{(320 + j90)}{(1.6 - j1.2 + 2)}$$

Thus Impedance seen from port 1 =. $\dfrac{V_1}{I_1} = \dfrac{332.4\angle 15.71^0}{3.795\angle -18.43^0} = 87.59\angle 34.14^0\ \Omega$ **2 marks**

Alternate Method

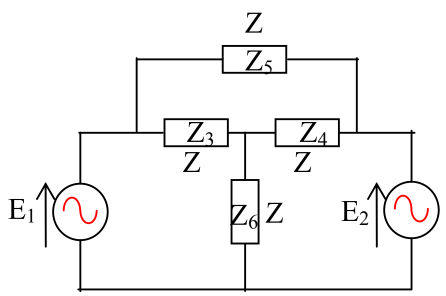
$V_1/I_1 = (40+j30) + (40+j30)//(140+j30)$

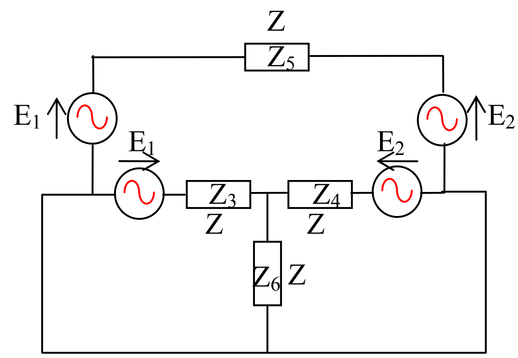
$$= (40 + j30) + \frac{(40 + j30)(140 + j30)}{180 + j60} = (40 + j30) + \frac{50\angle 36.87^0 \times 143.18\angle 12.09^0}{189.74\angle 18.43^0}$$

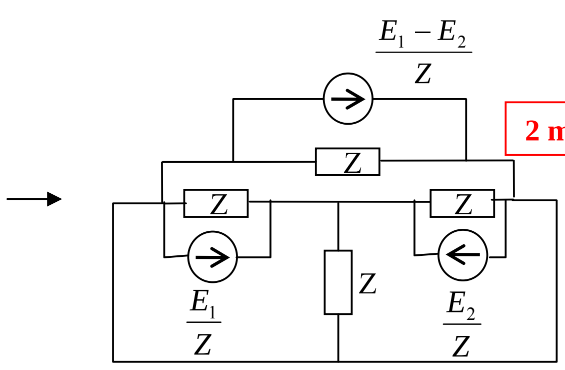
$$= (40 + j30) + \frac{7159\angle 48.96^0}{189.74\angle 18.43^0} = (40 + j30) + 37.73\angle 30.53^0 = 40 + j30 + 32.5 + j19.17$$

$= 72.5 + j49.17 = 87.6\angle 34.15^o\ \Omega = 72.5 + j\ 49.2\ \Omega$

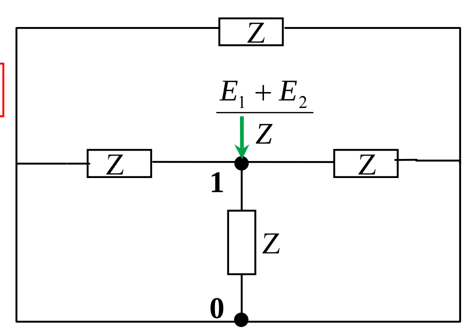
4. (a)







2 marks



Taking node 0 as reference and eliminating it from equations

$$[Y_b] = \begin{bmatrix} 1/Z & 0 & 0 & 0 \\ 0 & 1/Z & 0 & 0 \\ 0 & 0 & 1/Z & 0 \\ 0 & 0 & 0 & 1/Z \end{bmatrix}, \ \tilde{I}_{Sb} = \begin{bmatrix} E_1 / Z \\ E_2 / Z \\ (E_1 - E_2)/Z \\ 0 \end{bmatrix}$$

1 mark

1 mark

The branch-node incidence matrix is given by

$$\begin{bmatrix} -1 \\ -1 \\ 0 \\ 1 \end{bmatrix}$$

1 mark

(c) The nodal admittance matrix is given by

$$\begin{bmatrix} -1 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1/Z & 0 & 0 & 0 \\ 0 & 1/Z & 0 & 0 \\ 0 & 0 & 1/Z & 0 \\ 0 & 0 & 0 & 1/Z \end{bmatrix} \begin{bmatrix} -1 \\ -1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 3/Z \end{bmatrix}$$

2 marks

and the nodal current vector is given by

$$\begin{bmatrix} \frac{E_1 + E_2}{Z} \end{bmatrix}$$

1 mark

(d) When $E_1 = E_2 = E$,

$$\begin{bmatrix} \frac{2E}{Z} \end{bmatrix} = \begin{bmatrix} 3/Z \end{bmatrix} \begin{bmatrix} V_1 \end{bmatrix} \longrightarrow V_1 = \tfrac{2}{3} E$$

The currents in the original branches are

$I_3 = (E - \tfrac{2}{3}E)/Z = E/3Z$, $I_4 = (E - \tfrac{2}{3}E)/Z = E/3Z$, $I_5 = (E - E)/Z = 0$

$I_1 = I_3 + I_5 = E/3Z$, $I_2 = I_4 - I_5 = E/3Z$, $I_6 = I_3 + I_4 = 2E/3Z$

2 marks

Alternate Method convert delta containing Z, Z and Z to an equivalent star and proceed.

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5. Mechanical output = 10.5 kW

Efficiency = 84%, ∴ Input electrical power = 10.5/0.84 = 12.5 kW

Power factor = 0.70 lag. Terminal voltage = 400 V, line resistance = 0.2 Ω each phase

(a) Line current = 12500/(√3×400×0.7) = 25.77 A, phase angle = $\cos^{-1}0.7$ = 45.57 lag **1 mark**

Supply voltage = 400+0.2×√3×25.77∠– 45.57^0 = 400+6.25 – j6.37 = 406.30∠– 0.90^0 V **1 mark**

kVA input = √3×406.3×25.77 = 18.14 kVA **1 mark**

reactive power taken by the motor = 18.14×sin(– 0.90^0+45.57^0) = 12.75 kvar **1 mark**

(b) Total active power = 18.14×cos(– 0.90^0+45.57^0) = 12.90 kW

[Alternate: Loss is the 3 wires = 3×25.77^2×0.2 = 0.398 kW ⟶ total = 12.5 + 0.398]

New power factor = 0.96 ⟶ ϕ = $\cos^{-1}0.95$ = 18.19^0

Assuming the total active power remains constant and supply voltage remains unchanged, even after the connection of the capacitors,

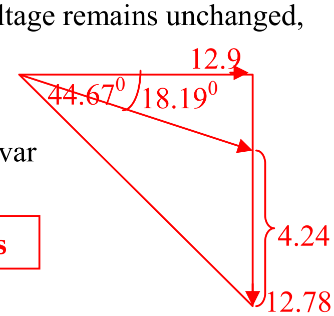
New reactive power = 12.90×tan18.19^0 = 4.24 kvar

∴ reactive power supplied by capacitors = 12.75 – 4.24 = 8.51 kvar

∴ 3×C×100π×406.3^2 = 8510

C = 54.7 μF in each arm of the delta

2 marks



(c) $I_{A0} = 1.0\angle{-30^o}$A, $I_{A1} = 2.0\angle 0^o$A and $I_{A2} = 1.0\angle 30^o$A

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix} \begin{bmatrix} 1.0\angle -30^0 \\ 2.0\angle 0^0 \\ 1.0\angle 30^0 \end{bmatrix}$$

$I_a = 1.0\angle{-30^o} + 2.0\angle 0^o + 1.0\angle 30^o = 0.866 - j0.5 + 2 + 0.866 + j0.5 = 3.732\angle 0^o$ A

$I_b = 1.0\angle{-30^o} + 2.0\angle 240^o + 1.0\angle 150^o = 0.866 - j0.5 - 1.0 - j1.732 - 0.866 + j0.5$

$= 2.0\angle 240^o$ A

$I_c = 1.0\angle{-30^o} + 2.0\angle 120^o + 1.0\angle 270^o = 0.866 - j0.5 - 1.0 + j1.732 - j1.0$

$= -0.134 + j0.232 = 0.268\angle 120^o$ A

2 marks

(d) $$[Z_p] = \begin{bmatrix} z_s & z_m & z_m \\ z_m & z_s & z_m \\ z_m & z_m & z_s \end{bmatrix}$$

the symmetrical component impedance matrix can be written as

$$[Z_s] = \tfrac{1}{3}[\Lambda]^*.[Z_p].[\Lambda] = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix} \begin{bmatrix} z_s & z_m & z_m \\ z_m & z_s & z_m \\ z_m & z_m & z_s \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix} \begin{bmatrix} z_s + 2z_m & z_s + (\alpha + \alpha^2)z_m & z_s + (\alpha + \alpha^2)z_m \\ z_s + 2z_m & \alpha^2 z_s + (1+\alpha)z_m & \alpha z_s + (1+\alpha^2)z_m \\ z_s + 2z_m & \alpha z_s + (1+\alpha^2)z_m & \alpha^2 z_s + (1+\alpha)z_m \end{bmatrix}$$

which can be simplified using the property $1+\alpha+\alpha^2 = 0$ as follows

$$[Z_s] = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix} \begin{bmatrix} z_s + 2z_m & z_s - z_m & z_s - z_m \\ z_s + 2z_m & \alpha^2(z_s - z_m) & \alpha(z_s - z_m) \\ z_s + 2z_m & \alpha(z_s - z_m) & \alpha^2(z_s - z_m) \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix} 3(z_s + 2z_m) & 0 & 0 \\ 0 & (1+\alpha^3+\alpha^3)(z_s - z_m) & 0 \\ 0 & 0 & (1+\alpha^3+\alpha^3)(z_s - z_m) \end{bmatrix}$$

i.e. $$[Z_s] = \begin{bmatrix} (z_s + 2z_m) & 0 & 0 \\ 0 & (z_s - z_m) & 0 \\ 0 & 0 & (z_s - z_m) \end{bmatrix} = \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix}$$

2 marks

6.

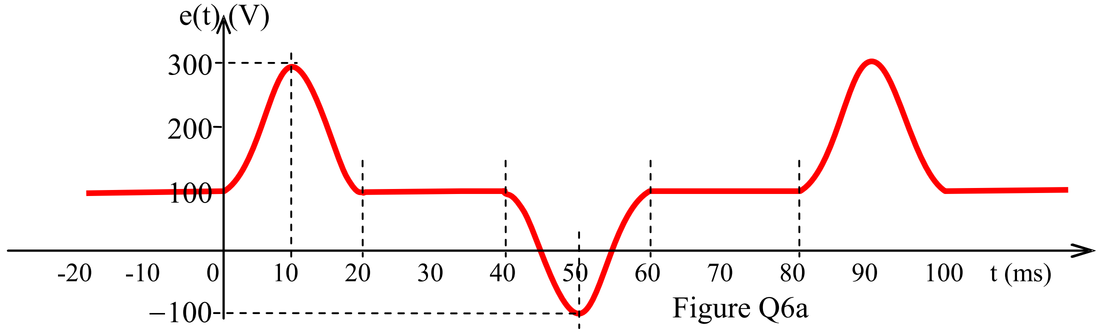


Figure Q6a

$$e(t) = 200 - 100\cos 100\pi t \qquad 0 \le t \le 20 \text{ ms}$$
$$= 100 \qquad 20 \le t \le 40 \text{ ms}$$
$$= 100\cos 100\pi t \qquad 40 \le t \le 60 \text{ ms}$$
$$= 100 \qquad 60 \le t \le 80 \text{ ms}$$

Period = 80 ms = 0.08 s, $\omega = 2\pi/0.08 = 25\pi$ rad/s

(a) waveform may be shifted as follows to obtain a more symmetrical waveform where $e(t) = f(t - 0.01) + 100$, or $f(t) = e(t+0.01) - 100$

$$f(t) = 100 - 100\cos(100\pi t + \pi) = 100 + 100\cos 100\pi t \qquad 0 \le t \le 10 \text{ ms}$$
$$= 0 \qquad 10 \le t \le 20 \text{ ms}$$

1 mark

$f(t) = A_0/2 + \sum A_n \cos n\omega t + B_n \sin n\omega t$

function f(t) has mean value = 0 → $A_0/2 = 0$

f(t) is even → $B_n = 0$ for all n

1 mark

f(t) has half-wave symmetry → even harmonics = 0

$$\therefore A_n = 4\times\frac{2}{T}\int_0^{0.02} f(t).\cos n\omega t.dt =$$

$$\frac{8}{0.08}\left[\int_0^{0.01}(100+100\cos 100\pi t).\cos n25\pi t.dt + \int_{0.01}^{0.02} 0.\cos n\omega t.dt\right]$$

$$A_n = 100\times\left[(100+100\cos 100\pi t).\frac{\sin n25\pi t}{n25\pi}\Bigg|_0^{0.01} - \int_0^{0.01}(100\times(-100\pi\times\sin 100\pi t).\frac{\sin n25\pi t}{n25\pi}.dt + 0\right]$$

$$= 100^2\times\left[0 - 0 + \int_0^{0.01}(\frac{2}{n}[\cos(100\pi - 25n\pi)t - \cos(100\pi + 25n\pi)t].dt\right]$$

$$= \frac{20000}{n}\times\left[\frac{\sin(100\pi - 25n\pi)t}{100\pi - 25n\pi} - \frac{\sin(100\pi + 25n\pi)t}{100\pi + 25n\pi}\right]_0^{0.01}$$

$$= \frac{800}{n}\times\left[\frac{\sin(\frac{n\pi}{4})}{(4-n)\pi} - \frac{\sin(\frac{n\pi}{4})}{(4+n)\pi}\right] = \frac{800\sin(\frac{n\pi}{4})}{n}\times\frac{2n}{(4-n^2)\pi} = \frac{1600\sin(\frac{n\pi}{4})}{(4-n^2)\pi}$$

2 marks

$A_1 = 120.04$, $A_3 = 72.03$, A_5 …. = 17.14, –8.00, –4.68, ….

$\therefore$ $f(t) = 120.04\cos 25\pi t + 72.03\cos 75\pi t + ...$ **1 mark**

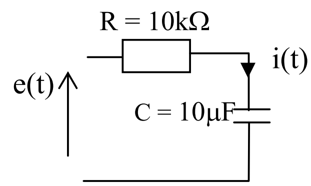
and $e(t) = f(t - 0.01) + 100 = 100 + 120.04\cos(25\pi t - \pi/4) + 72.03\cos(75\pi t - 3\pi/4) + ...$

(b) mean value = 100 V

$$\text{rms value} = \sqrt{100^2 + \frac{120.04^2}{2} + \frac{72.03^2}{2} + ...} = 140.7\text{ V}$$ **2 marks**

(limited accuracy due to low terms – actual value around 141.4 V)

(c) $I_n = E_n/(1\times10^4 + 1/jn25\pi\times10\times10^{-6})$



$$I_n = \frac{10^{-4}}{1 - \dfrac{j}{2.5n\pi}} E_n$$ **1 mark**

there will be no d.c. current flow due to the capacitance

$$I_1 = \frac{10^{-4}}{1 - j0.1273} E_1 = \frac{10^{-4}}{1.008\angle -7.25^0} \times 120.04 = 0.0119\angle 7.25^0$$

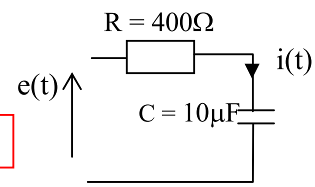
$$I_3 = \frac{10^{-4}}{1 - j0.0424} E_1 = \frac{10^{-4}}{1.0009\angle -2.43^0} \times 72.03 = 0.0072\angle 2.43^0$$ **1 mark**

$\therefore$ $i(t) = 0.0119\cos(25\pi t - 45^0 + 7.25^0) + 0.0072\cos(75\pi t - 135^0 + 2.43^0) + ...$

i.e. $i(t) = 0.0119\cos(25\pi t - 37.75^0) + 0.0072\cos(75\pi t - 132.57^0) + ...$ **1 mark**

If value of R was 400 Ω instead of 10 kΩ

$I_n = E_n/(400 + 1/jn25\pi\times10\times10^{-6})$



$$I_n = \frac{jn25\pi}{10^5 + j10000n\pi} E_n$$ **1 mark**

there will be no d.c. current flow due to the capacitance

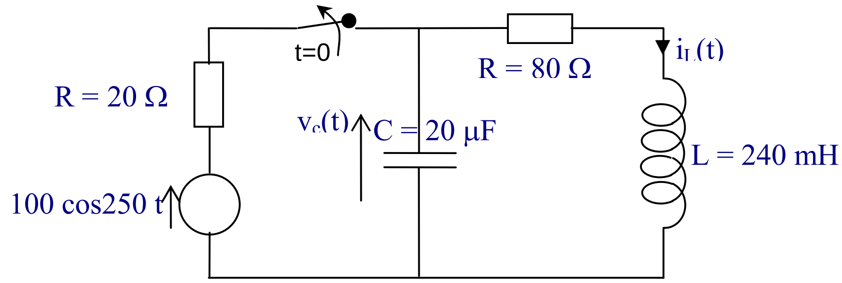
$$I_1 = \frac{j25\pi}{10^5 + j10000\pi} E_1 = \frac{25\pi\angle 90^0}{104.8\times10^3\angle 17.44^0} E_1 = 0.0007494\angle 72.56^0 E_1$$

$$I_3 = \frac{j75\pi}{10^5 + j30000\pi} E_3 = \frac{75\pi\angle 90^0}{137.4\times10^3\angle 43.30^0} E_3 = 0.001715\angle 46.70^0 E_3$$ **1 mark**

$\therefore$ $i(t) = 0.090\cos(25\pi t - 45^0 + 72.56^0) + 0.124\cos(75\pi t - 135^0 + 46.70^0) + ...$

i.e. $i(t) = 0.090\cos(25\pi t + 27.56^0) + 0.124\cos(75\pi t - 88.30^0) + ...$ **1 mark**

7. (a)



C → 1/j 250×20×10^{-6} = – j 200 Ω, L → j 250×240×10^{-3} = j 50 Ω

Working with peak values and complex numbers rather than with r.m.s. values to retain peak magnitudes

$$V_{C\max} = 100 \times \frac{-j200//(80+j60)}{20 - j200//(80+j60)}$$

$$V_{C\max} = 100 \times \frac{-j200 \times (80+j60)}{20 \times (-j200+80+j60) - j200 \times (80+j60)}$$

$$V_{C\max} = \frac{1000\angle -90^0 \times 100\angle 36.87^0}{-j200+80+j60-j800+600} = \frac{100000\angle -53.13^0}{680 - j940} = \frac{100000\angle -53.13^0}{1160\angle -54.11^0}$$

$= 86.2\angle 0.98^0$ V

$v_c(t) = 86.2 \cos(250t + 0.98^0)$

$I_{Lmax} = 86.2\angle 0.98^0/(80+j60) = 0.862\angle -35.89^0$ A **4 marks**

$i_L(t) = 0.862 \cos(250t - 35.89^0)$

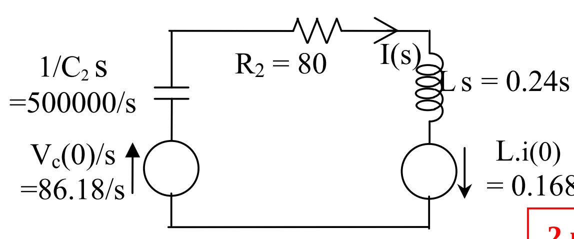
Thus at t = 0, initial conditions

$v_c(0) = 86.2 \cos(0.98^0) = 86.18$ V **1 mark**

$i_L(0) = 0.862 \cos(-35.89^0) = 0.698$ A

(c) The transformed equivalent circuit, after the switch is opened, is given by

Thus the circuit current can be calculated as



$$I(s) = \frac{\frac{86.18}{s} + 0.168}{\frac{500000}{s} + 80 + 0.24s}$$

2 marks

$$I(s) = \frac{0.168s + 86.18}{0.24s^2 + 80s + 50000} = \frac{0.7s + 359.1}{s^2 + 333.3s + 20833} = \frac{0.7s + 359.1}{(s+166.7)^2 + 424.9^2}$$

i.e. $$I(s) = \frac{0.7(s+166.7) + 242.4}{(s+166.7)^2 + 424.9^2} = 0.7\frac{s+166.7}{(s+166.7)^2 + 424.9^2} + 0.57\frac{424.9}{(s+166.7)^2 + 424.9^2}$$

3 marks

∴ curent is given by $i(t) = 0.7e^{-166.7t} \cos 424.9t + 0.57\, e^{-166.7t} \sin 424.9t$