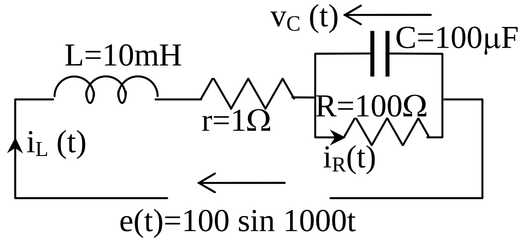
Level 2 Semester 1 Examination - January 2009



1. (a) $Z = j\omega L + r + \dfrac{R \times \frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = j\omega L + r + \dfrac{R}{1 + j\omega CR}$ ***[1 mark]***

(b) ω = 1000 rad/s

$L\omega = 0.01 \times 1000 = 10\Omega$, $1/C\omega = 1/0.0001 \times 1000 = 10\Omega$

$$Z = j10 + 1 + \frac{100}{1 + j10} = (1.9901 + j0.0990) = 1.9926\angle 2.85^0\ \Omega$$

Working with peak values instead of rms values,

$I_{Lm} = 100/1.9926\angle 2.85^0 = 50.186\angle{-2.85^0}$ A $= 50.186\angle{-0.0497}$ A

$i_L(t) =$ <u>$50.186 \sin (1000t - 2.85^0)$ A</u> ***[1 mark]***

$V_{Cm} = 100 - (1+j10) \times 50.186\angle{-2.85^0} = 100 - 10.0499\angle 84.29^0) \times 50.186\angle{-2.85^0}$

$= 100 - 504.36\angle 81.44^0 = 100 - (75.08+j498.74) = 24.92 - j498.74$

$= 499.36\angle{-87.14^0}$V $= 499.36\angle{-1.5209}$ V

$v_C(t) =$ <u>$499.36 \sin (1000t - 87.14^0)$ A</u> ***[1 mark]***

(c) Energy stored = $½Li^2 + ½Cv^2$

$W = 0.5 \times 0.01 \times 50.186^2 \times \sin^2 (1000t - 2.85^0) + 0.5 \times 0.0001 \times 499.36^2 \times \sin^2 (1000t - 87.14^0)$

$= 12.593\ \sin^2 (1000t - 2.85^0) + 12.468 \sin^2 (1000t - 87.14^0)$ ***[1 mark]***

For maximum energy stored, $dW/dt = 0$

$dW/dt = 12593 \times \sin (2000t - 5.70^0) + 12468 \times \sin (2000t - 174.28^0) = 0$

i.e. $12531 \sin 2000 t - 1251 \cos 2000t - 12406 \sin 2000t - 1242.6 \cos 2000t = 0$

$125 \sin 2000 t = 2494 \cos 2000t$

$\tan 2000t = 19.952$, $2000t = 1.5207$ rad or 4.6623 rad giving $t = 0.7604$ ms OR $t = 2.3312$ ms

∴ maximum energy stored $= 12.593 \times \sin^2 (0.7604 - 0.0497) + 12.468 \times \sin^2 (0.7604 - 1.5209)$

$= 5.359 + 5.924 = 11.28$ J

OR maximum energy stored $= 12.593 \times \sin^2 (2.3312 - 0.0497) + 12.468 \times \sin^2 (2.3312 - 1.5209)$

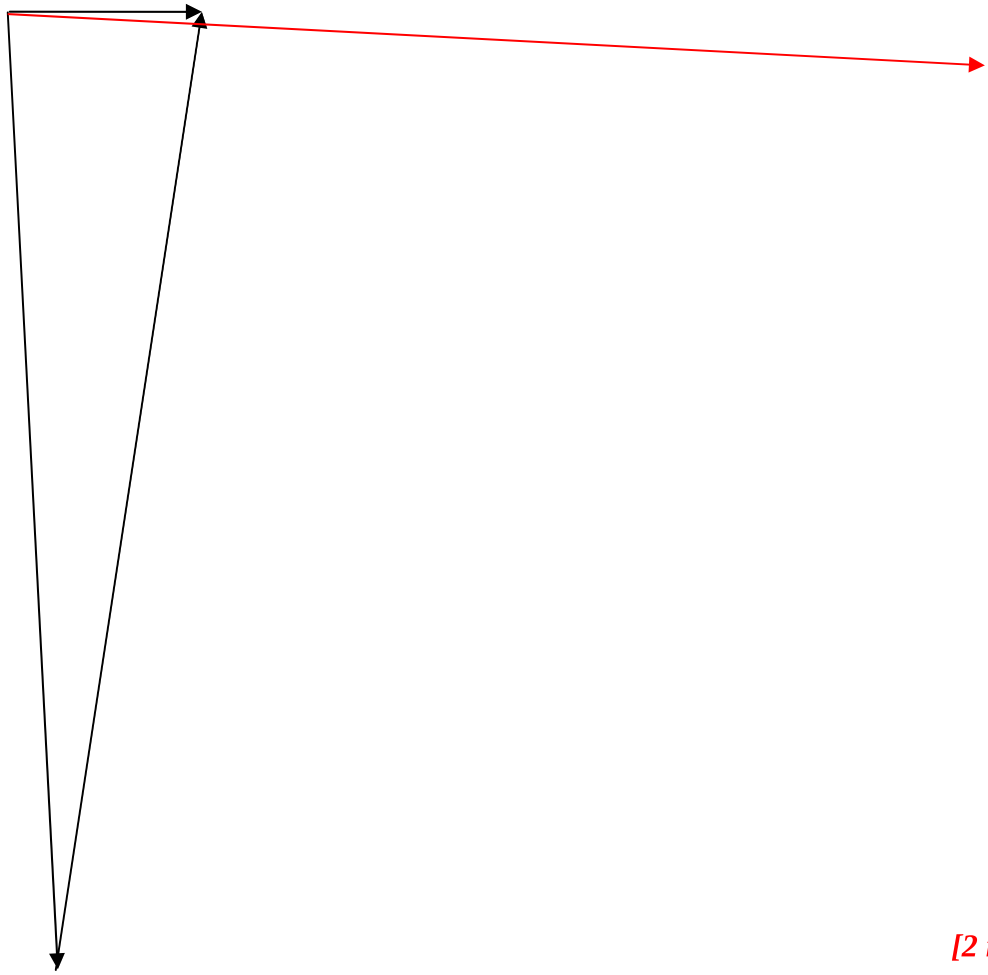
$= 7.234 + 6.544 =$ <u>13.78 J</u> ***[1 mark]***

(d) Power loss in the circuit $= (50.186/\sqrt{2})^2 \times 1 + (499.36/\sqrt{2})^2/100 = 2506$ W

Energy loss per cycle $= 5012 \times 2\pi/1000 =$ <u>15.75 J</u> ***[1 mark]***

Q factor of circuit $= 2\pi \times 13.78/15.75 =$ <u>5.50</u> ***[1 mark]***

(e)



[2 mark]

(f) Resonance frequency (when power factor is unity)

$$j\omega L + r + \frac{R}{1 + j\omega CR} = j\omega L + r + \frac{R(1 - j\omega CR)}{1 + \omega^2 C^2 R^2}$$

i.e. $\omega L = \dfrac{\omega CR^2}{1 + \omega^2 C^2 R^2}$ or $L + \omega^2 LC^2R^2 = CR^2$, giving $\omega = \sqrt{\dfrac{CR^2 - L}{LC^2R^2}}$ OR

$$\omega = \sqrt{\frac{10^{-4} \times 100^2 - 0.01}{0.01 \times 10^{-8} \times 100^2}} = \sqrt{\frac{0.99}{10^{-6}}} = \underline{995.0 \text{ rad/s}} \text{ (or 158.4 Hz)}$$

[1 mark]

2. (a) Reluctance = $l/\mu A$

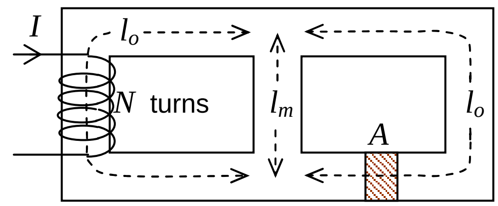


Figure Q2a

outer limb reluctance $S_o = l_o/\mu_o\mu_r A$

middle limb reluctance $S_m = l_m/\mu_o\mu_r A$

effective reluctance seen by coil = $S_o + S_m // S_o$

$$= S_o + \frac{S_m S_o}{S_m + S_o} = \frac{l_o}{\mu_o\mu_r A} + \frac{l_o l_m}{\mu_o\mu_r A(l_o + l_m)}$$

$$= \frac{0.2}{4\pi\times10^{-7}\times1000\times4\times10^{-4}} + \frac{0.2\times0.06}{4\pi\times10^{-7}\times1000\times4\times10^{-4}\times(0.2+0.06)}$$

= 397,887 + 91,820 = $\underline{\underline{489{,}707\ H^{-1}}}$ **[3 marks]**

(b) at 50 Hz, $P_e = P_h = 50$ W, with t = 0.5 mm

When operated at 60 Hz,

$P_h \propto f$,

Hysteresis loss $P_h = 50\times60/50 = \underline{\underline{60\ W}}$

$P_e + P_h = 100$

Eddy current loss $P_e = \underline{\underline{40\ W}}$ **[1 mark]**

$P_e \propto f^2 t^2$, $t \propto \frac{\sqrt{P_e}}{f}$,

Required thickness $t = 0.5\times\frac{50}{60}\times\sqrt{\frac{40}{50}} = \underline{\underline{0.373\ mm}}$ **[1 mark]**

(c) r_1 = 2 mm and r_2 =3 mm, l = 25 mm, ε_r = 4.5, W = 1 mJ

Capacitance $\boldsymbol{C} = \frac{2\pi\varepsilon \boldsymbol{l}}{\ln[r_2/r_1]} = \frac{2\times\pi\times4.5\times8.854\times10^{-12}\times0.025}{\ln(3/2)}$ = 15.435 pF

Energy stored $W = 1\times10^{-3} = ½\times15.435\times10^{-12}\times V^2$

Required voltage V = √129,575,639. = $\underline{\underline{11{,}383\ V}}$ **[2 marks]**

(d) Voltage in absence of load V_{oc} = 230 V

Short circuit current I_{sc} = 10 kA

∴ internal impedance = $230/10\times10^3 = 0.023\ \Omega$

Theoretical maximum power deliverable = $V^2/4r = 230^2/4\times0.023$ = 575 kW **[2 marks]**

Internal impedance is purely resistive is assumed.

The two reasons that make this value unachievable are

(i) Source impedance is not purely resistive

(ii) A load voltage of half value is not acceptable. **[1 mark]**

3. (a) The two port admittance matrix is given by

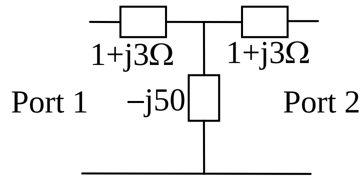


Figure Q3a

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_2 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}$$

$$z_{11}\big|_{V_2=0} = 1+j3+(-j50)//(1+j3) = 1+j3+\frac{-j50\times(1+j3)}{-j50+(1+j3)} = 1+j3+\frac{150-j50}{1-j47} = 1+j3+\frac{158.11\angle-18.43^0}{47.01\angle-88.78}$$

$$= 1+j3+3.363\angle 70.35^0 = 1+j3+1.1309+j3.1672 = 2.1309+j6.1672 = 6.525\angle 70.94^0$$

$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = \frac{1}{z_{11}} = 0.1532\angle{-70.94^0}\ \text{S}\ \ (\text{or } 0.0500 - j\ 0.1449)$$

$$y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = \frac{I_1}{V_1}\times\left.\frac{I_2}{I_1}\right|_{V_2=0} = -0.153\angle-70.94^0\times\frac{-j50}{1-j47} = -0.153\angle-70.94^0\times\frac{50\angle-90^0}{47.01\angle-88.78^0}$$

$$y_{21} = \frac{7.65\angle 19.06^0}{47.01\angle-88.78^0} = 0.163\angle 107.84^0\,S\ \ \text{or}\ \ (-0.0499 + j\ 0.1549)$$

$y_{12} = y_{21} = 0.163\angle 107.84^0$ S

$y_{22} = y_{11} = 0.153\angle{-70.940}$ S

The two port admittance matrix is

$$[Y] = \begin{bmatrix} 0.153\angle-70.9^0 & 0.163\angle 107.8^0 \\ 0.163\angle 107.8^0 & 0.153\angle-70.9^0 \end{bmatrix}$$

[2 marks]

(b) Characteristic impedance Z_0 is given when

$V_1/I_1 = Z_o = V_2/(-I_2)$

i.e. using two port admittance parameters,

$I_1 = y_{11}Z_oI_1 - y_{12}Z_oI_2$

$I_2 = y_{21}Z_oI_1 - y_{22}Z_oI_2$

$$\therefore\ \frac{I_2}{I_1} = \frac{y_{11}Z_0 - 1}{y_{12}Z_0} = \frac{y_{21}Z_0}{y_{22}Z_0 + 1}$$

Since $y_{22} = y_{11}$, and $y_{12} = y_{21}$

$y_{11}^2\ Z_0^2 - 1 = y_{12}^2\ Z_0^2$

$$Z_0 = \sqrt{\frac{1}{y_{11}^2 - y_{12}^2}} = \sqrt{\frac{1}{0.1532^2\angle-70.94\times 2^0 - 0.1630^2\angle 107.84\times 2^0}}$$

$$= \sqrt{\frac{1}{0.0235\angle -141.87^0 - 0.0266\angle 215.69^0}} = \sqrt{\frac{1}{0.00310 - j0.0010}} = \sqrt{\frac{1}{0.00326\angle 17.88^0}}$$

Z_0 = $\underline{17.51\angle -8.9^0\ \Omega}$ **[2 marks]**

Alternate Solution

$$Z_0 = 1 + j3 + \frac{(-j50)(1 + j3 + Z_o)}{-j50 + 1 + j3 + Z_o}$$

$$Z_0^2 + (1 + j3)Z_0 - j50Z_0 = (1 + j3)Z_0 + (1 + j3)^2$$
$$- j50(1 + j3) - j50Z_0$$

$$Z_0^2 = 292 - j94$$

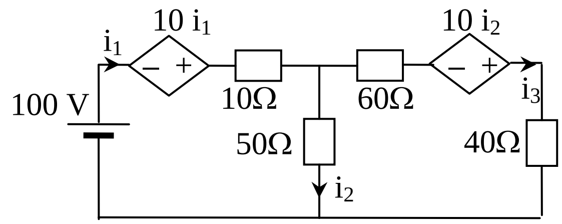
$Z_0 = 17.51\angle -8.92^0\ \Omega$

(c) Kirchoff's law equations

$i_1 = i_2 + i_3$

$100 = -10\, i_1 + 10\, i_1 + 50\, i_2 \rightarrow 100 = 50\, i_2$

$0 = 60\, i_3 - 10\, i_2 + 40\, i_3 - 50\, i_2 \rightarrow 6\, i_2 = 10\, i_3$ **[2 marks]**



(d) $i_2 = \underline{\underline{2\ A}}$

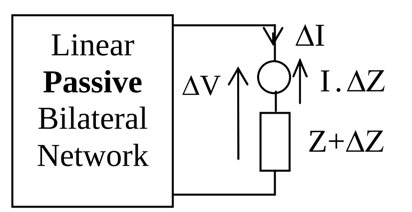
$i_3 = \underline{\underline{1.2\ A}}$

$i_1 = i_2 + i_3 = \underline{3.2\ A}$ **[2 marks]**

(e) $Z_s = 1+j3\ \Omega$, $Z_{load} = 40 + j30\ \Omega$, $I_{load} = 10.5\angle 30^0$A

For new current $I_{load} = 10\angle 30^0$A, load impedance = $Z_{load} + \Delta Z_{load}$

Using compensation theorem,



$\Delta I = 10.5\angle 30^0 - 10\angle 30^0 = -0.5\angle 30^0$ A

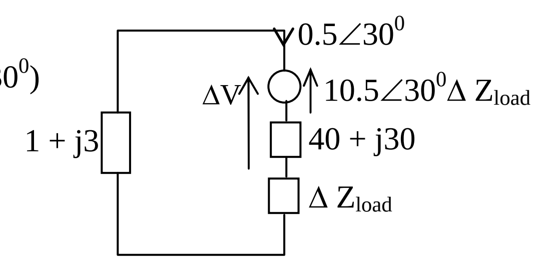
$I\,.\Delta Z = -[Z_s + (Z_{load} + \Delta Z_{load})]\,\Delta I$

i.e. $10.5\angle 30^0 \Delta Z_{load} = -[1 + j3 + (40 + j30 + \Delta Z_{load})]\,(-0.5\angle 30^0)$

i.e. $21\Delta Z_{load} = [41 + j33 + \Delta Z_{load}]$

$20\,\Delta Z_{load} = 41 + j33 \rightarrow \Delta Z_{load} = 2.05 + j1.65$

$Z_{load} = 42.05 + j31.65 = \underline{\underline{52.63\angle -36.97^0}}$ **[2 marks]**



4. (a) E = 100V, ω=250 rad/s,

$R_1 = R_2 = 10\,\Omega$, $Y_{R1} = 0.1$, $E/R_1 = 10A$

$L = 10mH$, $X_L = 250\times0.01 = 2.5\,\Omega$,

$Y_{LR2} = 1/(10+j2.5) = 0.09411 - j0.02353$

$C = 100\mu F$, $Y_C = j250\times10^{-4} = j0.025$ S

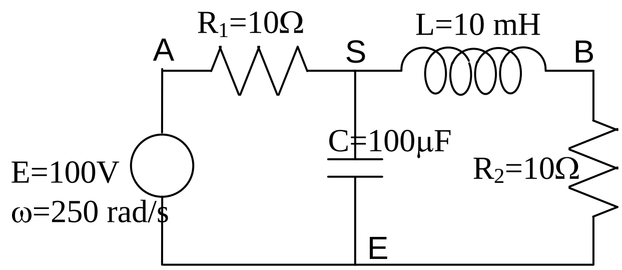


Figure Q4

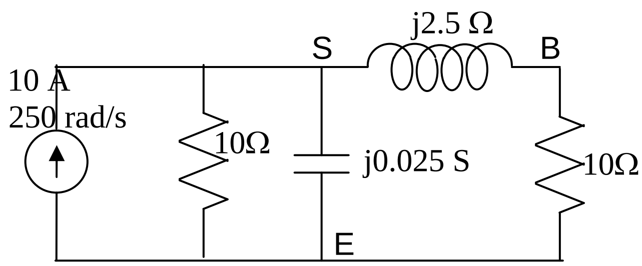


Figure Q4

$$\frac{1}{10+j2.5} = 0.0941 - j0.02353$$

[1 mark]

(b) Selecting node E as reference, and S as node 1 and combining L and R_2 into 1 branch

$[Y_N] = [0.1 + j0.025 + 0.09411 - j0.02353] = [0.19411+j0.00147] = 0.19412\angle0.434^0$ ***[1 mark]***

$[I_{gN}] = [10]$ ***[1 mark]***

(c) using nodal analysis, $[I_{gN}] = [Y_N]\,[V_N]$

i.e. $V_S = [0.19412\angle0.434^0]^{-1} \times10 = 51.51\angle-0.434^0$

$\therefore$ $I_{AS} = [100 - 51.51\angle-0.434^0]/10 = [10 - 5.151 + j0.039] = 4.849 + j0.039 = 4.849\angle0.46^0$ A ***[1 mark]***

$I_{SE} = 51.51\angle-0.434^0\times j0.025 = 1.288\angle89.57^0$ A ***[1 mark]***

$I_{SBE} = 51.51\angle-0.434^0\times(0.09411 - j0.02353) = 51.51\angle-0.434^0\times0.0970\angle-14.04^0$

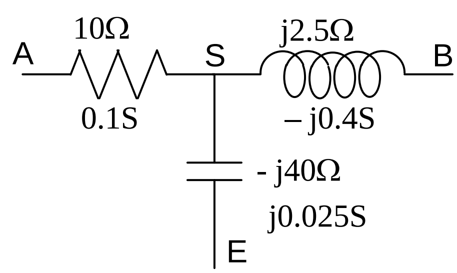
$= 4.996\angle-14.47^0$ A ***[1 mark]***

(d) $$Y_{AE} = \frac{0.1\times j0.025}{0.1 + j0.025 - j0.4} = \frac{j0.0025}{0.1 - j0.375}$$

$Z_{AE} = -150 - j40\ \Omega = 155.24\angle-165.07^0\ \Omega$ ***[1 mark]***

$$Y_{AB} = \frac{0.1\times(-j0.4)}{0.1 + j0.025 - j0.4} = \frac{-j0.04}{0.1 - j0.375}$$

$Z_{AB} = 9.375+j2.5\ \Omega = 9.703\angle14.93^0\ \Omega$ ***[1 mark]***



(e) $I_{AE} = 100/155.24\angle-165.07^0 = 0.644\angle165.07^0$ A

$I_{AB} = (100 - 49.96\angle-14.47^0)/9.703\angle14.93^0 = (51.62 + j12.48)/9.703\angle14.93^0$

$= 53.11\angle13.59^0/9.703\angle14.93^0 = 5.474\angle-1.34^0$ A ***[1 mark]***

(f) Thus $I_{AS} = 5.474\angle-1.34^0$ A $+ 0.644\angle165.07^0$ A $= 5.473 - j0.128 - 0.6223 + j0.1659$

$I_{AS} = 4.850 + j\,0.038 = 4.850\angle0.45^0$ A ***[1 mark]***

5. Load (i) = star connected, resistive, 40 Ω each

Load (ii) = 2 kW, 0.7 lag, 3 phase motor

(a) Line current of load (i) = $400/(\sqrt{3}\times 80) = 2.886\angle 0^0$ A

Line current of load (ii) = $2000/(\sqrt{3}\times 400\times 0.7) = 4.124\angle -45.57^0$ A

$\therefore$ total line current = $2.886 + 4.124\angle -45.57^0 = 2.886 + 2.887 - j2.945 = 5.873 - j2.945$

$= \underline{6.48\angle -27.02^0 \text{ A}}$ **[2 mark]**

(b) Supply power factor = cos 27.32 = $\underline{0.891 \text{ lag}}$ **[1 mark]**

(c) Total active power of load = $\sqrt{3}\times 400\times 6.48\times\cos(27.02^0) = 4000$ W

Total reactive power of load = $\sqrt{3}\times 400\times 6.48\times\sin(27.02^0) = 2040$ var

Total reactive power supplied by capacitor = 2040 var

$\therefore 3\times 400^2\times C\times 100\pi = 2040$

Each capacitor C of delta = $\underline{13.53\ \mu\text{F}}$ **[2 mark]**

(d) $I_{A0} = 1.0\angle 90^o$A, $I_{A1} = 10.0\angle 0^o$A and $I_{A2} = 3.0\angle -90^o$A

Since supply is balanced, only positive sequence voltage is present

$V_{A0} = 0$ V, $V_{A1} = 230\angle 0^o$V and $V_{A2} = 0$ V

Power associated with zero sequence = $\underline{0 \text{ W}}$

Power associated with positive sequence = $\sqrt{3}\times 400\times 10.0\times\cos 0^0 = \underline{6928 \text{ W}}$

Power associated with negative sequence = $\underline{0 \text{ W}}$ **[2 mark]**

(e) Phase currents are given by

$$\begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix} \begin{bmatrix} 1\angle 90^0 \\ 10\angle 0^0 \\ 3\angle -90^0 \end{bmatrix}$$

$I_A = j1 + 10 - j3 = 10 - j2 = \underline{10.20\angle -11.31^0 \text{ A}}$

$I_B = j1 + 10\angle 240^0 + 3\angle 30^0 = j1 - 5 - j\,8.660 + 2.598 + j1.5 = -2.402 - j\,6.160$

$= \underline{6.612\angle -111.3^0 \text{ A}}$

$I_C = j1 + 10\angle 120^0 + 3\angle 150^0 = j1 - 5 + j\,8.660 - 2.598 + j1.5 = -7.598 + j\,11.160$

$= \underline{13.5\angle 124.2^0 \text{ A}}$ **[3 marks]**

6. $5\cos 6\omega_o t$ in the region $-T_1$ to T_1

$5\cos 6\omega_o T_1 = 0$ gives $6\omega_o T_1 = \pi/2$

i.e. $6\times 2\pi/T\times T_1 = \pi/2$, or $24\,T_1 = T$

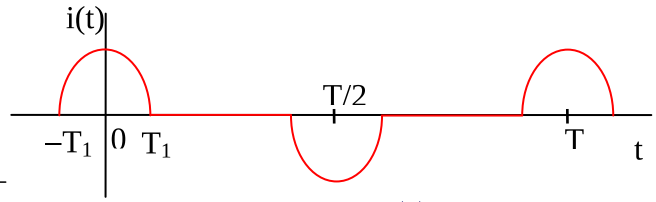


Figure Q6(a) $\omega_o T = 2\pi$

(a) rms current = $\sqrt{\frac{1}{T}\int_0^T i^2(t)dt} = \sqrt{\frac{4}{T}\int_0^{T_1} i^2(t)dt}$

i.e. $I_{rms} = \sqrt{\frac{4}{T}\int_0^{T_1} 25\cos^2 6\omega_0 t\,dt} = \sqrt{\frac{4}{T}\int_0^{T_1} 12.5(1+\cos 12\omega_0 t)\,dt} = \sqrt{\frac{4}{T}\left[12.5\left(t+\frac{\sin 12\omega_0 t}{12\omega_0}\right)\right]_0^{T/24}}$

$$= \sqrt{\frac{4}{T}\left[12.5\left(\frac{T}{24}+\frac{\sin\frac{\omega_0 T}{2}}{12\omega_0}\right)\right]} = \sqrt{50\left(\frac{1}{24}+\frac{\sin\pi}{24\pi}\right)} = \underline{1.4433\ \text{A}}$$

[2 mark]

(b) waveform is even-symmetric, half-wave symmetric and mean value is zero.

$\therefore$ $B_n = 0$ for all n, $A_n = 0$ for even harmonics, $A_0 = 0$

and $A_n = \frac{4\times 2}{T}\int_0^{T/24} 5\cos 6\omega_0 t\cdot\cos n\omega_0 t\cdot dt = \frac{20}{T}\int_0^{T/24}\left[\cos(6-n)\omega_0 t + \cos(6+n)\omega_0 t\right]\cdot dt$

$$A_n = \frac{20}{T}\left[\frac{\sin(6-n)\omega_0 t}{(6-n)\omega_0}+\frac{\sin(6+n)\omega_0 t}{(6+n)\omega_0}\right]_0^{T/24} = \frac{20}{\omega_0 T}\left[\frac{\sin\frac{(6-n)\omega_0 T}{24}}{(6-n)}+\frac{\sin\frac{(6+n)\omega_0 T}{24}}{(6+n)}\right]$$

$$= \frac{10}{\pi}\left[\frac{\sin\frac{(6-n)\pi}{12}}{(6-n)}+\frac{\sin\frac{(6+n)\pi}{12}}{(6+n)}\right]$$

$A_1 = 1.0542$

$A_3 = 1.0003$

$A_5 = 0.8987$ **[4 marks]**

Thus $i(t) = 1.0542\cos\omega_0 t + 1.0003\cos 3\omega_0 t + 0.8987\cos 5\omega_0 t$

(c) $v(t) = 100\cos(\omega_o t+\pi/12) - 1\times(1.0542\cos\omega_0 t + 1.0003\cos 3\omega_0 t + 0.8987\cos 5\omega_0 t + \ldots)$

$v(t) = (96.5926-1.0542)\cos\omega_0 t - 25.8819\sin\omega_0 t - 1.0003\cos 3\omega_0 t - 0.8987\cos 5\omega_0 t + \ldots$

$= 98.98\cos(\omega_0 t - 15.16^0) - 1.0003\cos 3\omega_0 t - 0.8987\cos 5\omega_0 t + \ldots$ **[2 marks]**

(d) power supplied = $98.98\times 1.0542\times\cos 15.16^0$

= $\underline{100.7\ \text{W}}$ **[2 marks]**

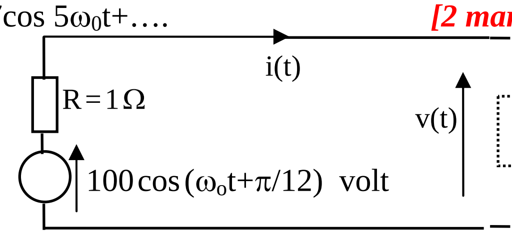


Figure Q6(b)

7. (a) Determination of Laplace transform of the causal sinusoidal waveform **[2 marks]**

(b) Em = 100 V, f = 50 Hz → ω = 100π rad/s,

R = 100 Ω, C = 40 μF, L =1.8 H

With switch S open,

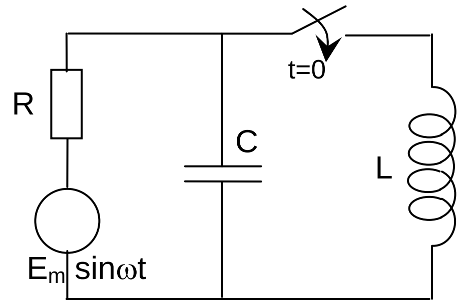


Figure Q7

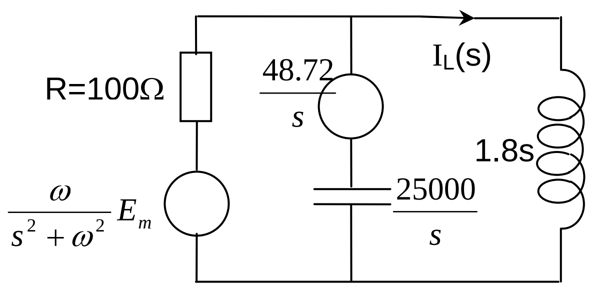
$$V_C = \frac{\frac{1}{j100\pi\times 40\times 10^{-6}}}{100+\frac{1}{j100\pi\times 40\times 10^{-6}}}E$$

$$= \frac{1}{100\times j100\pi\times 40\times 10^{-6}+1}E$$

$$= \frac{1}{1+j0.4\pi}E = 0.6227\angle 51.49^0\times E \rightarrow$$ gives $v_c(t)$ = 62.27 sin (100πt+51.49°)

At t= 0, initial value of voltage across C = $V_c(0)$ = 62.27 sin 51.49^0 = 48.72 V **[2 marks]**

(c)



[2 marks]

(d)

$$I_L(s) = \frac{\omega}{s^2+\omega^2}\cdot E_m\cdot\frac{1}{100+\frac{1.8s\times 25000/s}{1.8s+25000/s}}\times\frac{25000/s}{1.8s+25000/s}+\frac{48.72}{s}\times\frac{1}{\frac{25000}{s}+\frac{100\times 1.8s}{100+1.8s}}\times\frac{100}{100+1.8s}$$

$$= E_m\cdot\frac{\omega}{s^2+\omega^2}\times\frac{25000/s}{180s+\frac{2500000}{s}+1.8\times 25000}+\frac{48.72}{s}\times\frac{100s}{25000\times 100+25000\times 1.8s+180s^2}$$

$$= E_m\cdot\frac{\omega}{s^2+\omega^2}\times\frac{2500}{18s^2+4500s+250000}+\frac{487.2}{18s^2+4500s+250000}$$

$$= \frac{2500}{18s^2+4500s+250000}\times\left[100\frac{100\pi}{s^2+(100\pi)^2}+487.2\right]$$

$$= \frac{138.889}{(s+244.315)(s+5.685)}\times\left[\frac{31415.9}{s^2+(314.159)^2}+487.2\right]$$

(e)