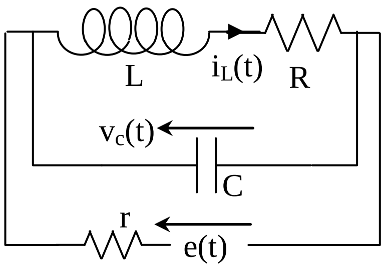
EE 2010 - THEORY OF ELECTRICITY – Short Answers

Level 2 Semester 1 Examination - August 2010

1. (a)



Impedance at angular frequency ω = r + 1/jωC//(R+ jωL)

$$Z = r + \frac{\frac{1}{j\omega C}\times(R + j\omega L)}{\frac{1}{j\omega C} + R + j\omega L} = r + \frac{R + j\omega L}{1 + j\omega C(R + j\omega L)}$$

1 mark

(b) L=10 mH, R=100 Ω, C = 100 μF, r = 1 Ω and e(t) = 100 sin 1000t
Calculating with peak values instead of with rms values,

$$I_m = E/\left(r + \frac{R + j\omega L}{1 + j\omega C(R + j\omega L)}\right) = \frac{E(1 + j\omega C(R + j\omega L))}{R + r - \omega^2 LCr + j\omega(CrR + L)}$$

$$I_m = \frac{100(1 + j1000\times 100\times 10^{-6}(100 + j1000\times 0.01))}{100 + 1 - 1000^2\times 0.01\times 100\times 10^{-6}\times[1 + j1000\times(100\times 10^{-6}\times 1\times 100 + 0.01)]}$$

$$= \frac{(100 + j1000 - 100))}{100 + j(10 + 10)} = \frac{j1000}{101 + j20} = \frac{1000\angle 90^o}{101.98\angle 11.31^o} = 9.806\angle 78.69^o$$

$$I_{Lm} = I_m \times \frac{\frac{1}{j\omega C}}{R + j\omega L + \frac{1}{j\omega C}} = I_m \times \frac{1}{1 + j\omega C(R + j\omega L)}$$

$$= \frac{j1000}{100 + j20}\times\frac{1}{1 + j1000\times 100\times 10^{-6}\times(100 + j1000\times 0.01)}$$

$$I_{Lm} = \frac{j1000}{100 + j20}\times\frac{1}{j10} = \frac{100}{101.98\angle 11.31^o} = 0.9806\angle -11.31^o$$

$i_L(t) = 0.980 \sin(1000t - 11.31^o)$

1 mark

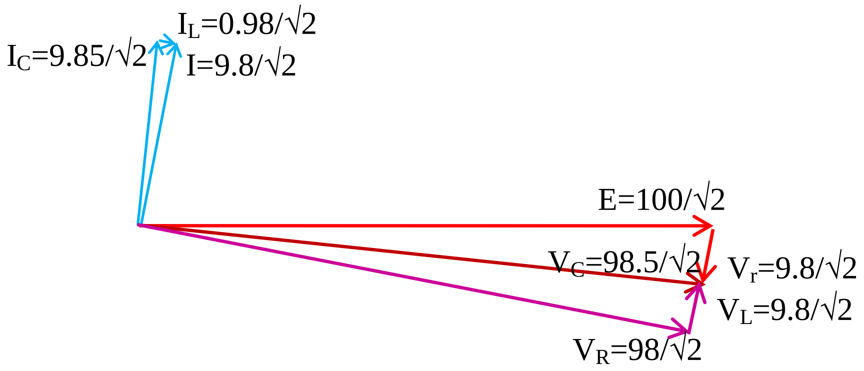
$$V_{Cm} = I_{Lm}(R + j\omega L) = \frac{j100}{100 + j20}\times(100 + j1000\times 0.01)$$

$= 0.9806\angle -11.31^o \times 100.5\angle 5.71^o = 98.55\angle -5.60^o$

1 mark

$v_C(t) = 98.55 \sin(1000t - 5.60^o)$

(c)



2 marks

(d) Energy stored $W = ½ L i^2 + ½ C v^2$

$= ½×0.01×0.9806^2×\sin^2(1000t – 11.31^o) + ½×10^{-4}×98.55^2×\sin^2(1000t – 5.59^o)$ **1 mark**

Maximum occurs at t = 1.67 ms when the two terms have values 0.00476 and 0.48560 **1 mark**

Maximum Energy stored = 0.490 J

(e) Energy loss per cycle = $[(9.8/\sqrt{2})^2×1+(0.98/\sqrt{2})^2×100]×2\pi/1000 = 0.604$ J **1 mark**

Q-factor of circuit = 2π× Maximum Energy stored/ Energy loss per cycle

$= 2\pi×0.49/0.604 = 5.1$ **1 mark**

(f) Resonance frequency = ω_0

$$Z = r + \frac{R + j\omega L}{1 + j\omega C(R + j\omega L)} = r + \frac{R + j\omega L}{1 - \omega^2 LC + j\omega CR}$$

Considering unity power factor resonance, imaginary part = 0

$$\therefore \frac{R}{1-\omega^2 LC} = \frac{\omega L}{\omega CR} \text{ gives } 1-\omega^2 LC = \frac{R^2 C}{L}, \quad \omega = \sqrt{\frac{L - R^2 C}{L^2 C}}$$ **1 mark**

$$\omega_o = \sqrt{\frac{0.01-100^2 \times 10^{-4}}{0.01^2 \times 10^{-4}}} = \sqrt{\frac{0.01-100^2 \times 10^{-4}}{0.01^2 \times 10^{-4}}} = \text{imaginary.}$$

Thus resonance does not occur for the particular values of components.

2. (a)

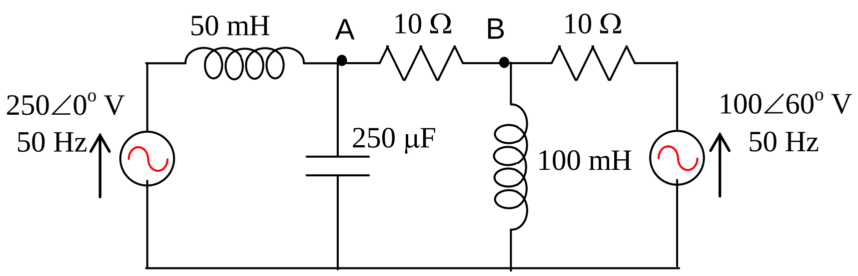


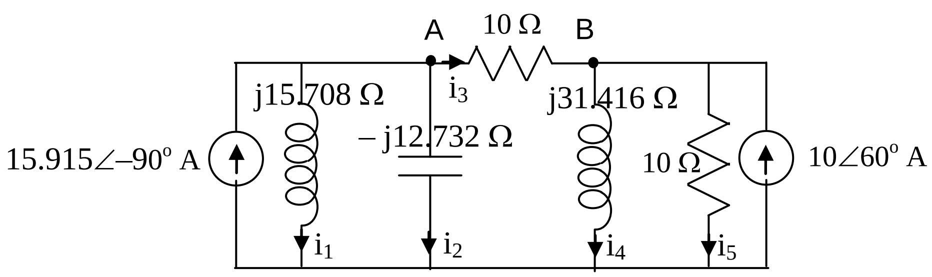
Figure Q2

50 mH → 2π×50×0.05 → j 15.708 Ω, 100 mH → j 31.416 Ω

250 μF → $1/(2\pi×50×250×10^{-6})$ → – j 12.732 Ω

Equivalent current sources are 250∠0°/ j15.708 = 15.915∠–90° A, 100∠60°/10 = 10∠60° A

The converted circuit is as follows



2 marks

(b) Taking the common node as reference and eliminating it from equations

The branch-node incidence matrix and the branch admittance matrix are given by

1 mark

$$\begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 1 & -1 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}, \quad [Y_b] = \begin{bmatrix} -j0.0637 & 0 & 0 & 0 & 0 \\ 0 & j0.0785 & 0 & 0 & 0 \\ 0 & 0 & 0.1 & 0 & 0 \\ 0 & 0 & 0 & -j0.0318 & 0 \\ 0 & 0 & 0 & 0 & 0.1 \end{bmatrix}$$

1 mark

(c) The nodal admittance matrix is given by

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} -j0.0637 & 0 & 0 & 0 & 0 \\ 0 & j0.0785 & 0 & 0 & 0 \\ 0 & 0 & 0.1 & 0 & 0 \\ 0 & 0 & 0 & -j0.0318 & 0 \\ 0 & 0 & 0 & 0 & 0.1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 1 & -1 \\ 0 & 1 \\ 0 & 1 \end{bmatrix} =$$

1 mark

$$[Y_N] = \begin{bmatrix} -j0.0637 + j0.0785 + 0.1 & -0.1 \\ -0.1 & 0.1 - j0.0318 + 0.1 \end{bmatrix}$$

1 mark

(d) The nodal current vector is given by

$$[I_N] = \begin{bmatrix} 15.915\angle -90^0 \\ 10\angle 60^0 \end{bmatrix}$$

$$\begin{bmatrix} 15.915\angle -90^0 \\ 10\angle 60^0 \end{bmatrix} = \begin{bmatrix} -j0.0637 + j0.0785 + 0.1 & -0.1 \\ -0.1 & 0.01 - j0.0318 + 0.1 \end{bmatrix} \begin{bmatrix} V_A \\ V_B \end{bmatrix}$$

$$\begin{bmatrix} 15.915\angle -90^0 \\ 10\angle 60^0 \end{bmatrix} = \begin{bmatrix} 0.1 + j0.0148 & -0.1 \\ -0.1 & 0.2 - j0.0318 \end{bmatrix} \begin{bmatrix} V_A \\ V_B \end{bmatrix}$$

2 marks

Determinant = 0.02 + 0.000470 – j0.00318 + j0.00296 – 0.01 = 0.010470 – j0.00022

$\Delta = 0.0105\angle -1.3^o$

$$\begin{bmatrix} V_A \\ V_B \end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} 0.2 - j0.0318 & 0.1 \\ 0.1 & 0.1 + j0.0148 \end{bmatrix} \begin{bmatrix} 15.915\angle -90^0 \\ 10\angle 60^0 \end{bmatrix}$$

$V_A = 221.2\angle -89.0^o$

2 marks

$V_B = 71.6\angle -59.2^o$

3. (a)

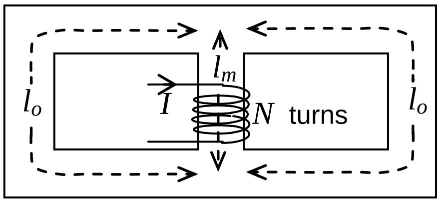


Figure Q3a

N =100 turns, A = 4 cm^2, l_o = 15 cm, l_m = 4 cm, μ_r = 1000

Reluctance $S = l/\mu A$

$$S_0 = \frac{0.15}{4\pi \times 10^{-7} \times 1000 \times 4 \times 10^{-4}} = 298.4\times10^3 \text{ H}^{-1}$$

$$S_m = \frac{0.04}{4\pi \times 10^{-7} \times 1000 \times 4 \times 10^{-4}} = 79.6\times10^3 \text{ H}^{-1}$$

Effective reluctance of magnetic circuit as seen from coil $= 79.6\times10^3 + 298.4\times10^3/2$
$= 228.8\times10^3 \text{ H}^{-1}$

Thus, effective inductance = $N^2/S = 100^2/228.8\times10^3$ = 0.0437 H = 43.7 mH   **3 marks**

(b)

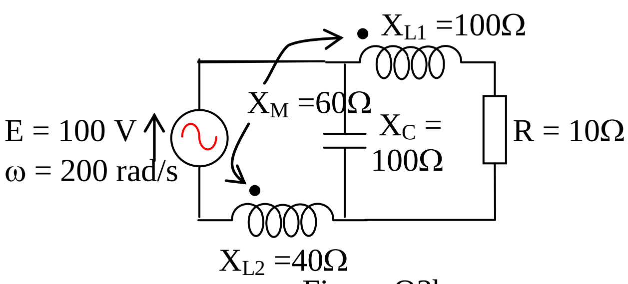
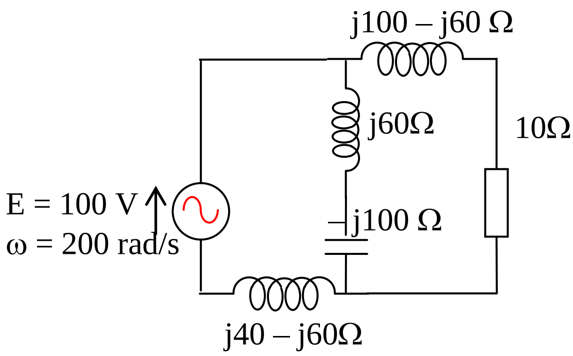


Figure Q3b

The non-coupled equivalent circuit is as follows



The current supplied from the source is given as

$$I_S = \frac{100\angle 0^0}{-j20 + (-j40)//(10+j40)} \qquad I_S = \frac{100\angle 0^0}{-j20 + \dfrac{(-j40)\times(10+j40)}{-j40+10+j40}} = \frac{100\angle 0^0}{-j20 - j40 + 160}$$

$$I_S = \frac{100\angle 0^0}{160 - j60} = \frac{5\angle 0^0}{8 - j3} = \frac{5\angle 0^0}{8.544\angle -20.556^0} = 0.585\angle 20.556^0 \text{ A}$$

3 marks

(c)

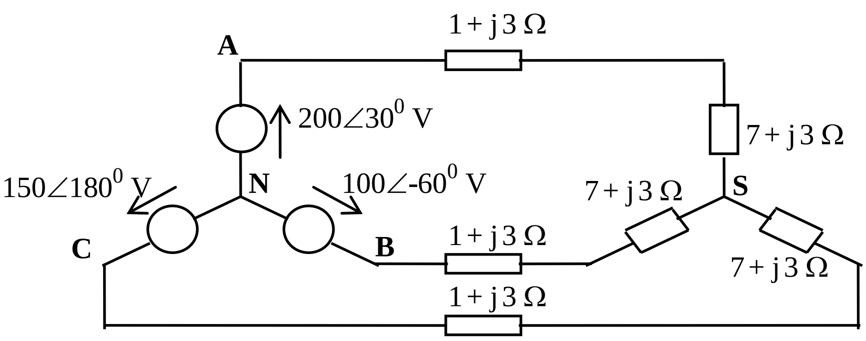


Figure Q3c

Considering the potential of the neutral N as zero, from Millmann's theorem

$$V_S = V_{SN} = \frac{\sum[Y][V]}{\sum[Y]} = \frac{\frac{1}{8+j6}\times 200\angle 30^0 + \frac{1}{8+j6}\times 100\angle -60^0 + \frac{1}{8+j6}\times 150\angle 180^0}{\frac{1}{8+j6}+\frac{1}{8+j6}+\frac{1}{8+j6}}$$

$$V_S = \frac{200\angle 30^0 + 100\angle -60^0 + 150\angle 180^0}{3} = \frac{173.2 + j100 + 50 - j86.6 - 150}{3} = 24.4 + j4.47$$

$V_s = 24.8\angle 10.4^0$ V

4 marks

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4. (a) A = D = (1 + j 0.1), B = j 10 Ω, and C = (0.02 + j 0.001) S.

Considering the direction of I_2 corresponding to impedance parameters,

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} = \begin{bmatrix} 1+j0.1 & j10 \\ 0.02+j0.001 & 1+j0.1 \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}, \text{ and } \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Expansion gives

$V_1 = (1+j0.1)V_2 - j10\, I_2$, $I_1 = (0.02+j0.001)V_2 - (1+j0.1)\, I_2$

Re-arranging gives

$(0.02+j0.001)V_2 = I_1 + (1+j0.1)\, I_2$

Giving $z_{21} = 1/C = 1/(0.02+j0.001) = 49.94\angle -2.86^0$,

$z_{22} = D/C = (1+j0.1)/(0.02+j0.001) = 50.19\angle 8.57^0$

Also

$V_1 = (1+j0.1)[1/(0.02+j0.001)\times I_1 + (1+j0.1)/(0.02+j0.001)\times I_2 - j10\, I_2$

Giving $z_{11} = A/C = (1+j0.1)/(0.02+j0.001) = 50.19\angle 8.57^o$,

$z_{21} = -B + AD/C = 49.94\angle -2.86^0$

$$[Z] = \begin{bmatrix} 50.19\angle 8.57^0 & 49.94\angle -2.86^0 \\ 49.94\angle -2.86^0 & 50.19\angle 8.57^0 \end{bmatrix}$$

3 marks

(b) For the characteristic impedance Z_0, $V_1 = Z_0 \times I_1$, $V_2 = Z_0 \times (-I_2)$

i.e. $$\begin{bmatrix} Z_0 I_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 1+j0.1 & j10 \\ 0.02+j0.001 & 1+j0.1 \end{bmatrix} \begin{bmatrix} -Z_0 I_2 \\ -I_2 \end{bmatrix}$$

$$\therefore \frac{Z_0 I_1}{I_1} = \frac{(1+j0.1)(-Z_0 I_2) + j10(-I_2)}{(0.02+j0.001)(-Z_0 I_2) + (1+j0.1)(-I_2)}$$

i.e. $$Z_0 = \frac{(1+j0.1)Z_0 + j10}{(0.02+j0.001)Z_0 + (1+j0.1)}$$

$$(0.02+j0.001)Z_0^2 + (1+j0.1)Z_0 = (1+j0.1)Z_0 + j10$$

$$Z_0^2 = \frac{1}{(0.0001 - j0.002)} = \frac{1}{0.0020025\angle -87.14^0} = 499.4\angle 87.14^0,\ Z_0 = 22.35\angle 43.57^0\ \Omega$$ **2 marks**

(c)

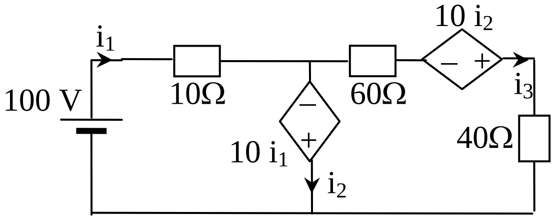

Figure Q4b

$100 = 10\ i_1 - 10\ i_1$, $i_1 = i_2 + i_3$, $-10\ i_1 = (60+40)\ i_3 - 10\ i_2$ **2 marks**

(d) Solution is not possible. **3 marks**

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5. 3 phase, 400 V, 50 Hz, balanced supply.

Balanced star connected load R= 100 Ω and L= 150 mH in series.

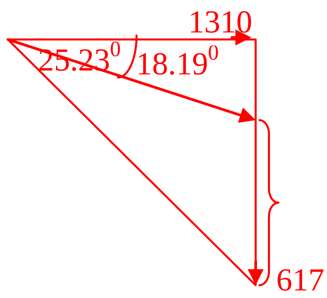
(a) 150 mH → 100π×0.15 → j 47.12 Ω

Line current = $(400\angle 0^0/\sqrt{3})/(100+j47.12) = 230.9/110.55\angle 25.23^0 = 2.089\angle -25.23^0$ A

Power factor of the load = $\cos 25.23^0 = 0.905$ lag

Total active power supplied to load = $\sqrt{3}\times 400\times 2.089\times 0.905 = 1310$ W **3 marks**

(b) Value of 3 capacitors in delta to increase power factor 0.95 lag.



P remains unchanged at 1310 W

Original Q = $\sqrt{3}\times 400\times 2.089\times \sin 25.23^0 = 617$ var

New power factor angle corresponding to 0.95 lag = $\cos^{-1}0.95 = 18.19^0$

New reactive power = $1310\times \tan 18.19^0 = 430$ kvar

$\therefore$ reactive power supplied by capacitors = 617 – 430 = 187 kvar

$\therefore\ 3\times C\times 100\pi\times 400^2 = 187$

C = 1.24 μF in each arm of the delta **2 marks**

Phase sequence ABC,

$I_{A0} = 1.0\angle 90^o$A, $I_{A1} = 10.0\angle 0^o$A and $I_{A2} = 3.0\angle -90^o$A, and

$V_{AN} = 200\angle 0^o$ V, $V_{BN} = 100\angle -60^o$ V and $V_{CN} = 200\angle 120^o$ V.

(c) The Symmetrical Components of the phase “A” voltage are given by

$$\begin{bmatrix} V_{A0} \\ V_{A1} \\ V_{A2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}\begin{bmatrix} 200\angle 0^0 \\ 100\angle -60^0 \\ 200\angle 120^0 \end{bmatrix}$$

V_{A0} = 1/3 ($200\angle 0^o$ + $100\angle{-60^o}$ + $200\angle 120^o$)
= (200 + 50 – j 86.6 – 100 + j 173.2)/3 = 50 + j 28.87 = $57.74\angle 30^o$ V

V_{A1} = 1/3 ($200\angle 0^o$ + $100\angle(120^o-60^o)$ + $200\angle(120^o+240^o)$)
= (200 + 50 + j 86.6 + 200)/3 = 150 + j 28.87 = $152.75\angle 10.89^o$ V

V_{A2} = 1/3 ($200\angle 0^o$ + $100\angle(240^o-60^o)$ + $200\angle(120^o+120^o)$)
= (200 – 100 – 100 – j 173.2)/3 = – j 57.74 = $57.74\angle{-90^o}$ V **3 marks**

(d) Power associated with each sequence component.

$P_0 = 3\times57.74\times1.0\times\cos 60^o = 86.6$ W
$P_1 = 3\times152.75\times10.0\times\cos 10.89^o = 4500$ W
$P_2 = 3\times57.74\times3.0\times\cos 0^o = 519.6$ W **2 marks**

6.

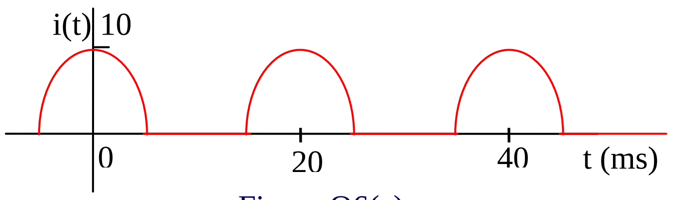


Figure Q6(a)

Period = 20 ms = 0.02 s, ω = 2π/0.02 = 100π rad/s

(a) i(t) = 10 cos100πt   –5ms ≤ t ≤ 5 ms
= 0   10ms ≤ t ≤ 15 ms

rms value = $\sqrt{\frac{2}{0.02}\int_0^{0.005} 10^2\cos^2 100\pi t.dt} = \sqrt{\frac{100^2}{2}\int_0^{0.005}(1+\cos 200\pi t).dt}$

$= \sqrt{\frac{100^2}{2}\left(t+\frac{\sin 200\pi t}{200\pi}\right)\Bigg|_0^{0.005}} = \sqrt{\frac{100^2}{2}\left(0.005+\frac{\sin\pi}{200\pi}\right)} = 5$ A **2 marks**

(b) f(t) = $A_0/2 + \sum A_n \cos n\omega t + B_n \sin n\omega t$
function f(t) has mean value = 10/π → $A_0/2$ = 3.18 A **1 mark**
f(t) is even → B_n = 0 for all n

$\therefore A_n = 2\times\frac{2}{0.02}\int_0^{0.005} 10\cos 100\pi t.\cos n100\pi t.dt + 0$

$= 1000\int_0^{0.005}[\cos 100\pi(n+1)t + \cos n100\pi(n-1)t).dt$

$$A_n = 1000 \times \left[\frac{\sin 100\pi(n+1)t}{100\pi(n+1)} + \frac{\sin 100\pi(n-1)t}{100\pi(n-1)} \right]_0^{0.005}$$

$$= 10 \times \left[\frac{\sin(n+1)\frac{\pi}{2}}{(n+1)\pi} + \frac{\sin(n-1)\frac{\pi}{2})}{(n-1)\pi} \right]$$

2 marks

Cannot be calculated for n = 1.

$$A_1 = 2 \times \frac{2}{0.02} \int_0^{0.005} 10\cos 100\pi t . \cos 100\pi t . dt = 1000 \int_0^{0.005} (1 + \cos 200\pi t).dt = 1000 \times \left[t + \frac{\sin 200\pi t}{200\pi} \right]_0^{0.005}$$

A_1 = 1000[0.005+0] = 5

$$A_2 = 10 \times \left[\frac{\sin\frac{3\pi}{2}}{3\pi} + \frac{\sin\frac{\pi}{2}}{\pi} \right] = \frac{10}{3\pi}(-1+3) = \frac{20}{3\pi} = 2.122,$$

$$A_3 = 10 \times \left[\frac{\sin\frac{4\pi}{2}}{4\pi} + \frac{\sin\frac{2\pi}{2}}{2\pi} \right] = 0,$$

$$A_4 = 10 \times \left[\frac{\sin\frac{5\pi}{2}}{5\pi} + \frac{\sin\frac{3\pi}{2}}{3\pi} \right] = \frac{10}{15\pi}(3-5) = -\frac{4}{3\pi} = -0.424$$

1 mark

∴ i(t) = 3.18 + 5 cos 25πt + 2.122 cos 50πt – 0.424 cos 100πt + ...

(c)

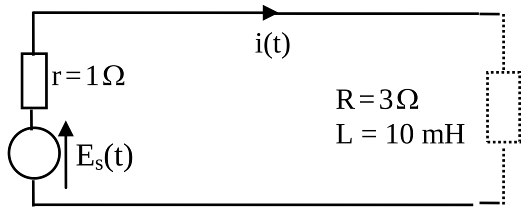


Figure Q6(b)

$E_s(t) = 4\, i(t) + 0.01\, di(t)/dt$

∴ E_s (t) = 12.72 + 20 cos 25πt + 8.488 cos 50πt – 1.796 cos 100πt + ...

+ 1.25 sin 25πt + 1.061 sin 50πt – 0.424 cos 100πt + ...

3 mark

Average power supplied = 12.72× 3.18 + (20×5 + 8.488×2.122 + 1.796×0.424 + ….)/(√2×√2)

= 40.45 + 50 + 0.52 + 0.38 + … = 91.5 W

1 mark

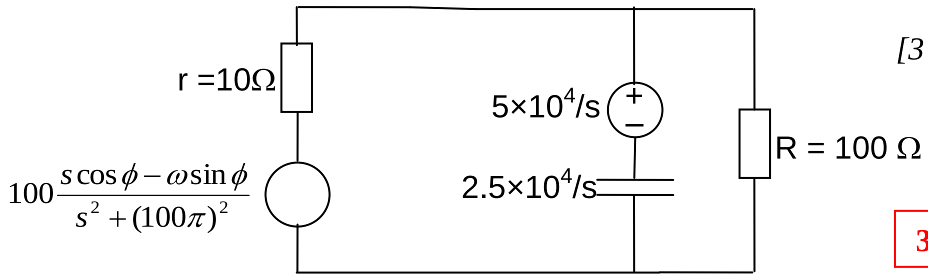
EE 2010 - THEORY OF ELECTRICITY – Short Answers

7. (a) Derivation from first principles of Laplace transform of causal waveform cos(ωt+ϕ)

$$L[\cos(\omega t+\phi).H(t)] = \frac{s}{s^2+\omega^2}\cos\phi - \frac{\omega}{s^2+\omega^2}\sin\phi$$

3 marks

(b)



[3 marks]

3 marks

q_0 = 2 coulomb, E_m = 100 kV, f = 50 Hz, r = 10 Ω, R = 100 Ω, C = 40 μF

Since ϕ is not specified, assuming it as 0^o, ω = 100π

(c) Laplace transform for the current supplied from the source can be obtained from superposition.

$$I(s) = \frac{\dfrac{100s}{s^2+(100\pi)^2}}{10+\dfrac{100\times\dfrac{2.5\times10^4}{s}}{100+\dfrac{2.5\times10^4}{s}}} - \frac{\dfrac{100}{110}\times\dfrac{5\times10^4}{s}}{\dfrac{2.5\times10^4}{s}+\dfrac{100\times10}{100+10}}$$

$$I(s) = \frac{100s}{s^2+(100\pi)^2}\times\frac{100s+2.5\times10^4}{10(100s+2.5\times10^4)+250\times10^4} - \frac{500\times10^4}{275\times10^4+1000s} -$$

$$= \frac{10s^2+2500s-5000s^2-50\times10^6\pi^2}{(s^2+100^2\pi^2)(s+2750)} = \frac{9.10s-29.1}{(s^2+100^2\pi^2)} - \frac{4999}{(s+2750)}$$

2 marks

(d) Time domain solutions for the current through the Resistor R

i(t) = 9.10 cos 100πt – (29.1/100π) sin 100πt – 4999 e^{-2750t}

i(t) = 9.10 cos 100πt – 0.093 sin 100πt – 4999 e^{-2750t}

2 marks