



EE 2010 - THEORY OF ELECTRICITY – Short Answers

Level 2 (Repeat) Examination – November 2011

1. $e(t) = 150 \sin 100\pi t$ volt.

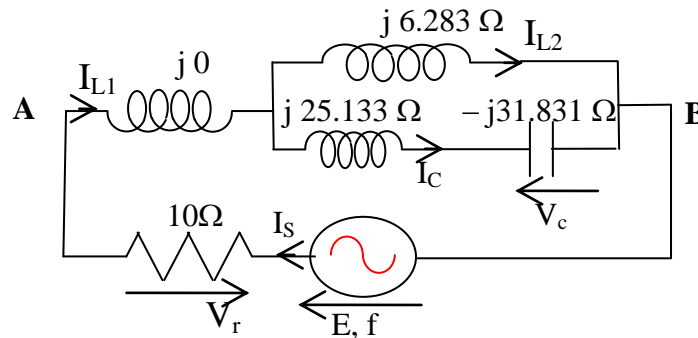
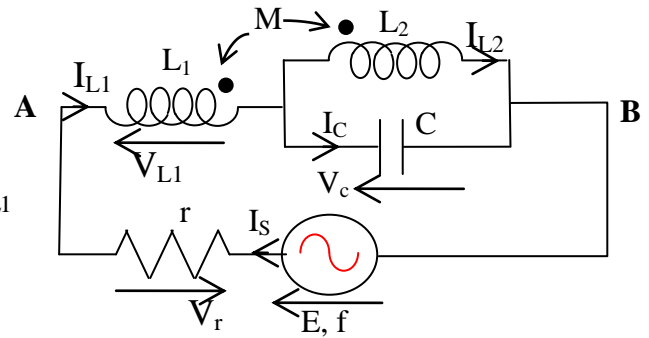
(a) Voltage drop across AB = V_{AB}

$$j\omega L_1 \cdot I_{L1} - j\omega M \cdot I_{L2} + j\omega L_2 \cdot I_{L2} - j\omega M \cdot I_{L1}$$

$$= j100\pi \times 0.08 \times I_{L1} - j100\pi \times 0.08 \times I_{L2}$$

$$+ j100\pi \times 0.10 \times I_{L2} - j100\pi \times 0.08 \times I_{L1}$$

$$= -j100\pi \times 0.08 \times I_{L2} + j100\pi \times 0.10 \times I_{L2} = -j25.133 I_{L2} + j31.416 I_{L2} = j6.283 I_{L2} \quad [1 \text{ mark}]$$



(b) $Z_{AB} = j6.283 // (j25.133 - j31.831) = \frac{j6.283 \times (-j6.698)}{j6.283 - j6.698} = j101.4 \Omega$ [2 marks]

(c) $I_s = \frac{150/\sqrt{2}}{40 + j101.4} = \frac{106.1}{109.0} \angle -68.47^\circ = 0.973 \angle -68.47^\circ \text{ A}$ [1 mark]

(d) $I_{L2} = 0.973 \angle -68.47^\circ \times \frac{-j6.698}{j6.283 - j6.698} = 15.70 \angle -68.47^\circ \text{ A}$

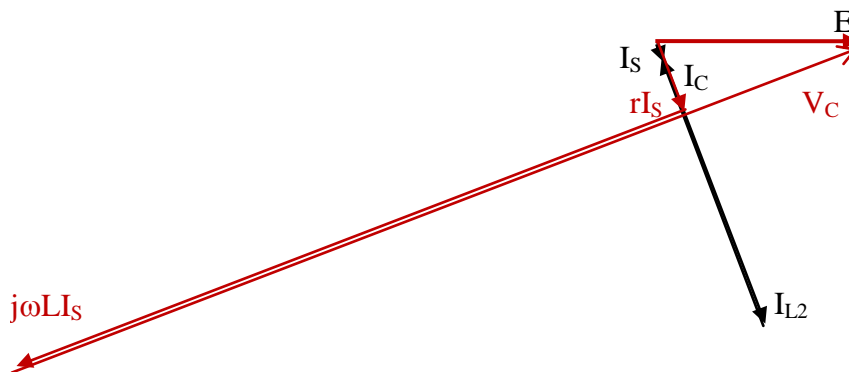
$I_C = 0.973 \angle -68.47^\circ - 15.70 \angle -68.47^\circ = 14.73 \angle 111.53^\circ \text{ A}$ [2 marks]

(e) $V_r = 40 \times I_s = 38.92 \angle -68.47^\circ \text{ V}$

$V_{L1} = j25.13 I_{L1} - j25.13 I_{L2} = j25.13 (0.973 \angle -68.47^\circ - 15.70 \angle -68.47^\circ)$
 $= -25.13 \times 14.73 \angle 21.53^\circ = 370.2 \angle -158.47^\circ \text{ V}$

$V_C = -j31.83 I_C = -j31.83 \times 14.70 \angle 111.4^\circ = 467.9 \angle 21.4^\circ \text{ V}$ [2 marks]

(f) phasor diagram showing all the voltages and currents in the circuit.



[2 marks]



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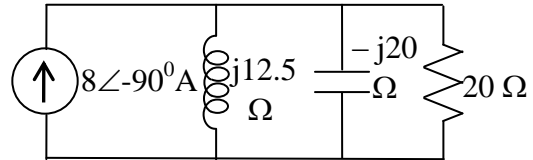
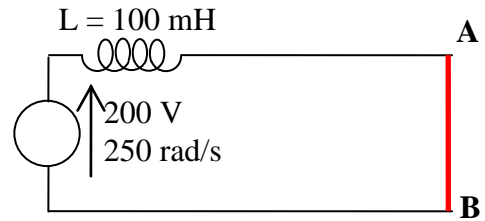
2. (a) $\frac{1}{j\omega C} = \frac{1}{j250 \times 200 \times 10^{-6}} = -j20 \Omega$

$j\omega L = j250 \times 0.05 = j12.5 \Omega$

$I_N = \frac{100}{j12.5} = -j8 \text{ A}$

$Y_N = \frac{1}{j12.5} = -j0.08 \text{ S}$ or $Z_N = j12.5 \Omega$

The Norton's equivalent circuit thus becomes



[2 marks]

voltage across R = $\frac{8 \angle -90^\circ}{-j0.08 + j0.05 + 0.05} = \frac{8 \angle -90^\circ}{0.05 - j0.03} = \frac{8 \angle -90^\circ}{0.0583 \angle -30.96^\circ}$

voltage across R = $137.2 \angle -59.04^\circ \text{ V}$

[1 mark]

power delivered to resistor = $137.2^2 / 20 = 941.5 \text{ W}$

[1 mark]

(b)

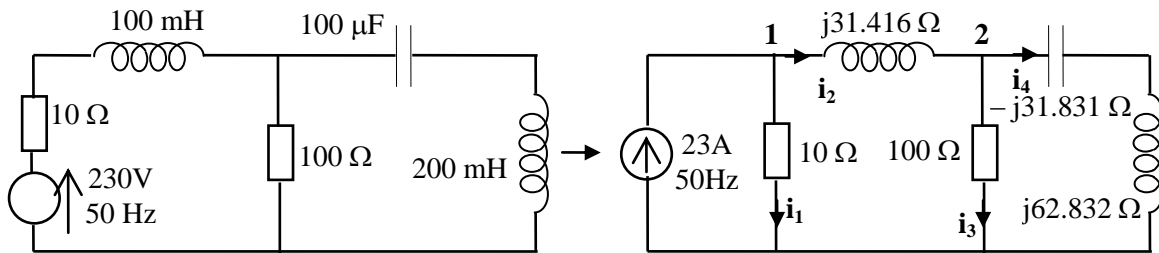


Figure Q2(b)

[1 mark]

(c) branch-node incidence matrix

$$[A] = \begin{bmatrix} 1 & 0 \\ 1 & -1 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}$$

[1 mark]

branch admittance matrix

$$[Y_b] = \begin{bmatrix} 0.10 & 0 & 0 & 0 \\ 0 & -j0.03183 & 0 & 0 \\ 0 & 0 & 0.01 & 0 \\ 0 & 0 & 0 & -j0.03226 \end{bmatrix}$$

[1 mark]

(d) nodal admittance matrix

$$[Y_N] = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0.10 & 0 & 0 & 0 \\ 0 & -j0.03183 & 0 & 0 \\ 0 & 0 & 0.01 & 0 \\ 0 & 0 & 0 & -j0.03226 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & -1 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}$$



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$$[Y_N] = \begin{bmatrix} 0.10 - j0.03183 & j0.03183 \\ j0.03183 & 0.01 - j0.06409 \end{bmatrix}$$

[1 mark]

(e) obtain the node voltages

$$\begin{bmatrix} 23 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.10 - j0.03183 & j0.03183 \\ j0.03183 & 0.01 - j0.06409 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$\begin{aligned} \text{Discriminant } \Delta &= (0.10 - j0.03183)(0.01 - j0.06409) - (j0.03183)^2 \\ &= (0.001 - 0.00204 + 0.00101) + j(-0.006409 - 0.003183) \\ &= -0.00003 - j0.009592 \approx -j0.009592 \end{aligned}$$

$$\begin{aligned} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} &= \frac{1}{-j0.009592} \begin{bmatrix} 0.10 - j0.03183 & j0.03183 \\ j0.03183 & 0.01 - j0.06409 \end{bmatrix} \begin{bmatrix} 23 \\ 0 \end{bmatrix} \\ &= \frac{23}{0.009592 \angle -90^\circ} \begin{bmatrix} 0.1049 \angle -17.66^\circ \\ 0.03183 \angle 90^\circ \end{bmatrix} = \begin{bmatrix} 251.5 \angle 72.34^\circ \\ 76.3 \angle 180^\circ \end{bmatrix} \end{aligned}$$

$$V_1 = 251.5 \angle 72.3^\circ \text{ V}$$

$$V_2 = 76.3 \angle 180^\circ \text{ V}$$

[2 marks]

3. (a) Admittance parameter matrix

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$y_{11} = \left. \frac{I_1}{V_1} \right|_{V_2=0} = \frac{1}{5} + \frac{1}{3+j4}$$

$$= 0.2 + 0.12 - j0.16 = 0.32 - j0.16 = 0.3578 \angle -26.57^\circ \text{ S}$$

$$y_{21} = \left. \frac{I_2}{V_1} \right|_{V_2=0} = \frac{I_1}{V_1} \times \frac{I_2}{I_1} \bigg|_{V_2=0} = -\frac{5}{8+j4} \times 0.3578 \angle -26.57^\circ = \frac{1.789 \angle 153.43^\circ}{8.944 \angle 26.57^\circ}$$

$$= 0.200 \angle 126.86^\circ \text{ S}$$

$$\begin{aligned} y_{12} &= y_{21} = 0.200 \angle 126.86^\circ \text{ S} \\ y_{22} &= y_{11} = 0.3578 \angle -26.57^\circ \text{ S} \end{aligned} \quad \begin{bmatrix} 0.3578 \angle -26.6^\circ & 0.200 \angle 126.9^\circ \\ 0.200 \angle 126.9^\circ & 0.3578 \angle -26.6^\circ \end{bmatrix}$$

[3 marks]

(b) Kirchoff's Law equations are

$$-60 = 10 i_1 + (20+40) (i_1 - 0.1 v_2) - 10 i_1$$

$$i_1 - 0.1 v_2 = v_2/40$$

Simplification gives

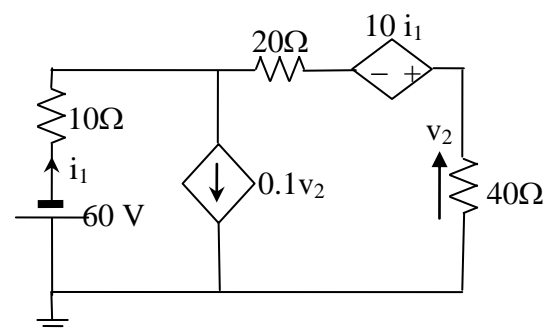
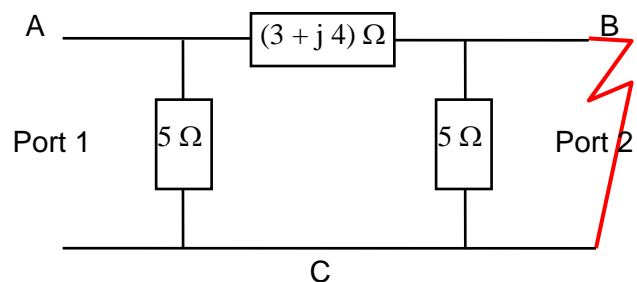
$$i_1 - 0.1 v_2 = -1 \text{ and } i_1 = 0.125 v_2$$

elimination of v_2 gives

$$i_1 - 0.8 i_1 = -1 \rightarrow i_1 = -5 \text{ A}$$

[2 marks]

[1 mark]



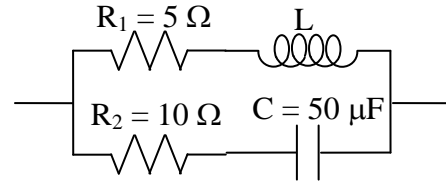


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- (c) Resonance frequency = 50 Hz
Effective impedance is given by

$$Z = Z_1 // Z_2 = \frac{Z_1 \cdot Z_2}{Z_1 + Z_2}$$

$$= \frac{(5 + jX)(10 - j63.662)}{5 + jX + 10 - j63.662} = \frac{(50 + 63.662X) + j(10X - 318.31)}{15 + j(X - 63.662)}$$



Considering unity power factor resonance, $Z = \text{real}$

$$\therefore \frac{(50 + 63.662X)}{15} = \frac{(10X - 318.31)}{(X - 63.662)}$$

$$\text{i.e. } 50X - 3183.1 - 4052.9X + 63.662X^2 = 150X - 4775$$

$$\text{i.e. } 63.662X^2 + (50 - 4052.9 - 150)X + 1592 = 0$$

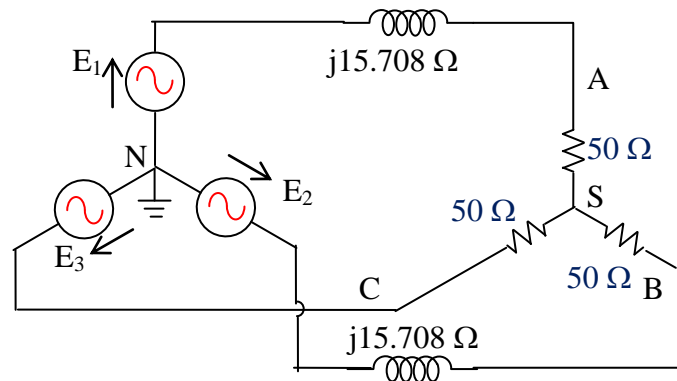
$$X = \frac{4153 \pm \sqrt{4153^2 - 4 \times 63.662 \times 1592}}{2 \times 63.662} = \frac{4153 \pm 4104}{127.324} = 64.85 \text{ or } 0.385$$

$$L = 206.4 \text{ mH or } 1.225 \text{ mH}$$

[4 marks]

4. (a) Using Millmann's theorem

$$V_{SN} = \frac{\sum Y.V}{\sum Y}$$



$$V_{SN} = \frac{\frac{1}{50 + j15.708} \times 230 \angle 0^\circ + \frac{1}{50 + j15.708} \times 230 \angle -120^\circ + \frac{1}{50} \times 230 \angle 120^\circ}{\frac{1}{50 + j15.708} + \frac{1}{50 + j15.708} + \frac{1}{50}}$$

$$= \frac{\frac{1}{52.41 \angle 17.44^\circ} \times 230 \angle 0^\circ + \frac{1}{52.41 \angle 17.44^\circ} \times 230 \angle -120^\circ + \frac{1}{50} \times 230 \angle 120^\circ}{\frac{1}{52.41 \angle 17.44^\circ} + \frac{1}{52.41 \angle 17.44^\circ} + \frac{1}{50}}$$

$$= \frac{4.388 \angle -17.44^\circ + 4.388 \angle -137.44^\circ + 4.6 \angle 120^\circ}{0.01907 \angle -17.44^\circ + 0.01907 \angle -17.44^\circ + 0.02}$$

$$= \frac{(4.186 - j1.315) + (-3.232 - j2.968) + (-2.3 + j3.984)}{2 \times (0.01819 - j0.0057) + 0.02} = \frac{-1.346 - j0.299}{0.05638 - j0.0114}$$

$$= \frac{1.379 \angle -167.5^\circ}{0.0575 \angle -11.4^\circ} = 23.97 \angle -156.1^\circ \text{ V}$$

[3 marks]



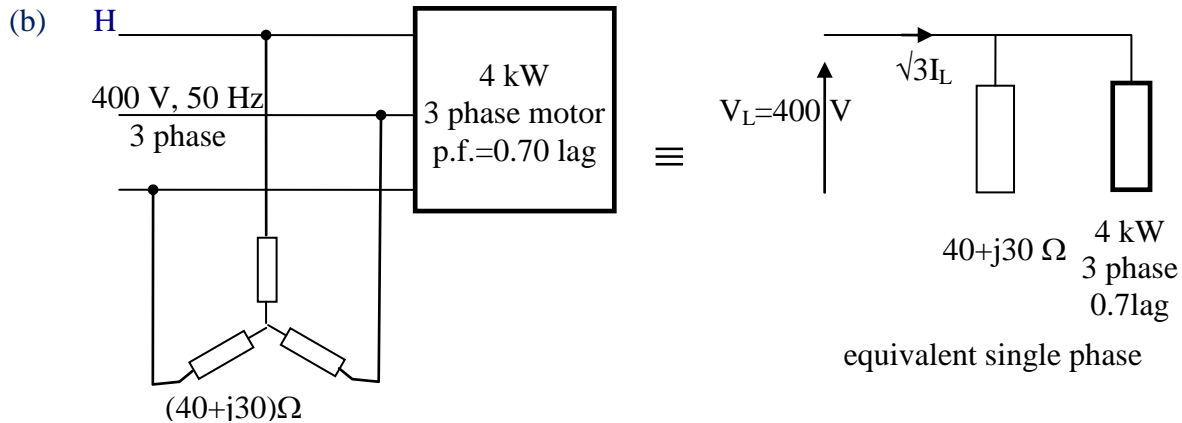
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$$\text{Thus current through } R_1 = \frac{230\angle 0^\circ - 23.97\angle -156.1^\circ}{50 + j15.708} = \frac{230 - 21.85 - j9.68}{50 + j15.708}$$

$$= \frac{208.15 - j9.68}{50 + j15.708} = \frac{208.38\angle -2.66^\circ}{52.41\angle 17.44^\circ} = 3.975\angle -20.10^\circ \text{ A}$$

$$\text{Potential of A} = 230\angle 0^\circ - 3.975\angle -20.1^\circ \times j15.708 = 230 - 62.44\angle 69.9^\circ$$

$$= 230 - 21.458 + j58.637 = 208.54 + j58.637 = 216.6\angle 15.70^\circ \text{ V} \quad [1 \text{ mark}]$$



$$\therefore \sqrt{3}I_L \text{ for star connected load} = 400/(40 + j30) = 8\angle -36.87^\circ$$

$$\text{and } \sqrt{3}I_L \text{ for motor load} \rightarrow 4000/400 = 10 \text{ at lagging p.f. of } 0.7 \rightarrow 5\angle -45.57^\circ$$

$$\therefore \text{total } \sqrt{3}I_L \text{ of load} = 8\angle -36.87^\circ + 10\angle -45.57^\circ = 6.4 - j4.8 + 7.0 - j7.14 = 13.4 - j11.94$$

$$\therefore \text{line current supplied from supply} = (13.4 - j11.94)/\sqrt{3} = 10.36\angle -41.70^\circ \text{ A}$$

$$\text{Power factor} = \cos 41.70^\circ = 0.747 \text{ lag}$$

$$\text{Active power supplied} = \sqrt{3} \times 400 \times 10.36 \times 0.747 = 5362 = 5.36 \text{ kW} \quad [2 \text{ marks}]$$

$$\text{Reactive power supplied} = \sqrt{3} \times 400 \times 10.36 \times \sin(41.70^\circ) = 4.775 \text{ kvar}$$

$$\text{Reactive power requirement at } 0.95 \text{ lag} = 5.362 \times \tan(18.19^\circ) = 1.762 \text{ kvar}$$

$$\text{Rating of 3 phase capacitor bank} = 4.775 - 1.762 = 3.013 \text{ kvar} \quad [1 \text{ mark}]$$

$$\text{(or Rating of individual capacitor} = 3.013/3 = 1.004 \text{ kvar, } 1004/(400 \times 100\pi) = 19.98 \mu\text{F})$$

$$V_A = 200\angle 0^\circ \text{ A, } V_B = 150\angle 60^\circ \text{ A and } V_C = 200\angle -60^\circ \text{ A,}$$

$$\begin{bmatrix} V_{A0} \\ V_{A1} \\ V_{A2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix} \begin{bmatrix} 200\angle 0^\circ \\ 150\angle 60^\circ \\ 200\angle -60^\circ \end{bmatrix}$$

$$V_{A0} = (200\angle 0^\circ + 150\angle 60^\circ + 200\angle -60^\circ)/3 = (200 + 75 + j129.9 + 100 - j173.2)/3$$

$$= 125 - j14.4 = 125.8\angle -6.57^\circ \text{ V}$$

$$V_{A1} = (200\angle 0^\circ + 150\angle 60^\circ \times 1\angle 240^\circ + 200\angle -60^\circ \times 1\angle 120^\circ)/3 = (200 + 75 - 129.9 + 100 + j173.2)/3$$

$$= 125 + j14.4 = 125.8\angle 6.57^\circ \text{ V}$$

$$V_{A2} = (200\angle 0^\circ + 150\angle 60^\circ \times 1\angle 120^\circ + 200\angle -60^\circ \times 1\angle 240^\circ)/3 = (200 - 150 - 200)/3$$

$$= -50 = 50\angle 180^\circ \text{ V} \quad [3 \text{ marks}]$$



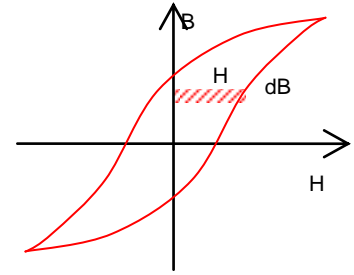
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5. (a)
$$P_h = \frac{1}{T} \int v.i.dt = f \cdot \int N \frac{d\phi}{dt} i.dt = f \cdot \int A.dB.H.l$$

$$= f \cdot \int H.dB \cdot \text{volume}$$

Hysteresis loss per unit volume $P_h \propto \int H.dB$
 = area of loop

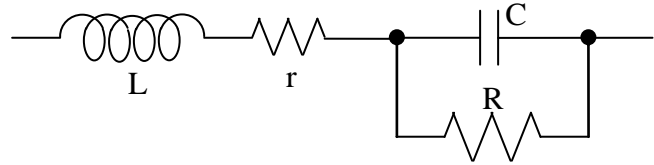
[2 marks]



(b) Eddy current loss per unit volume $P_e \propto \text{thickness}^2$

∴ reducing thickness, hence using small thickness sheets (laminations) drastically reduces the eddy current losses in the transformer core. [1 mark]

(c) Using the definition that resonance occurs when the impedance of the circuit is purely real (or power factor is unity),

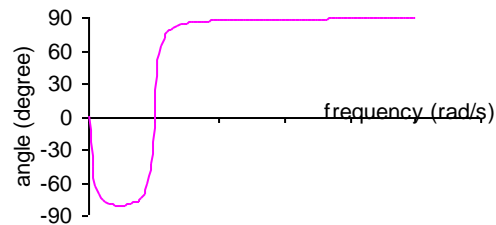
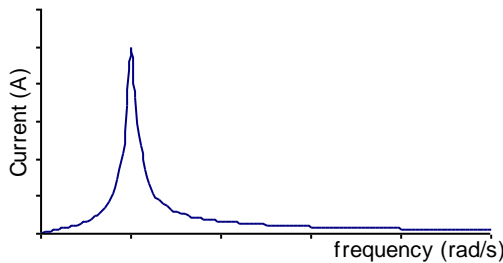


$$Z = j\omega L + r + \frac{R \cdot \frac{1}{j\omega C}}{R + \frac{1}{j\omega C}}$$

$$= j\omega L + r + \frac{R}{1 + j\omega CR} = \frac{R}{1 + \omega^2 C^2 R^2} + r + j\omega \left(L - \frac{CR^2}{1 + \omega^2 C^2 R^2} \right)$$

∴ at resonance $L = \frac{CR^2}{1 + \omega^2 C^2 R^2}$

[4 marks]



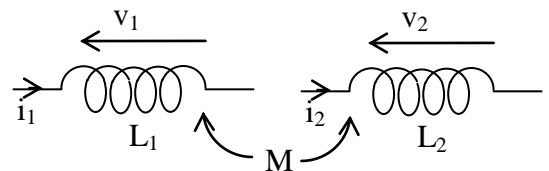
[1 mark]

(d) energy stored = $\int v_1.i_1 dt + \int v_2.i_2 dt$

$$= \int \left(L_1 \frac{di_1}{dt} \pm M \frac{di_2}{dt} \right) i_1 .dt + \int \left(L_2 \frac{di_2}{dt} \pm M \frac{di_1}{dt} \right) i_2 .dt$$

$$= \int L_1.i_1 d i_1 \pm (M.i_1 di_2 + M.i_2 di_1) + L_2.i_2 d i_2$$

$$= \frac{1}{2} L_1 i_1^2 \pm M.i_1 i_2 + \frac{1}{2} L_2 i_2^2$$



[2 marks]



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6.

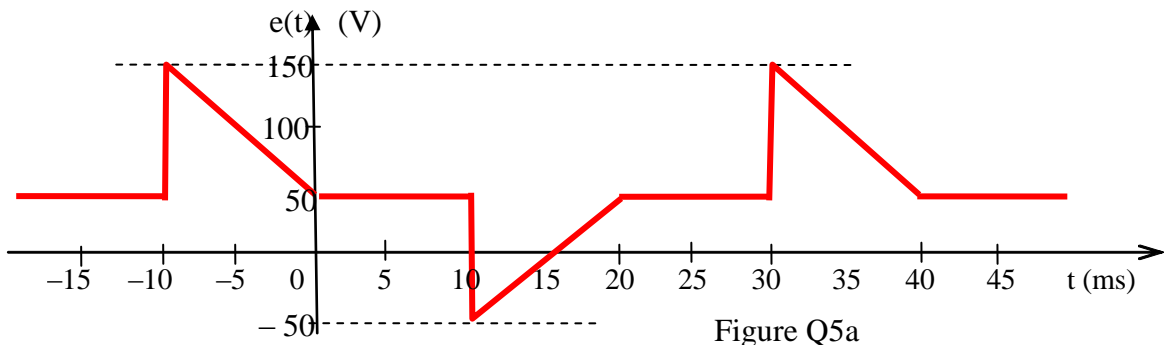
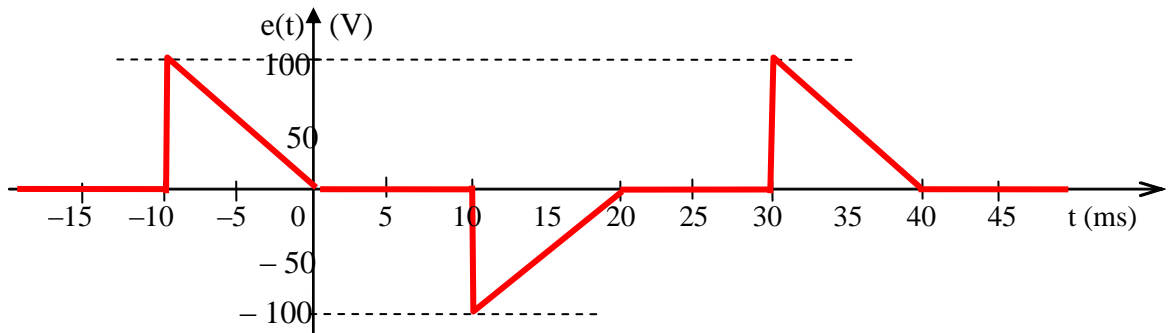


Figure Q5a

(a) Waveform has a mean value of 50 V.

Period $T = 40 \text{ ms}$, frequency $f = 1/40 \times 10^{-3} = 25 \text{ Hz}$, $\omega_0 = 2\pi \times 25 = 50\pi \text{ rad/s}$

If the waveform is shifted down by 50V, it has half-wave symmetry

Select $e(t) = e_1(t) + 50$.Modified waveform has $A_n, B_n = 0$ for all even n .The Fourier coefficients of $e_1(t)$ are calculated as

$$\begin{aligned}
 A_n &= 2 \times \frac{2}{0.04} \int_0^{0.02} e_1(t) \cdot \cos n\omega_0 t \cdot dt \\
 &= \frac{4}{0.04} \left[0 + \int_{0.01}^{0.02} (-200 + 10000t) \cdot \cos 50n\pi t \cdot dt \right] \\
 &= 100 \left[-\frac{200 \sin 50n\pi t}{50n\pi} \Big|_{0.01}^{0.02} + 10000 \left[\frac{\sin 50n\pi t}{50n\pi} \cdot t \Big|_{0.01}^{0.02} - \int_{0.01}^{0.02} \frac{\sin 50n\pi t}{50n\pi} dt + \right] \right] \\
 &= 100 \left[\frac{4}{n\pi} \cdot \left[\sin \frac{n\pi}{2} - \sin n\pi \right] + \frac{200}{n\pi} \left[\left[\sin n\pi \times 0.02 - \sin \frac{n\pi}{2} \times 0.01 \right] + \frac{\cos 50n\pi t}{50n\pi} \Big|_{0.01}^{0.02} \right] \right] \\
 &= 100 \left[\frac{4}{n\pi} \cdot \sin \frac{n\pi}{2} + \frac{2}{n\pi} \left[-\sin \frac{n\pi}{2} + \frac{2(\cos n\pi - \cos \frac{n\pi}{2})}{n\pi} \right] \right] \\
 \therefore \frac{A_0}{2} &= \text{mean} = 50 \rightarrow A_0 = 100 \text{ V}
 \end{aligned}$$



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$$A_1 = 100 \left[\frac{4}{\pi} \cdot \sin \frac{\pi}{2} + \frac{2}{\pi} \left[-\sin \frac{\pi}{2} + \frac{2(\cos \pi - \cos \frac{\pi}{2})}{\pi} \right] \right] = \frac{200}{\pi} \left[2 - 1 + \frac{2}{\pi} \right] = 104.15 \text{ V}$$

$$A_2 = 0$$

$$A_3 = 100 \left[\frac{4}{3\pi} \cdot \sin \frac{3\pi}{2} + \frac{2}{3\pi} \left[-\sin \frac{3\pi}{2} + \frac{2(\cos 3\pi - \cos \frac{3\pi}{2})}{3\pi} \right] \right] = \frac{200}{3\pi} \left[-2 + 1 - \frac{2}{3\pi} \right]$$

$$= -25.72 \text{ V}$$

Similarly obtain B_n and then obtain the Fourier Series.

[8 marks]

- (b) $e(t) = 10 + 100 \sin 100t + 30 \sin (300t + \pi/3) + 20 \sin(500t - \pi/6)$ is applied across a resistor $R = 10 \Omega$ and capacitor $C = 100 \mu\text{F}$,

Fourier series of the resulting current $i(t)$ is given by

$$i(t) = \frac{e(t)}{R} + C \frac{d e(t)}{dt} = 1 + 10 \sin 100t + 3 \sin (300t + \pi/3) + 2 \sin(500t - \pi/6)$$

$$+ 10^{-4} (0 + 100 \times 100 \cos 100t + 30 \times 300 \cos (300t + \pi/3) + 20 \times 500 \cos(500t - \pi/6))$$

$$= 1 + [10 \sin 100t + \cos 100t] + [3 \sin (300t + \pi/3) + 0.9 \cos (300t + \pi/3)]$$

$$+ [2 \sin(500t - \pi/6) + \cos(500t - \pi/6)]$$

$$= 1 + 10.05 \sin (100t + 0.100) + 3.13 \sin (300t + 1.339) + 2.23 \sin (500t - 0.06)$$

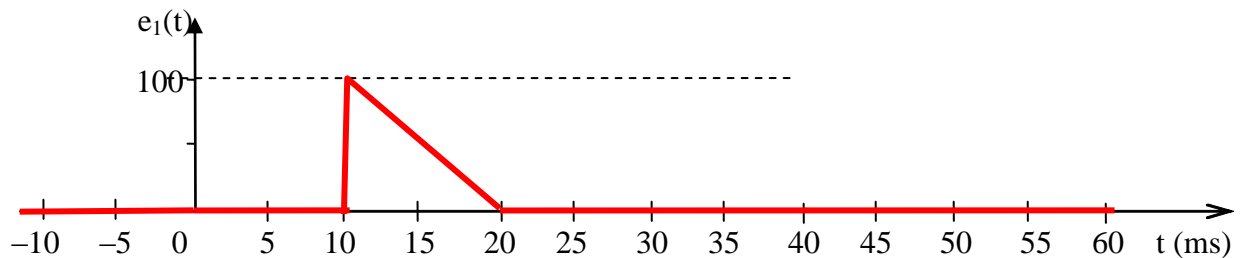
$$[\text{or } = 1 + 10.05 \sin (100t + 5.73^\circ) + 3.13 \sin (300t + 76.72^\circ) + 2.23 \sin (500t - 3.44^\circ)]$$

[2 marks]

7.

- (a) Laplace transform of waveform $e(t)$ is obtained as follows.

Waveform is a repetitive waveform of the waveform $e_1(t)$ from 0 to 20 ms.



Laplace transform of $e_1(t)$ is given by

$$L[e_1(t)] = \int_0^{0.01} 0 \cdot e^{-st} \cdot dt + \int_{0.01}^{0.02} (200 - 10000t) \cdot e^{-st} \cdot dt$$

$$= 0 + (200 - 10000t) \cdot \frac{e^{-st}}{-s} \Big|_{0.01}^{0.02} - \int_{0.01}^{0.02} (0 - 10000) \cdot \frac{e^{-st}}{-s} \cdot dt$$



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$$= \frac{(200 - 200) \cdot e^{-0.02s} - (200 - 100) \cdot e^{-0.01s}}{-s} + 10000 \cdot \frac{e^{-st}}{(-s)^2} \Big|_{0.01}^{0.02}$$

$$= \frac{100 \cdot e^{-0.01s}}{s} + 10000 \cdot \frac{(e^{-0.02s} - e^{-0.01s})}{s^2}$$

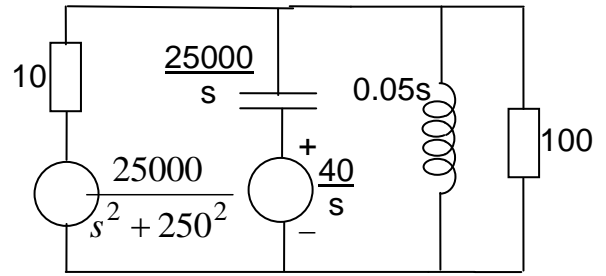
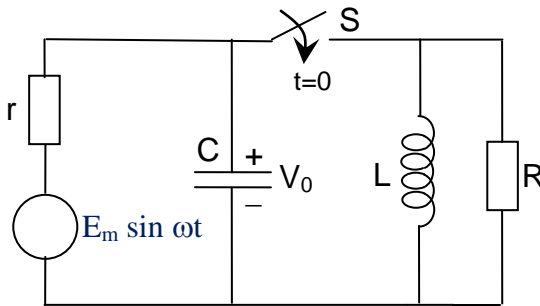
Laplace transform of $e(t)$ = addition of shifted waveforms $e_1(t)$

$$= H(t) \cdot e_1(t) + H(t-0.02) \cdot e_1(t-0.02) + H(t-0.04) \cdot e_1(t-0.04) + H(t-0.06) \cdot e_1(t-0.06) + \dots$$

$$\mathcal{L}[e(t)] = \mathcal{L}[e_1(t)] + \mathcal{L}[e_1(t)] \cdot e^{-0.02s} + \mathcal{L}[e_1(t)] \cdot e^{-0.04s} + \mathcal{L}[e_1(t)] \cdot e^{-0.06s} + \dots$$

$$= \frac{1}{1 - e^{-0.02s}} \left[\frac{100 \cdot e^{-0.01s}}{s} + 10000 \cdot \frac{(e^{-0.02s} - e^{-0.01s})}{s^2} \right] \quad [6 \text{ marks}]$$

(b)



[4 marks]