University of Moratuwa, Sri Lanka
Faculty of Engineering
Department of Electrical Engineering
B. Sc. Engineering Honours Degree Course
Level 2 – Semester 2 Examination

EE2010 – THEORY OF ELECTRICITY

Time Allowed: 3 Hours | January 2009

Additional Material

Graph Paper will be provided if required.
A table of Laplace transforms is provided on the other side of this page.

Instructions to Candidates

This paper contains 7 questions in 6 pages, including the cover page.
Answer All Questions.
This examination accounts for 70% of the module assessment.
Each question carries a total 10 marks. Maximum marks allocated for each part of a question is indicated in square brackets at the end of the part.
Total allocation for the paper is 70 marks.
This is a closed book examination and only authorised calculators will be permitted.

Technical Data:

Permeability of free space μ_0 $= 4\pi \times 10^{-7}$ H/m
Permittivity of free space ε_0 $= 8.854 \times 10^{-12}$ F/m
Velocity of light in free space $= 2.998 \times 10^{8}$ m/s

Table of Laplace Transforms of common causal functions *f(t)*

$f(t)$	$F(s)=\mathrm{L}[f(t)]$
Unit impulse – δt	1
Unit step – $U(t)$	$\frac{1}{s}$
t	$\frac{1}{s^2}$
t^n	$\frac{n!}{s^{n+1}}$
e^{-at}	$\frac{1}{(s+a)}$
$1-e^{-at}$	$\frac{a}{s(s+a)}$
$t\,e^{-at}$	$\frac{1}{(s+a)^2}$
$t^n\,e^{-at}$	$\frac{n!}{(s+a)^{n+1}}$
$e^{-at}-e^{-bt}$	$\frac{b-a}{(s+a)(s+b)}$
$\sin(\omega t)$	$\frac{\omega}{(s^2+\omega^2)}$
$\sin(\omega t+\phi)$	$\frac{\omega\cos(\phi)+s\sin(\phi)}{(s^2+\omega^2)}$
$t\sin(\omega t)$	$\frac{2\omega s}{(s^2+\omega^2)^2}$
$\cos(\omega t)$	$\frac{s}{(s^2+\omega^2)}$
$\cos(\omega t+\phi)$	$\frac{s\cos(\phi)-\omega\sin(\phi)}{(s^2+\omega^2)}$
$t\cos(\omega t)$	$\frac{s^2-\omega^2}{(s^2+\omega^2)^2}$
$e^{-at}\sin(\omega t)$	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{-at}\cos(\omega t)$	$\frac{s+a}{(s+a)^2+\omega^2}$
$\sinh(\omega t)$	$\frac{\omega}{(s^2-\omega^2)}$
$\cosh(\omega t)$	$\frac{s}{(s^2-\omega^2)}$

Question 1

(a) A circuit consists of a series combination of (i) an inductance L, (ii) a resistance r, and (iii) a capacitor C with resistor R in parallel. Sketch the circuit and write down an expression for the impedance of the circuit at an angular frequency ω. *[1 marks]*

(b) If L=10 mH, r = 1 Ω, C = 100 μF, R=100 Ω and an emf of e(t) = 100 sin 1000t is applied across the circuit, determine expressions for the current $i_L(t)$ through the inductor and the voltage $v_c(t)$ across the capacitor. *[2 marks]*

(c) Write an expression for the energy stored in the circuit, and determine its maximum value *[2 marks]*

(d) Determine the energy loss per cycle and the Q factor of the circuit. *[2 marks]*

(e) Sketch a phasor diagram showing all the voltages and the supply current. *[2 marks]*

(f) Determine also an expression for the resonance frequency of the circuit. *[1 mark]*

Question 2

(a) Obtain an expression for the effective reluctance seen by the coil of N=100 turns in figure Q2a, if the effective cross-section of each limb is A=4cm^2, and the effective magnetic lengths are l_o=20 cm and l_m=6 cm. The relative permeability of the magnetic material is μ_r = 1000. *[3 marks]*

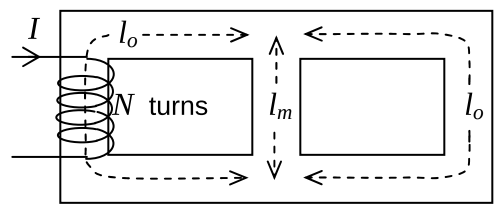

Figure Q2a

(b) A certain transformer has a core loss of 100 W (with equal hysteresis and eddy current loss) when operating at a frequency of 50 Hz with laminations of thickness 0.5 mm. What should be the thickness of laminations required when operated at 60 Hz to keep the total core loss constant. Determine the contributions of eddy current loss and hysteresis loss under this condition at 60 Hz. *[2 marks]*

(c) A capacitor is made up of two concentric electrodes of radii 2 mm and 3 mm. The length of each cylindrical electrode is 25 mm and the dielectric material has a relative permittivity of 4.5. Determine the capacitance of the arrangement. What is the potential difference that has to be applied across the capacitor to obtain an energy stored of 1 mJ. *[2 marks]*

(d) The terminal voltage of a certain 50 Hz, alternating current, socket outlet, in the absence of a load, is measured to be 230 V. The short-circuit current across the terminals is known to be 10 kA. What is the theoretical maximum active power that could be delivered to a resistive load from this socket outlet ? State the assumption you have made in obtaining this value? State the two main practical restrictions that would limit the actual maximum power delivered to a practical load ? *[3 marks]*

Question 3

For the 2-port network shown in figure Q3a, determine

(a) The two-port admittance matrix *[2 mark]*

(b) the impedance Z_o which when connected at port 2 as a load would give the input impedance at port 1 also as Z_o *[2 marks]*

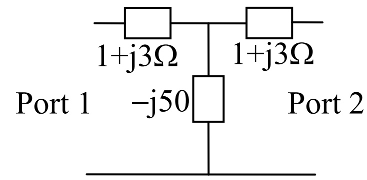

Figure Q3a

For the circuit shown in figure Q3b,

(c) write down the Kirchoff's circuit equations *[2 marks]*

(d) determine the currents in all the branches *[2 marks]*

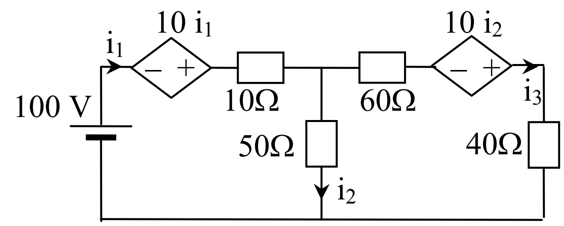

Figure Q3b

(e) In a certain system with source impedance (1+j3) Ω, the load current is measured to be $10.5\angle 30^0$A when the load impedance is (40+j30) Ω. Using the compensation theorem, determine the required load impedance if the current required is $10\angle 30^0$A. *[2 marks]*

Question 4

(a) For the circuit shown in figure Q4, if E = 100V, ω=250 rad/s, $R_1 = R_2 = 10\,\Omega$, L = 10mH and C = 100μF convert the voltage source to an equivalent current source and redraw the circuit. *[1 mark]*

(b) Selecting suitable nodes, write down the nodal admittance matrix and the nodal current source. *[2 mark]*

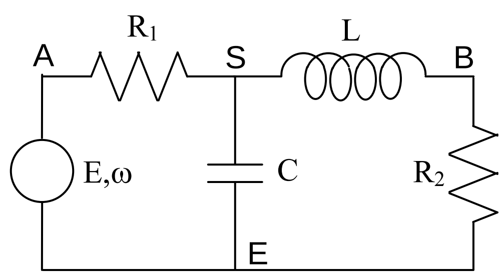

Figure Q4

(c) Determine the currents in all the branches. *[3 marks]*

(d) Obtain the impedances Z_{AE} and Z_{AB} of the equivalent delta connected network corresponding to the star connected network Z_{AS}, Z_{BS} and Z_{ES}. *[2 marks]*

(e) Hence obtain the currents flowing in Z_{AB} and Z_{AE} in the converted circuit. *[1 mark]*

(f) Using the results of Q4(e) confirm the value of the current in Z_{AS}. *[1 mark]*

Question 5

A balanced 3-phase, 400 V, 50 Hz supply feeds (i) a 3-phase star-connected balanced resistive load consisting of arms of value 80 Ω each, and (ii) a 3-phase motor consuming 2 kW at a power factor of 0.7 lag at a voltage of 400 V. Determine

(a) the line current supplied from the source, *[2 marks]*

(b) the supply power factor. *[1 mark]*

(c) It is now required to improve the load power factor to unity be connecting a bank of delta-connected capacitors across the load. Determine the values of the capacitors. *[2 marks]*

For a system with phase sequence ABC, when a balanced 3 phase supply of voltage 400V is applied, the symmetrical component currents in phase A of the star connected system are measured to be $I_{A0} = 1.0\angle 90^o$A, $I_{A1} = 10.0\angle 0^o$A and $I_{A2} = 3.0\angle -90^o$A.

(d) determine the power associated with each sequence component *[2 marks]*

(e) determine the actual currents in all three phases. *[3 marks]*

Question 6

(a) Determine the rms value of the periodic current i(t) shown in figure Q6(a), if it has the value 5 cos $6\omega_o t$ ampere in the region $-T_1$ to T_1 . [$\omega_o T = 2\pi$] *[2 marks]*

(b) Determine the first 3 significant terms of the Fourier series of the current i(t). *[4 marks]*

(c) Determine an expression for the terminal voltage v(t) shown in figure Q6(b) when the current i(t) flows in the load. *[2 marks]*

(d) Determine the average power supplied from the source. *[2 marks]*

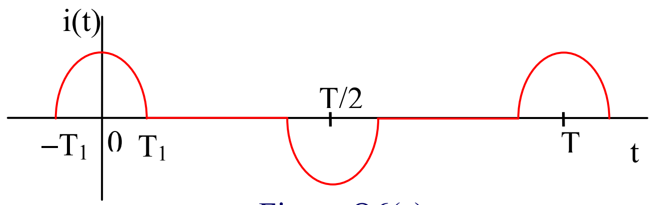

Figure Q6(a)

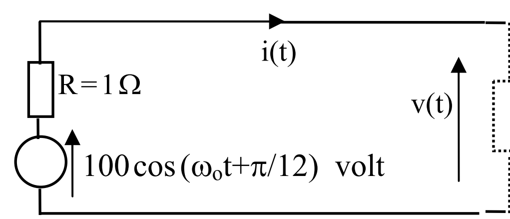

Figure Q6(b)

Question 7

(a) Determine from first principles the Laplace transform of the causal sinusoidal waveform.

[2 marks]

[2 marks]

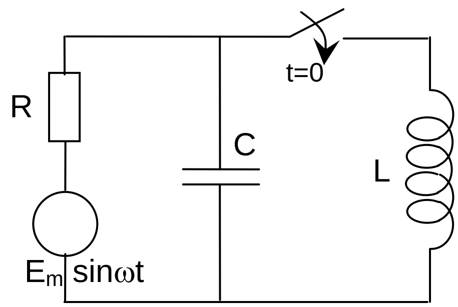

Figure Q7

Figure Q7 shows a circuit which has reached steady state with the switch open. If E_m = 100 V, R = 100 Ω, f = 50 Hz, C = 40 μF, L =1.8 H, and switch S is closed at time t = 0, determine

(b) The initial voltage across the capacitor at t=0. *[2 marks]*

(c) Draw the equivalent circuit in the Laplace domain. *[2 marks]*

(d) Determine the Laplace domain expression for the current through the inductor *[2 marks]*

(e) Determine the corresponding time domain solutions for the current through the inductor *[2 marks]*

[END OF QUESTION PAPER]