University of Moratuwa, Sri Lanka
Faculty of Engineering
Department of Electrical Engineering
B. Sc. Engineering Honours Degree Course
Level 2 – Semester 2 Examination

EE2010 – THEORY OF ELECTRICITY

Time Allowed: 3 Hours — August 2010

Additional Material

Graph Paper will be provided if required.
A table of Laplace transforms is provided on the other side of this page.

Instructions to Candidates

This paper contains 7 questions in 6 pages, including the cover page.
Answer All Questions.
This examination accounts for 70% of the module assessment.
Each question carries a total 10 marks. Maximum marks allocated for each part of a question is indicated in square brackets at the end of the part.
Total allocation for the paper is 70 marks.
This is a closed book examination and only authorised calculators will be permitted.

Technical Data:

Permeability of free space μ_0 $= 4\pi \times 10^{-7}$ H/m
Permittivity of free space ε_0 $= 8.854 \times 10^{-12}$ F/m
Velocity of light in free space $= 2.998 \times 10^{8}$ m/s

Table of Laplace Transforms of common causal functions *f(t)*

$f(t)$	$F(s) = \mathrm{L}[f(t)]$
Unit impulse – δt	1
Unit step – $U(t)$	$\frac{1}{s}$
t	$\frac{1}{s^2}$
t^n	$\frac{n!}{s^{n+1}}$
e^{-at}	$\frac{1}{(s+a)}$
$1-e^{-at}$	$\frac{a}{s(s+a)}$
$t\,e^{-at}$	$\frac{1}{(s+a)^2}$
$t^n\,e^{-at}$	$\frac{n!}{(s+a)^{n+1}}$
$e^{-at}-e^{-bt}$	$\frac{b-a}{(s+a)(s+b)}$
$\sin(\omega t)$	$\frac{\omega}{(s^2+\omega^2)}$
$\sin(\omega t+\phi)$	$\frac{\omega\cos(\phi)+s\sin(\phi)}{(s^2+\omega^2)}$
$t\sin(\omega t)$	$\frac{2\omega s}{(s^2+\omega^2)^2}$
$\cos(\omega t)$	$\frac{s}{(s^2+\omega^2)}$
$\cos(\omega t+\phi)$	$\frac{s\cos(\phi)-\omega\sin(\phi)}{(s^2+\omega^2)}$
$t\cos(\omega t)$	$\frac{s^2-\omega^2}{(s^2+\omega^2)^2}$
$e^{-at}\sin(\omega t)$	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{-at}\cos(\omega t)$	$\frac{s+a}{(s+a)^2+\omega^2}$
$\sinh(\omega t)$	$\frac{\omega}{(s^2-\omega^2)}$
$\cosh(\omega t)$	$\frac{s}{(s^2-\omega^2)}$

Question 1

(a) A circuit consists of a parallel combination of (i) an inductance L in series with a resistance R, and (ii) a capacitor C. If the circuit is supplied from an alternating source e(t) with internal resistance r, sketch the circuit and write down an expression for the impedance of the circuit at a supply frequency ω. *[1 mark]*

(b) If L=10 mH, R=100 Ω, C = 100 μF, r = 1 Ω, and e(t) = 100 sin 1000t, determine expressions for the current $i_L(t)$ through the inductor and the voltage $v_c(t)$ across the capacitor. *[2 marks]*

(c) Sketch a phasor diagram showing all the voltages and currents in the circuit. *[2 marks]*

(d) Write an expression for the energy stored in the circuit, and determine its maximum value. *[2 marks]*

(e) Determine the energy loss per cycle and the Q factor of the circuit. *[2 marks]*

(f) Determine the resonance frequency of the circuit. *[1 mark]*

Question 2

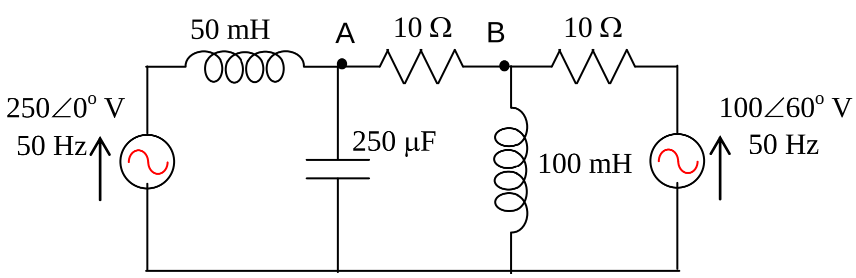

Figure Q2

(a) For the circuit shown in Figure Q2, convert the voltage sources to equivalent current sources and redraw the circuit. *[2 marks]*

(b) Write down the branch-node incidence matrix and the branch admittance matrix for the redrawn circuit. *[2 marks]*

(c) Write down the steps in determining the nodal admittance matrix. *[2 marks]*

(d) Using nodal matrix analysis, determine the voltages at nodes A and B. *[4 marks]*

Question 3

(a) Figure Q3a shows the magnetic circuit of a certain inductor. If N is100 turns, effective cross-section A of each limb is 4 cm^2, effective magnetic lengths are l_o = 15 cm and l_m = 4 cm, μ_r of magnetic material is 1000, Obtain an expression for the effective inductance of the coil. *[3 marks]*

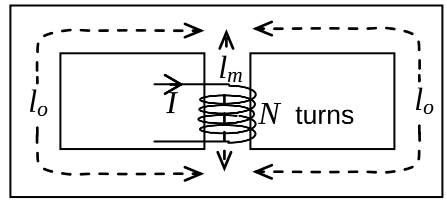

Figure Q3a

(b) Express the circuit shown in figure Q2b, as an uncoupled circuit. Hence obtain the current supplied from the source, taking the supply voltage as reference. *[3 marks]*

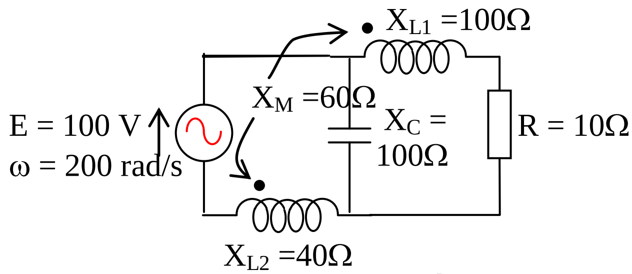

Figure Q3b

(c) For the circuit shown in Figure Q3c, determine the potential of the star point S using Millmann's theorem. *[4 marks]*

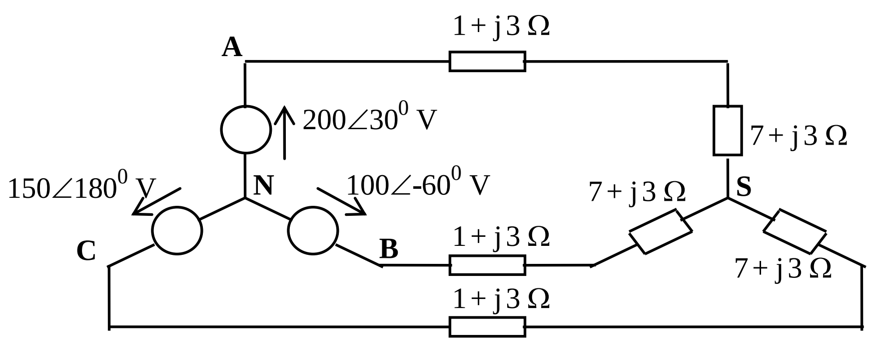

Figure Q3c

Question 4

The ABCD parameters of a certain two-port network are known to be

A = D = (1 + j 0.1), B = j 10 Ω, and C = (0.02 + j 0.001) S.

(a) Determine the corresponding impedance-parameters of the network. *[3 marks]*

(b) Determine the characteristic impedance Z_0 of the network. *[2 marks]*

For the circuit shown in figure Q4b,

(c) write down the Kirchoff's circuit equations *[2 marks]*

(d) determine the currents in all the branches *[3 marks]*

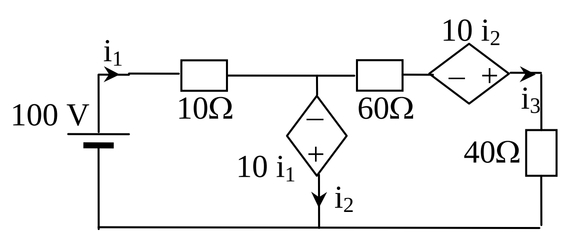

Figure Q4b

Question 5

A 3 phase, 400 V, 50 Hz balanced supply feeds a balanced star connected load, each arm consisting of a resistance of 100 Ω and an inductance of 150 mH in series.

(a) Determine the line current, the power factor of the load and the total active power supplied to the load. *[3 marks]*

(b) Determine the value of the 3 capacitors that must be connected across the load in delta in order to increase the overall power factor to 0.95 lag. *[2 marks]*

For a system with phase sequence ABC, the symmetrical component currents in phase A of a star connected system are $I_{A0} = 1.0\angle 90^o$A, $I_{A1} = 10.0\angle 0^o$A and $I_{A2} = 3.0\angle -90^o$A, and the phase voltages are $V_{AN} = 200\angle 0^o$ V, $V_{BN} = 100\angle -60^o$ V and $V_{CN} = 200\angle 120^o$ V.

(c) Determine the Symmetrical Components of the phase "A" voltage. *[3 marks]*

(d) Determine the power associated with each sequence component. *[2 marks]*

Question 6

(a) Determine the rms value of the half-wave-rectified sinusoidal current i(t), with peak value I_m, shown in figure Q6(a). *[2 marks]*

(b) Determine the first 3 significant terms of the Fourier series of the current i(t). *[4 marks]*

(c) Determine an expression for the supply voltage $E_s(t)$, when the current i(t) flows in the load, as shown in figure Q6(b). *[3 marks]*

(d) Determine the average power supplied from the source. *[1 mark]*

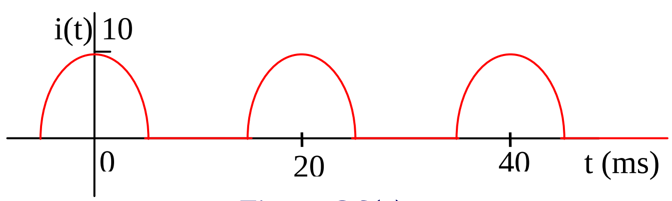

Figure Q6(a)

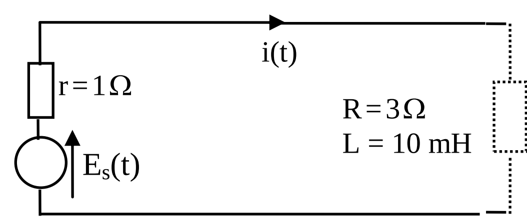

Figure Q6(b)

Question 7

(a) Determine from first principles the Laplace transform of the causal waveform cos ($\omega t+\phi$).

[3 marks]

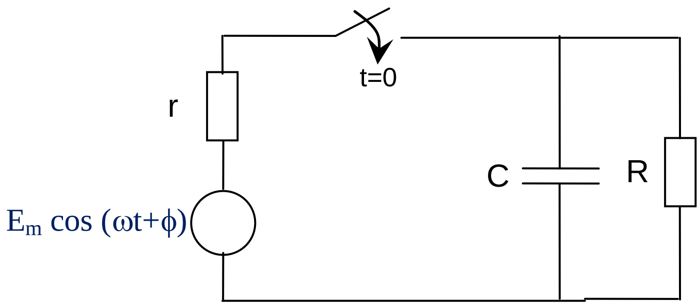

Figure Q7

Figure Q7 shows a circuit in which the capacitor C has an initial charge q_0 = 2 coulomb. If E_m = 100 kV, f = 50 Hz, r = 10 Ω, R = 100 Ω, C = 40 μF, and switch S is closed at time t = 0,

(b) draw the equivalent circuit in the Laplace domain, *[3 marks]*

(c) determine the Laplace expression for the current supplied from the source, *[2 marks]*

(d) determine the time domain solutions for the current through the Resistor R. *[2 marks]*

[END OF QUESTION PAPER]