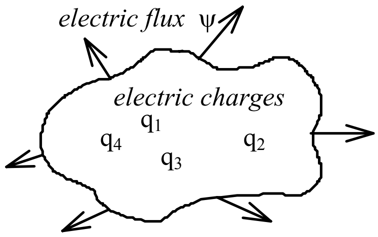
4.1 Electrostatic Theory

Electrostatics is based on two theories - Gauss's Law and the Inverse Square Law

4.1.1 Gauss's Law



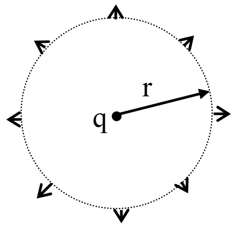
Gauss's law states that the total electric flux coming out of a closed surface is equal to the algebraic sum of the charge enclosed .

$$\psi = \Sigma q$$

[The Unit of both *electric flux* ψ and *charge* q is the ***coulomb (C)***]

Electric flux density D is the amount of electric flux coming out per unit area normal to the direction of the flux. [Unit: ***coulomb per meter***^{2} or C/m^2]

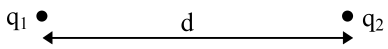
$$D = \psi/A$$



Consider an imaginary sphere of radius **r** surrounding a point charge **q** as shown. Electric flux $\psi = q$ will go out equally in all directions through a surface area of $4\pi r^2$, normal to this surface.

∴ electric flux density at radius *r* is $D_r = \dfrac{q}{4\pi r^2}$

4.1.2 Inverse Square Law



$$F \propto \frac{q_1 \cdot q_2}{d^2} = k.\frac{q_1 \cdot q_2}{d^2}$$

The Inverse square law of electrostatics states that the force exerted by two point charges, on each other, is proportional to the product of their charges and inversely proportional to the square of the distance between them.

The Electric field ξ at a point is defined as the force exerted on a unit positive charge placed at that point.

If the field is produced by a charge q_1, then the force acting on a charge $q_2 = +1$ will be $\xi_r = k.\dfrac{q_1 \times 1}{r^2}$ at a distance *r* from the charge.

It can be seen that the electric flux density D_r and the electric field ξ_r are both proportional to *q* and inversely proportional to r^2 and are thus proportional to each other.

The permittivity ε of the medium is thus defined as their ratio. i.e. $D = \varepsilon\, \xi$.

[Unit of permittivity is ***farad per meter*** or ***F/m***]

Thus the constant *k* in the inverse square law equation becomes 1/k = 4π ε and the ***inverse square law*** can be stated as $F = \dfrac{q_1 \cdot q_2}{4\pi\varepsilon\, d^2}$.

The *permittivity of free space* is $\varepsilon_0 = 8.854 \times 10^{-12}$ F/m.

It is usual to express the permittivity of a medium as a relative value by comparing it with that of free space. Thus the ***relative permittivity*** (also called ***dielectric constant***) ε_r of a material is defined using the relationship $\varepsilon_r = \varepsilon/\varepsilon_0$ or $\varepsilon = \varepsilon_0\, \varepsilon_r$. [$\varepsilon_r$ is always greater than 1.]

[Relative permittivity being a number, has no dimension and is thus without a unit].

[In practice the relative permittivity of air which is 1.00060 is taken as unity or same of that of free space]

Electric potential difference between two points A and B is (usually) defined as the work that must be done, against the forces of an electric field, in moving a unit positive charge from point A to point B.

The change in energy ***dW*** in moving a unit positive charge an elemental distance ***dx*** in the direction of the force ***F*** would be ***F. dx*** . The corresponding change in potential would be an elemental value $-dV$.

The force acting on a unit positive charge, by definition, is the electric field ξ.

$$\therefore\ \xi.\, dx = -\,dV \quad \text{or} \quad \xi = -\frac{dV}{dx}$$

[Unit for electric field is ***volt per meter*** or ***V/m***]

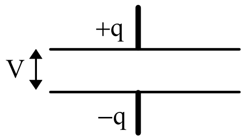
4.1.3 Capacitance of a dielectric (or insulating material) is the ability to store charge when placed between two electrodes across which a potential difference has been applied. Thus capacitance is defined as the ratio of the amount of charge transferred to the applied potential difference.

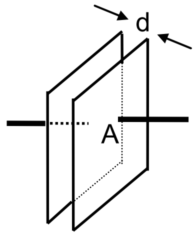
i.e. $C = q/V$ or $q = C.\, V$

[Unit of capacitance is the ***farad*** (***F***)]

Parallel plate capacitor

Simplest form of capacitor is the parallel plate capacitor where the dielectric (permittivity ε) of thickness **d** separates two parallel electrodes of cross-section area A.





From Gauss's theorem, $\psi = q$

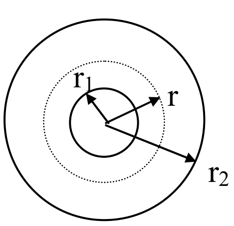
$\therefore\ D = \psi/A = q/A,$ also $D = \varepsilon\,\xi = q$

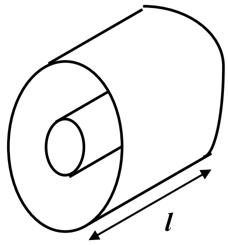
and for a uniform field, $\xi = -dV/dx = V/d$

$$\therefore\ D = \varepsilon\frac{V}{d} = \frac{q}{A},\quad C = \frac{q}{V} = \frac{\varepsilon A}{d}$$

Cylindrical capacitor

A common form of capacitance obtained, especially with co-axial cables, is the case where both electrodes are cylindrical and have the same axis .





From Gauss's theorem, $\psi = q$

$\therefore\ D_r = \psi/A = q/2\pi r\, l,$

also $D = \varepsilon\,\xi = q/A$ and $\xi = -dV/dr$

$$\therefore\ -\frac{dV}{dr} = \frac{q}{2\pi\varepsilon r l}$$

Integration gives $-\int_{V_1}^{V_2} dV = \int_{r_1}^{r_2} \frac{q}{2\pi\varepsilon l}\frac{dr}{r},\quad -V_1 + V_2 = \frac{q}{2\pi\varepsilon\, l}\ln\left[\frac{r_2}{r_1}\right]$

$$\therefore\ C = \frac{2\pi\varepsilon\, l}{\ln[r_2/r_1]}$$

4.1.4 Energy stored in an electric field

Consider a parallel plate capacitor of cross-section area A and spacing of electrodes d

Energy stored in a capacitor = $\int v\cdot i\cdot dt = \int v\cdot dq = \int v\cdot C\cdot dv = \tfrac{1}{2}Cv^2$

Energy stored in a unit volume in an electric field $= \int \frac{v\cdot dq}{\text{volume}} = \int \frac{v\cdot dq}{d.A} = \int \xi\cdot d\,D = \int \frac{D}{\varepsilon}\cdot d\,D$

$$= \tfrac{1}{2}\frac{D^2}{\varepsilon} = \tfrac{1}{2}D\cdot\xi \quad \text{J/m}^3$$

4.1.5 Force exerted in an electric field

Consider moving the electrodes of a parallel plate capacitor so that the spacing changes by dx

change in energy stored = $\tfrac{1}{2}\,D.\xi\times$ (change in volume) $= \tfrac{1}{2}\,D.\xi\times A.\,dx$

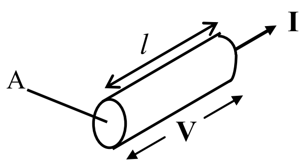
Also, change in energy stored = work done = $F\,.\,dx$

$\therefore\ F\,.\,dx = \tfrac{1}{2}\,D.\xi\,.\,A.\,dx$ or $F = \tfrac{1}{2}\,D.\xi\,.\,A$

i.e. Force exerted on unit area in an electric field $= F/A = \tfrac{1}{2}\,D.\xi$ N/m^2

4.1.6 Ohm's Law in an electric field

Consider a cylindrical volume in the direction of the current flow, as shown in the figure.



Current density in the conducting medium $J = I/A$

Field in the medium $\xi = -dV/dx = V/l$ for a uniform field

Also, since $R = \rho l/A$ and $V = R.I$, $\xi\cdot l = \frac{\rho\, l}{A}\cdot J\cdot A$

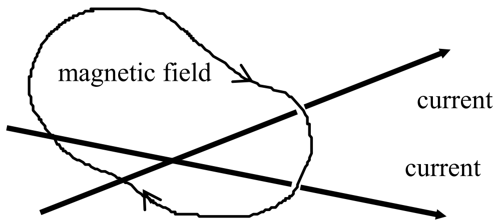
This gives Ohm's law for an electric field as $\xi = \rho\,.\,J$ or more commonly written as $J = \sigma\,.\,\xi$

where conductivity $\sigma = 1/\rho$

4.2 Electromagnetic Theory

One of the basic theorems in electromagnetism is the Ampere's Law which relates, the magnetic field produced by an electric current, to the current passing through a conductor.

4.2.1 Ampere's Law



Ampere's Law states that the line integral of the magnetic field ***H*** taken around a closed path is equal to the total current enclosed by the path.

$$\oint \boldsymbol{H} \cdot d\,\boldsymbol{l} = \sum \boldsymbol{I}$$

For a uniform field, ***H*** is a constant and we have $\boldsymbol{H} \,.\, \boldsymbol{l} = \Sigma \boldsymbol{I}$

or if ***H*** is constant over sections, with different sections having different ***H***, then $\Sigma \boldsymbol{H} \,.\, \boldsymbol{l} = \Sigma \boldsymbol{I}$

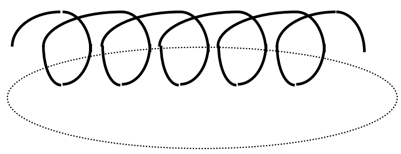
[Unit of magnetic field is ***ampere per meter (A/m)***]

4.2.2 Magnetomotive force (mmf)

Magnetomotive force is the flux producing ability of an electric current in a magnetic circuit. [It is something similar to electromotive force in an electric circuit].

[Unit of magnetomotive force is ***ampere (A)***] - *Note:* Although some books use the term *ampere-turns*, it is strictly not correct as *turns* is not a dimension]

mmf $\Im = \Sigma \boldsymbol{I}$

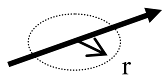


Consider a coil having ***N*** turns as shown.

It will link the flux path with each turn, so that total current linking with the flux would be $\Sigma \boldsymbol{I} = \boldsymbol{N}.\boldsymbol{I}$

Thus from Ampere's Law, the **mmf** produced by a coil of ***N*** turns would be ***N I***, and ***N I = H l***.

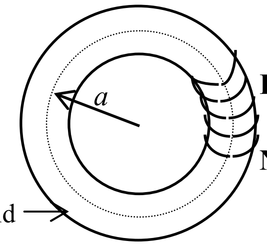
4.2.3 Field produced by a long straight conductor



If a circular path of radius ***r*** is considered around the conductor carrying a current ***I,*** then the field $\boldsymbol{H}_r$ along this path would be constant by symmetry.

$\therefore$ by Ampere's Law, $1.\boldsymbol{I} = \boldsymbol{H}_r . 2\pi \boldsymbol{r}$ or $\boldsymbol{H}_r = \dfrac{\boldsymbol{I}}{2\pi \boldsymbol{r}}$ at a radial distance ***r*** from the conductor.

4.2.4 Field produced inside a toroid



Consider a toroid (similar to a ring) wound uniformly with ***N*** turns.

If the mean radius of the magnetic path of the toroid is ***a***, then the magnetic path length would be $2\pi \boldsymbol{a}$, and the total mmf produced would be ***N I***.

Thus from Ampere's Law

magnetic field $\boldsymbol{H} = \dfrac{\boldsymbol{NI}}{2\pi \boldsymbol{a}}$ inside the toroid. [variation of the magnetic field inside the cross section of the toroid is usually not necessary to be considered and is assumed uniform].

4.2.5 Magnetic flux density

The magnetic field ***H*** gives rise to a magnetic flux ϕ, which has a magnetic flux density ***B*** for a given area ***A***. The relationship between ***B*** and ***H*** is given by the permeability of the medium μ..

$\boldsymbol{B} = \mu \boldsymbol{H}$, where $\mu = \mu_o \mu_r$, μ_r is the relative permeability and μ_o is the permeability of free space

$\mu_o = 4\pi \times 10^{-7}$ H/m [permeability of air is generally taken to be equal to that of free space in practice]

[Unit of permeability is ***henry per meter (H/m)***]. [Unit of magnetic flux density is the ***tesla (T)***]

$$\phi = \boldsymbol{B}.\boldsymbol{A}$$

[Unit of magnetic flux is the ***weber (Wb)***]

4.2.6 Reluctance of a magnetic path

A magnetic material presents a Reluctance ***S*** to the flow of magnetic flux when an mmf is applied to the magnetic circuit. [This is similar to the resistance shown by an electric circuit when an emf is applied]

Thus $\boldsymbol{mmf} = \boldsymbol{Reluctance} \times \boldsymbol{flux}$ or $\Im = \boldsymbol{S} \,.\, \phi$

For a uniform field, $\mathfrak{I} = N I = H.l$, and $\phi = B.A = \mu H.A$

$\therefore H.l = S.\mu H.A$ so that the magnetic reluctance $S = \dfrac{l}{\mu A}$, where l = length and A = cross-section

[Unit of magnetic reluctance is ***henry***$^{-1}$ (H^{-1})]

Magnetic Permeance Λ is the inverse of the magnetic reluctance. Thus $\Lambda = \dfrac{1}{S} = \dfrac{\mu A}{l}$

[Unit of magnetic permeance is ***henry (H)***]

4.2.7 Self Inductance

While the reluctance is a property of the magnetic circuit, the corresponding quantity in the electrical circuit is the inductance.

Induced emf $e = N\dfrac{d\phi}{dt} = L\dfrac{di}{dt}$, $N\phi = L\,i$, $L = \dfrac{N\phi}{i}$

The self inductance L of a winding is the flux linkage produced in the same winding due to unit current flowing through it.
For a coil of N turns, if the flux in the magnetic circuit is ϕ, the flux linkage with the coil would be $N.\phi$.

also since $N I = S\phi$, $L = \dfrac{N^2}{S} = \dfrac{N^2 \mu A}{l}$

Thus the inductance of a coil of N turns can be determined from the dimensions of the magnetic circuit.

4.2.8 Mutual Inductance

When two coils are present in the vicinity of each other's magnetic circuit, mutual coupling can take place. One coil produces a flux which links with the second coil, and when a current in the first coil varies, an induced emf occurs in the second coil.

Induced emf in coil 2 due to current in coil 1: $e_2 = N_2\dfrac{d\phi_{12}}{dt} = M_{12}\dfrac{di_1}{dt}$, $N_2\,\phi_{12} = M_{12}\,i_1$,

$M_{12} = \dfrac{N_2\phi_{12}}{i_1}$

The Mutual inductance M_{12}, of coil 2 due to a current in coil 1, is the flux linkage in the coil 2 due to unit current flowing in coil 1.
also since $N_1 I_1 = S\,\phi_1$, and a fraction k_{12} of the primary flux would link with the secondary, $\phi_{12} = k_{12}\,.\,\phi_1$

$\therefore M_{12} = \dfrac{k_{12}N_1N_2}{S} = \dfrac{k_{12}N_1N_2\mu A}{l}$, k_{12} is known as the coefficient of coupling between the coils.

$k_{12} = k_{21}$ so that $M_{12} = M_{21}$. For good coupling, k_{12} is very nearly equal to unity.

4.2.9 Energy stored in a magnetic field

Energy stored in an inductor = $\int v\cdot i\cdot dt = \int L\dfrac{di}{dt}\cdot i\cdot dt = \int L\cdot i\cdot di = \tfrac{1}{2}Li^2$

Energy stored in a unit volume in magnetic field = $\displaystyle\int \frac{N\cdot\frac{d\phi}{dt}\cdot i\cdot dt}{\text{volume}} = \int\frac{N\cdot i\cdot d\phi}{A.l} = \int\frac{N\cdot i}{l}\cdot d\frac{\phi}{A} = \int H\cdot dB$

$$= \int\frac{B}{\mu}\cdot dB = \frac{1}{2}\frac{B^2}{\mu} = \frac{1}{2}B\cdot H \qquad \text{J/m}^3$$

4.2.10 Force exerted in an magnetic field

Consider moving the electromagnet so that the spacing changes by dx

change in energy stored = ½ $B.H\times$ (change in volume) = ½ $B.H\times A.\,dx$

Also, change in energy stored = work done = $F\,.\,dx$, $\therefore$ $F\,.\,dx$ = ½ $B.H.A.\,dx$ or F = ½ $B.H.A$

i.e. Force exerted on unit area in an electric field = F/A = ½ $B.H$ N/m^2